Implementation of an ECC with M-511

CS448 - Introduction to IT Security

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Outline

- Why is non-elliptic curve cryptography not enough?
- What is an elliptic curve?
- Why M-511?

The Discrete Logarithm Problem[7]

Finite abelian Group, with multiplication

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The Discrete Logarithm Problem[7]

Finite abelian Group, with addition

$$(P, Q) \mapsto P + Q$$

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$(\mathbb{F}_p, \cdot)$ is used in:

- Digital Signature Algorithm (DSA)
- Diffie-Hellman (DH)
- El-Gamal
- RSA (on IFC)

RSA & DSA Keysize

Security strength	Symmetric algorithm	FFC(DSA)	IFC(RSA)
≤ 80	2TDEA	$L = 1024$ $N = 160$	$K = 1024$
112	3TDEA	$L = 2048$ $N = 224$	$K = 2048$
128	AES-128	$L = 3072$ $N = 256$	$K = 3072$
192	AES-192	$L = 7680$ $N = 384$	$K = 7680$
256	AES-256	$L = 15360$ $N = 512$	$K = 15360$

Table: Security Strength of DSA and RSA from NIST[2]

Continuous Eliptic Curve

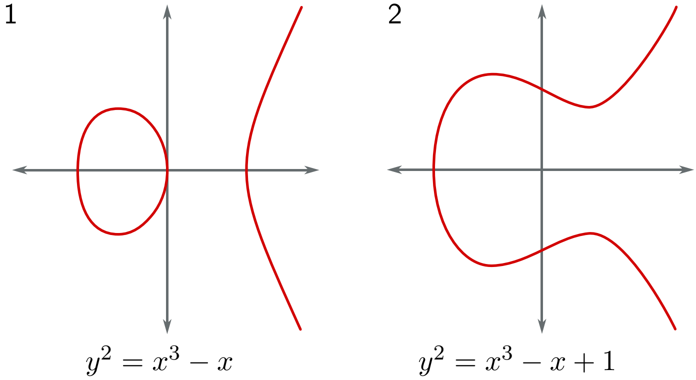
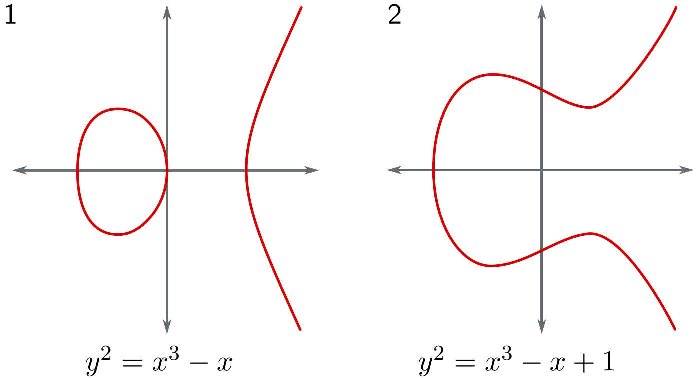


Figure: Two eliptic curves over $\mathbb{R}$ [6]

Continuous Eliptic Curve



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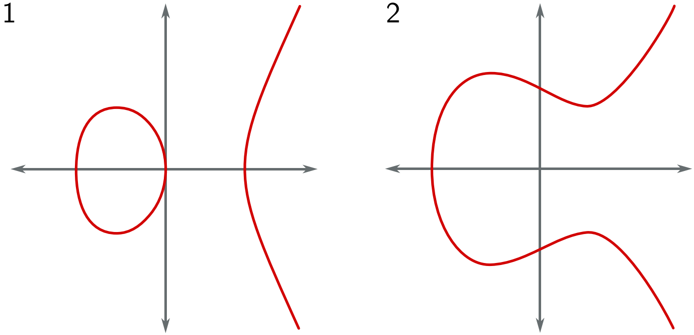
- Symmetry axis: x-axis

Continuous Eliptic Curve



$y^2 = x^3 - x$ $\qquad$ $y^2 = x^3 - x + 1$

Figure: Two eliptic curves over $\mathbb{R}$ [6]

- Symmetry axis: x-axis
- Every line intersecting two points has a third intersection point

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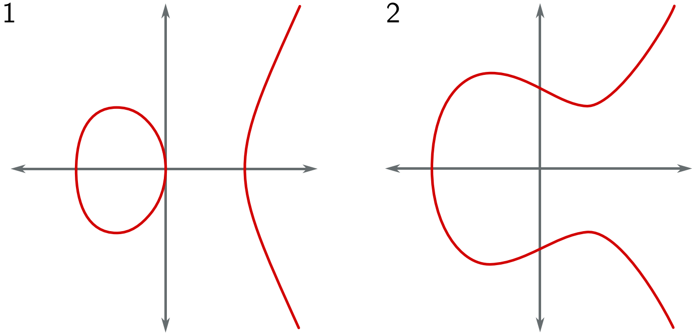

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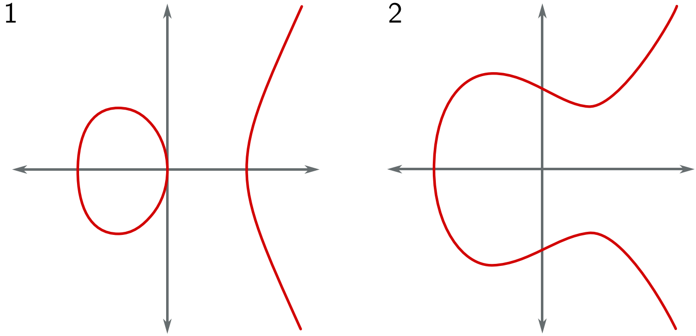
Figure: Two eliptic curves over $\mathbb{R}$ [6]

- Symmetry axis: x-axis
- Every line intersecting two points has a third intersection point
- Vertical lines intersect "infinity"

Continuous Eliptic Curve



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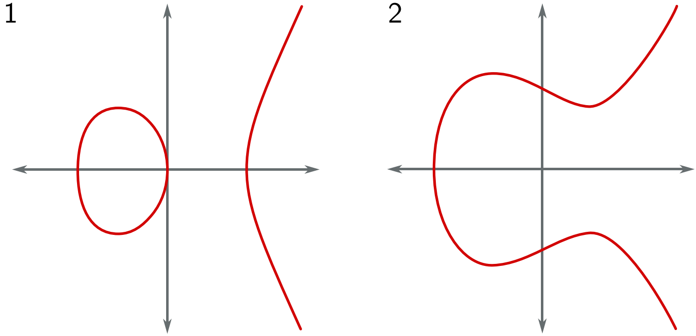

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- Element: point on the curve

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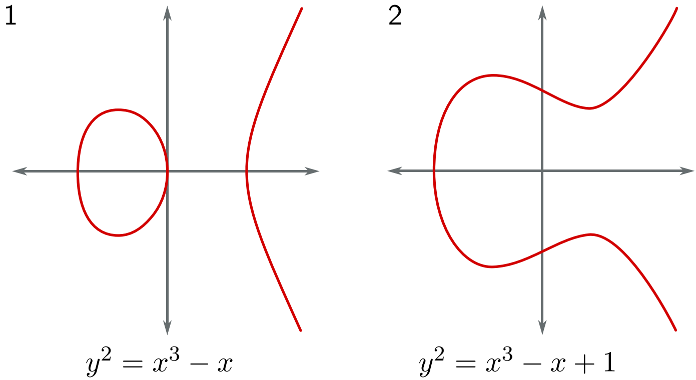

Figure: Two eliptic curves over $\mathbb{R}$ [6]

- Element: point on the curve
- $+$: 3rd intersection of a line, reflect over the x-axis

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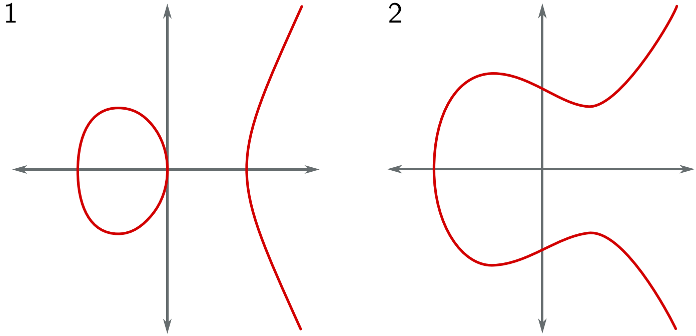

$$y^2 = x^3 - x \qquad y^2 = x^3 - x + 1$$

Figure: Two eliptic curves over $\mathbb{R}$ [6]

- Element: point on the curve
- $+$: 3rd intersection of a line, reflect over the x-axis
- Neutral element: "infinity", O

Continuous Eliptic Curve

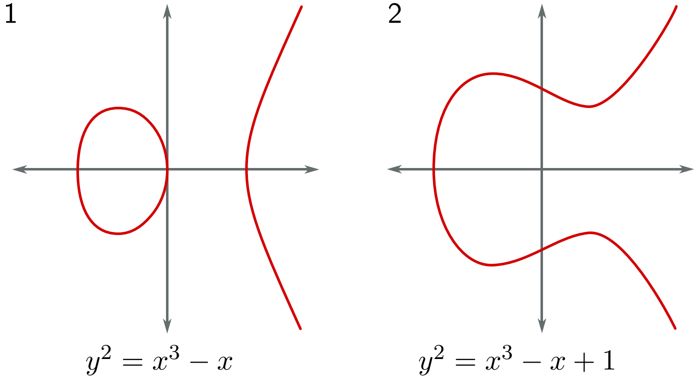

Figure: Two eliptic curves over $\mathbb{R}$ [6]

- Rounding errors
- Not suitable for cryptography

Eliptic curves over finite fields

- Make it discrete!
- "Random" jumps through a set of points

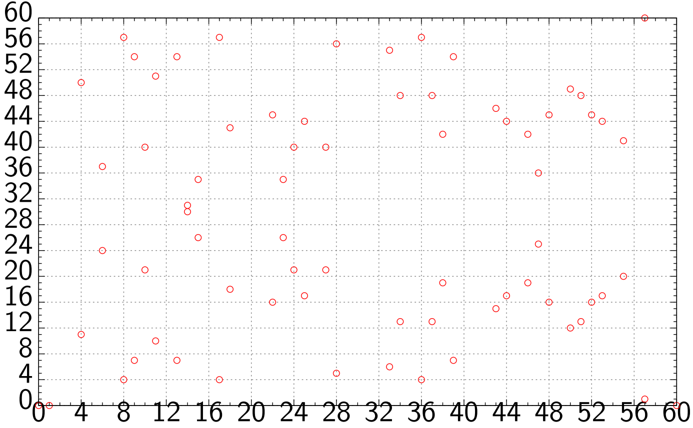

Figure: Set of affine points of elliptic curve $y^2 = x^3 - x$ over finite field $\mathbb{F}_{61}$.

ECC key sizes

Security strength	IFC(RSA)	ECC
≤ 80	$k = 1024$	$f = 160 - 223$
112	$k = 2048$	$f = 224 - 255$
128	$k = 3072$	$f = 256 - 383$
192	$k = 7680$	$f = 384 - 511$
256	$k = 15360$	$f = 512+$

Table: Security Strength of ECC compared to RSA[2]

Handshake size of ECC compared to RSA

RSA key (bits)	**X.509 certificate (bytes)**	**Handshake, no chain (bytes)**	**Handshake, chain (bytes)**
1024	589	1225	2073
2048	845	1481	2585
3072	1101	1737	3097
4096	1357	1993	
ECC key (bits)	**X.509 certificate (bytes)**	**Handshake, no chain (bytes)**	**Handshake, chain (bytes)**
160	291	959	1277
224	315	983	1317
256	331	999	1349
288	347	1015	

Table: Sizes of handshakes and certificates with ECC and RSA[5]

NIST-Curves

The National Institute for Standards and Technology (NIST) proposed some cryptographic curves in 1999.

- Special characteristics for efficiency
- Chosen "randomly"

ECC might not as hard as ECDLP!
Some attacks can be performed on special classes of curves.

- Attacks on NIST-Curves have been found
- NIST-Curves were probably not tru4ly chosen at random [4]

Alternatives: Curve25519, M-511, M-383

Curve25519:

- Proposed by Daniel Bernstein [3]
- No security flaws found until today
- De facto standard implemented in most libraries

M-511, M-383, M-221, E-521, E-382, E-222:

- Proposed by Diego F. Aranha et. al. [1]
- No security flaws found until today

Term Project

Goals for the semester:

- understand the maths behind ECC
- implement a library with M-511 with:
  - key generation
  - en-/decryption
  - signature and verification

"Never implement your own crypto"

We will not:

- Implement a library for real-world use
- Care about side-channel attacks

References I

- Diego F. Aranha et al. *A note on high-security general-purpose elliptic curves*. Cryptology ePrint Archive, Report 2013/647. http://eprint.iacr.org/2013/647. 2013.
- Elaine Barker et al. "Recommendation for key management part 1: General (revision 4)". In: *NIST special publication* 800.57 (2016), pp. 1–147.
- Daniel J Bernstein. "Curve25519: new Diffie-Hellman speed records". In: *International Workshop on Public Key Cryptography*. Springer. 2006, pp. 207–228.
- Daniel J. Bernstein and Tanja Lange. *SafeCurves: choosing safe curves for elliptic-curve cryptography*. URL: https://safecurves.cr.yp.to (visited on 03/13/2017).

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Annette Werner. *Elliptische Kurven in der Kryptographie*. Springer-Verlag, 2013.