PA3 Part A Documentation

Program Description

Solves systems of equations given in the first 3 columns of the input files. I will accomplish this through inverse matrices, the dot product theorem, and my custom homogeneous system solver.

Important Library Details

- Eigen
 - Library path: the headers for the Eigen library are located in /usr/include/eigen3 on my Linux machine.
 - o Library version: I have installed Eigen version 3.4.0.

Marginal Cases

- Invalid inputs:
 - System is underdetermined. System is inconsistent if $\vec{a_1} \cdot \vec{a_2} = \pm ||\vec{a_1}|| ||\vec{a_2}||$ and $\vec{a_1} \cdot \vec{b} = \pm ||\vec{a_1}|| ||\vec{b}||$ where $A = \begin{bmatrix} \vec{a_1} \\ \vec{a_2} \end{bmatrix}$. (derived from the dot product theorem, where $\theta = 0^\circ$ or 180°)
 - System is inconsistent. System is inconsistent if $\vec{a_1} \cdot \vec{a_2} = \pm ||\vec{a_1}|| \, ||\vec{a_2}||$ and $\vec{a_1} \cdot \vec{b} \neq \pm ||\vec{a_1}|| \, ||\vec{b}||$ where $A = \begin{bmatrix} \vec{a_1} \\ \vec{a_2} \end{bmatrix}$. (derived from the dot product theorem, where $\theta = 0^\circ$ or 180°)
 - The solution is trivial. Found with $det(A) \neq 0$.
- Invalid computations:
 - All important computations in the Eigen implementation methods were handled by Eigen, and the outputs have been checked.
 - All custom implementation methods simply added two doubles and stored them in the corresponding double element of a result matrix. The outputs have also been checked.

Design Choices

- \vec{x} will be solved via the inverse matrix: $\vec{x} = A^{-1}\vec{b}$. This can be done because the invalid input cases for the first part of this section cover all cases where A is non-invertible.
- For solving the homogeneous systems, the homogeneous system solver will be used.

Pseudocode

```
Matrix(2,6) GetInputAsMatrix(const string &input path)
Int main():
       CALL SolveFile() for each file
       RETURN 0
bool AreColinear(const vector &vec 1, const vector &vec 2):
       RETURN vec_1.dotProduct(vec_2) == vec_1.norm() * vec_2.norm() or
                     vec 1.dotProduct(vec 2) == vec 1.norm() * vec 2.norm() * -1
void SolveFile(const string &input path, const string &output path):
       DECLARE 2x6 matrix raw_input as CALL of GetInputAsMatrix with input_path
       OPEN output_file at output_path
       DECLARE matrix mat as the first two column vectors of raw_input
       DECLARE vector result as the third column vector of raw input
       RUN PartA(mat, result, output_file)
       RUN PartB(mat, output_file)
       CLOSE output file
void PartA(const matrix &mat, const vector &result, ofstream &output_file):
       IF IsColinear(mat.rowVector(0), mat.rowVector(1)):
              IF IsColinear(mat.rowVector(0), result):
                     PRINT "underdetermined" to output file
              ELSE:
                     PRINT "inconsistent" to output_file
       ELSE:
              vector solution = mat.inverse() * result
              PRINT solution.transpose() to output file
void PartB(const matrix &mat, ofstream &output file):
       DECLARE vector solution
       DECLARE bool is trivial as !SolveHomogeneousSystem(mat, solution)
       IF is_trivial:
              PRINT "trivial" to output file
       ELSE:
              PRINT solution.transpose() to output file
```