Air Skate Lift Characteristics

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Revision History

- October 25, 2015 – Patrick Mckeen (Harvey Mudd) pointed out that the equilibrium gap height is determined by the mass of the pod for the vibration analysis. Thus, there shouldn't be multiple curves for different pod masses, and the vibrational frequency is now plotted as functions of one variable.

Introduction

A report written by Witelski et al., titled "Analysis of Pressurized Porous Air Bearings," gives a very good introduction to the fluid mechanics of air bearings.[1] They start with the basic governing equations and arrive at the analytical solution for the lift force as a function of gap height. Because of the similarities between the air skates we are envisioning for the pod and traditional air bearings, we can, by the most part, follow their methodology.

Figure 1 shows a schematic of the air skates, which are imagined to be long and rectangular in shape, using the aluminum subtrack as the gliding surface. The dimensions of the skates lead to two assumptions in the fluid mechanics. First, the gap height is very small, allowing us to use lubrication flow theory. Second, the skates are much longer than they are wide ($L \gg W$), which makes the boundary effects of the y-direction negligible. The skates are assumed to be made of some type of porous media that can be modeled with Darcy's law, which states the velocity field is proportional to the pressure gradient.

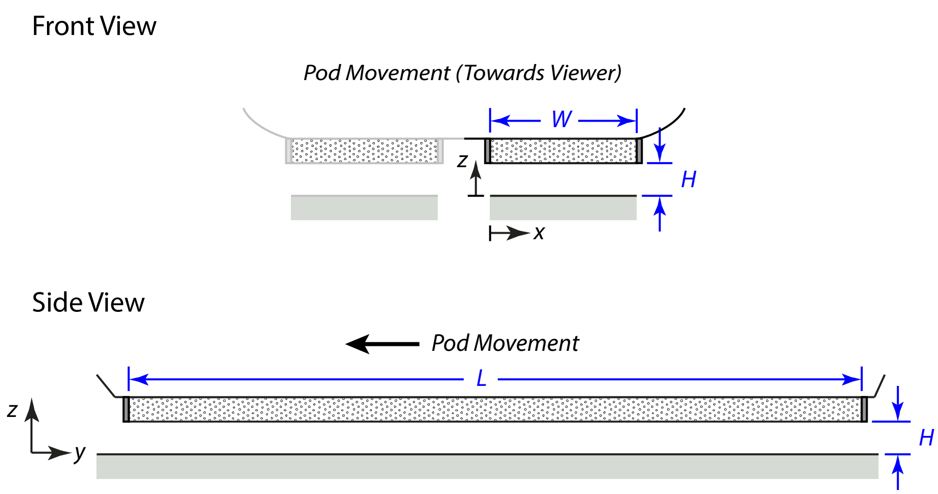

Figure 1: Front and side view schematic of the air skates with axes and dimensions labeled.

[1] T. Witelski, D. Schwendeman, and P. Evans. "Analysis of Pressurized Porous Air Bearings." 20th Annual Workshop on Mathematical Problems in Industry, University of Delaware, June 21–25, 2004.

Lift force vs gap height

Under these assumptions, the lift force (*F*) as a function of gap height (*H*) is given by

$$F(H) = 2LP_0\left[W - \frac{2}{\alpha}\tanh\left(\frac{\alpha W}{2}\right)\right] \qquad \alpha = \sqrt{\frac{12k}{H^3 D}}$$

where P_0 is the air pressure inside the skates, k is the air permeability, and D is the thickness of the porous layer. This is almost exactly the same as what Witelski et al. found, except multiplied by a ‘2L’ because we are dealing with two skates of length L.

Air flow vs gap height

Witelski et al. found the following pressure distribution for a long rectangular air bearing:

$$P(x) = P_0\left[1 - \frac{\cosh\left(\alpha\left(x - \frac{W}{2}\right)\right)}{\cosh\left(\frac{\alpha W}{2}\right)}\right]$$

The air flow required can be estimated by solving the lubrication flow equation for the velocity in the *x*-direction, and then integrating it over *z*.

$$\frac{\partial P}{\partial x} = \mu\frac{\partial^2 v_x}{\partial z^2}$$

The gas viscosity can be estimated using Sutherland’s formula.[2] It is approximately 2.33×10^{-5} Pa · s for air at a temperature of 400 °C. Applying no-slip boundary conditions at $z = 0$ and $z = H$, we end up with

$$v_x = \frac{LP_0\alpha}{2\mu}\frac{\sinh\left(\alpha\left(x - \frac{W}{2}\right)\right)}{\cosh\left(\frac{\alpha W}{2}\right)}z(H - z)$$

Integrating v_x over the edge of the gap (at $x = \frac{W}{2}$ from $z = 0$ to $z = H$) will yield the volumetric flow rate, which can be converted to a mass flow rate by multiplying by the density ($\rho = \frac{MP}{RT}$; $M_{air} = 28.97$ g/mol). The equation for mass flow rate is:

$$\dot{m} = \frac{MP_0^2 W\alpha}{RT\mu}\tanh\left(\frac{\alpha W}{2}\right)\left[\frac{H^3}{2} - \frac{H^3}{3}\right](W + L)$$

Note that the airflow out of the short edge is assumed to be the same as the airflow out of the long edge.

[2] http://www.lmnoeng.com/Flow/GasViscosity.php

Frequency of vibration

Small perturbations to the gap height will make the pod bounce like a harmonic oscillator. We want to find the frequency of this bounce, and since we already have an equation for the force as a function of gap height, we just need to do the linear approximation (first-order Taylor expansion) of that function about the equilibrium ride height, which we will call H_0. In other words H_0 is where the force exerted by the air skates exactly balances the force of gravity on the pod, $F(H_0) = F_g$, and the pod will oscillate about that.

$$\begin{aligned} F_{\text{net}}(H) &= F(H_0) - F(H) \\ &\approx F(H_0) - [F(H_0) - F'(H_0)(H - H_0)] \\ &= F'(H_0)(H - H_0) \\ &= \underbrace{-\frac{3nLP_0}{2\sqrt{H_0}}\left[\frac{2}{\alpha}\tanh\left(\frac{\alpha W}{2}\right) - W\operatorname{sech}^2\left(\frac{\alpha W}{2}\right)\right]}_{k_s}(H - H_0) \end{aligned}$$

$$\text{where } \alpha = \sqrt{\frac{12k}{H_0^3 D}}$$

The spring constant, k_s, is directly related to the frequency of oscillation, f:

$$f = \frac{1}{2\pi}\sqrt{\frac{k_s}{m_{\text{pod}}}}$$

The equilibrium height is directly related to the pod mass by the original force function. Thus, the frequency of oscillation can be expressed with either m_{pod} or H_0 exclusively.

$$m_{pod} = \frac{F(H_0)}{g}$$

Plotting the solutions

The MATLAB script in Listing 1 simulates a pair of skates, each 12 inches (30.48 cm) wide and 10 feet (3.048 m) long with a pressure of 11 kPa. As of this writing, SpaceX is considering increasing the subtrack width to 15 inches, which will allow us to make the skates wider. There is also the possibility of making the skates longer, since we are allowed a total pod length of 14 feet. Three pod masses (500 kg, 1000 kg, and 1500 kg) are provided to show how the vibrational frequency changes with mass. However, the biggest unknown right now is the porous media characteristics (the permeability, k, and thickness, D), as that has the greatest effect on the stiffness of the suspension. For now, k and D were chosen such that they give decent gap heights at 11 kPa, the pressure assumed in the Hyperloop Alpha whitepaper. The force, the flow rate, and vibration frequency curves are plotted in Figure 2.

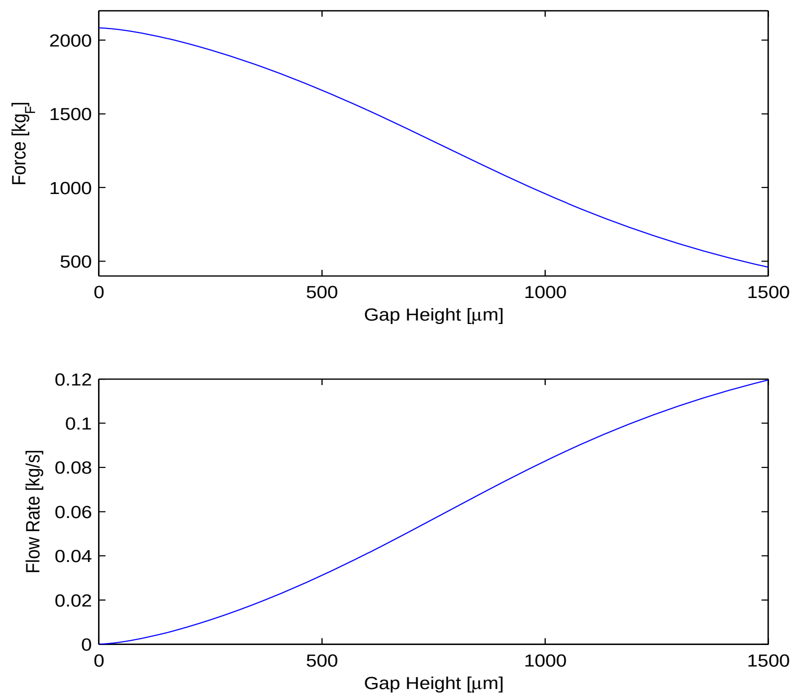

Figure 2: Plots of air skate lift force and air flow rate as functions of gap height.

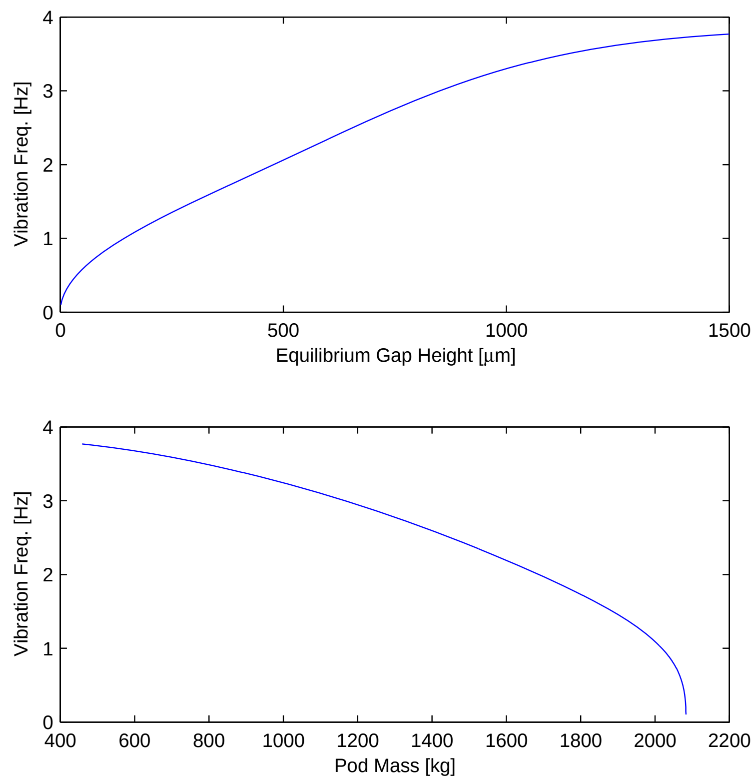

Figure 3: Plots of vibration frequency as a function of gap height and pod mass.

Listing 1: `twoskis.m`

```
% Simulates a pair of long, rectangular airskates

% Skate parameters
k = 8e-8*(2.54e-2)^2;        % Air permeability [m^2]
D = 0.187*2.54e-2;           % Thickness of porous layer [m]
P0 = 11e3;                   % Internal pressure of the skate [Pa]
n = 2;                       % Number of skates
W = 0.3048;                  % Skate width [m]
L = 10*0.3048;               % Skate length [m]
T = 400;                     % Nominal temperature of air through skate [K]

% Independent variable - Gap height
H = linspace(0,1500e-6,1e3); % Gap height [m]

% Other system parameters
P_supp = 2.1e3;              % Pressure of supply air to skate compressor [Pa]
T_supp = 300;                % Temperature of supply air to skate compressor
[K]
P_tube = 99;                 % Tube pressure [Pa]
m_pod = [500 1000 1500];     % Pod mass [kg]

% Physical constants
M_air = 28.97e-3;            % Molecular weight of air [kg/mol]
R = 8.3144598;               % Molar gas constant [J/K*mol]
g = 9.81;                    % Acceleration of gravity [m/s^2]
c = 343;                     % Speed of sound [m/s]

% Intermediate calculations
mu = 0.01827e-3*(291.15+120)/(T+120)... % Viscosity of air [Pa*s]
    *(T/291.15)^(1.5);
alpha = sqrt(12*k./(H.^3*D));    % Dimensionless parameter "alpha"
A = n*L*W;                       % Total skate area [m^2]

% Force as a function of gap height [N]
F = n*L*P0*(W-2./alpha.*tanh(alpha.*W/2));

% Flow rate as a function of gap height [kg/s]
m_flow = (P0*M_air/(R*T))*...
    W*P0*alpha./(2*mu).*tanh(alpha*W/2).*H.^3.*(1/2-1/3).*(2*(W+L));

% Derivative of force as a function of gap height [N/m]
F_H = n*L*P0*(2./alpha.^2.*tanh(alpha.*W/2) ...
              - W./alpha.*sech(alpha.*W/2).^2) ...
      .*alpha.*(-3/2).*(1./sqrt(H));

% Calculate frequency of oscillation
m_pod_H = F/g;      % Mass as a function of equilibrium ride height [kg]
freq_H = 1/(2*pi).*sqrt(-(1./m_pod_H).*F_H); % Frequency of oscillation as
                                             % a function of height [Hz]

% Plot carrying force vs gap height
```

```
figure(1)
subplot(2,1,1)
plot(H*1e6,F/g)
ylabel('Force [kg_F]')
xlabel('Gap Height [\mum]')

% Plot airflow as a function of gap height
subplot(2,1,2)
plot(H*1e6,m_flow)
ylabel('Flow Rate [kg/s]')
xlabel('Gap Height [\mum]')

% Plot vibration frequency as a function of equilibrium gap height
figure(2)
subplot(2,1,1)
plot(H*1e6,freq_H)
ylabel('Vibration Freq. [Hz]')
xlabel('Equilibrium Gap Height [\mum]')

% Plot vibration frequency as a function of pod mass
subplot(2,1,2)
plot(m_pod_H,freq_H)
ylabel('Vibration Freq. [Hz]')
xlabel('Pod Mass [kg]')

% Determine temperature of the uncooled air based on isentropic compression
V1 = 1;
V2 = (P_supp/P0)^(1/1.4);
T_uncooled = P0*V2/(P_supp*V1/T);   % Uncooled air temperature [K]

fprintf('%i skates at %g m^2 per skate\n',n,pi*R^2)
fprintf('Total skate area: %g m^2\n',A)
fprintf('Uncooled air temperature: %g K\n',T_uncooled)
```