Homogeneous System Solver Documentation

Program Description

Solves homogeneous systems of equations given the matrix of coefficients A and returns a normalized vector defining the null space. This has been separated from the rest of the code as it is used in both part A and part C.

Important Library Details

- Eigen
  - Library path: the headers for the Eigen library are located in /usr/include/eigen3 on my Linux machine.
  - Library version: I have installed Eigen version 3.4.0.

Marginal Cases

- Invalid inputs:
  - The solution is trivial. Found with $det(A) \neq 0$.
- Invalid computations:
  - Input matrix is all 0s: any solution is valid.
  - All important computations in the Eigen implementation methods were handled by Eigen, and the outputs have been checked.
  - Each output will be manually checked for validity.

Design Choices

- Seeing as 2x2 matrices are singular if the row vectors of a matrix are linearly dependent or at least 1 row vector is $[0\ 0]$. Therefore if you have an input matrix $A = \begin{bmatrix} \vec{r}_1 \\ \vec{r}_2 \end{bmatrix}$, then the normalized kernel is just the vector orthonormal to whichever row vector is non-zero: $\vec{x} = \frac{\vec{v}}{\|\vec{v}\|}$ where $\vec{v} = \begin{bmatrix} -r_{12} \\ r_{11} \end{bmatrix}$ or $\vec{v} = \begin{bmatrix} -r_{22} \\ r_{21} \end{bmatrix}$.
- When solving for input matrices of all zeros, any unit vector will do since the entire domain of $\mathbb{R}^2$ is valid.

Pseudocode

```
bool SolveHomogeneousSystem(const matrix &mat, vector &solution):
    IF A.determinant() != 0:
```

RETURN false

IF A.rowVector(0) == [0, 0]:
    IF A.rowVector(1) == [0, 0]:
        SET solution to $\begin{bmatrix} 0.6 \\ 0.8 \end{bmatrix}$
    ELSE:
        SET solution to $\begin{bmatrix} -A[1,1] \\ A[1,0] \end{bmatrix}$
ELSE:
    SET solution to $\begin{bmatrix} -A[0,1] \\ A[0,0] \end{bmatrix}$

NORMALIZE solution
RETURN true