

Implementation of an ECC with M-511

CS488 - Introduction to IT Security

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When? April 13, 2017

Outline

- Why is non-elliptic curve cryptography not enough?
- What is an elliptic curve?
- Why M-511?

The Discrete Logarithm Problem[7]

Finite abelian Group, with multiplication

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Finite abelian Group, with **addition**

$$(P, Q) \mapsto P + Q$$

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(\mathbb{F}_p, \cdot) is used in:

- Digital Signature Algorithm (DSA)
- Diffie-Hellman (DH)
- El-Gamal
- RSA (on IFC)

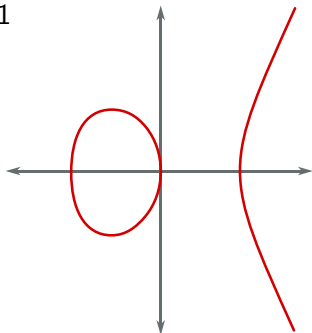
RSA & DSA Keysize

Security strength	Symmetric algorithm	FFC(DSA)	IFC(RSA)
≤ 80	2TDEA	$L = 1024$ $N = 160$	$K = 1024$
112	3TDEA	$L = 2048$ $N = 224$	$K = 2048$
128	AES-128	$L = 3072$ $N = 256$	$K = 3072$
192	AES-192	$L = 7680$ $N = 384$	$K = 7680$
256	AES-256	$L = 15360$ $N = 512$	$K = 15360$

Table: Security Strength of DSA and RSA from NIST[2]

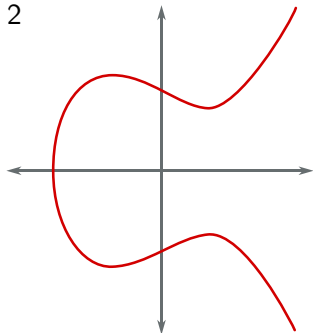
Continuous Elliptic Curve

1



$$y^2 = x^3 - x$$

2

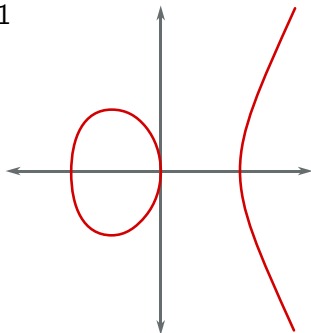


$$y^2 = x^3 - x + 1$$

Figure: Two elliptic curves over \mathbb{R} [6]

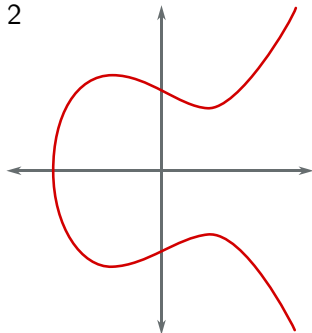
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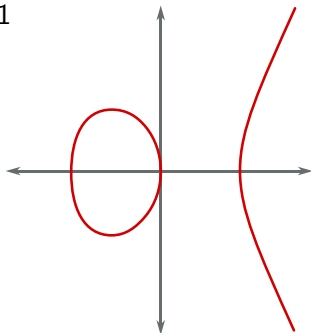
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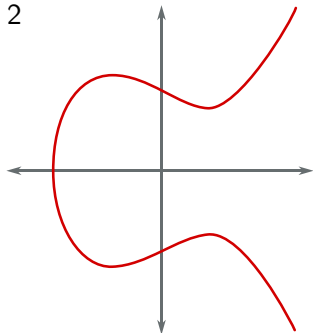
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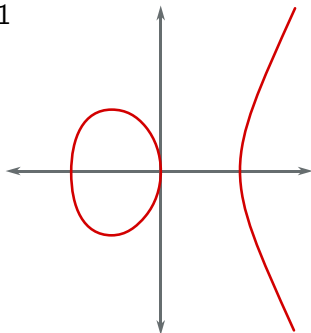
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Figure: Two elliptic curves over \mathbb{R} [6]

- Symmetry axis: x-axis
- Every line intersecting two points has a third intersection point

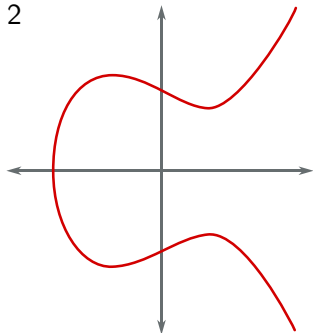
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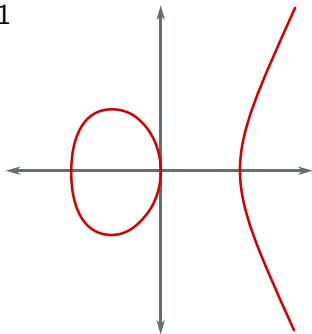
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Figure: Two elliptic curves over \mathbb{R} [6]

- Symmetry axis: x-axis
- Every line intersecting two points has a third intersection point
- Vertical lines intersect "infinity"

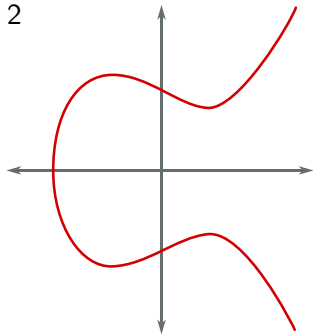
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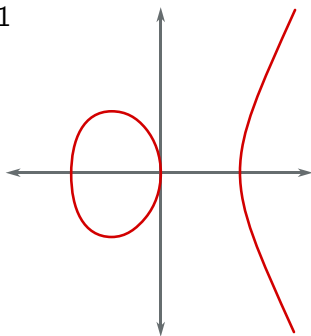


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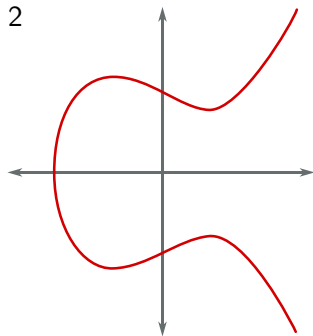
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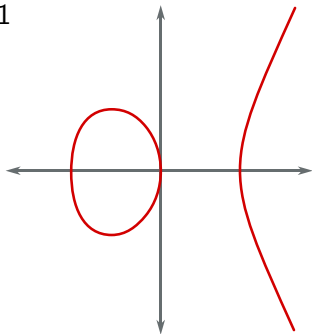
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Figure: Two elliptic curves over \mathbb{R} [6]

- Element: point on the curve

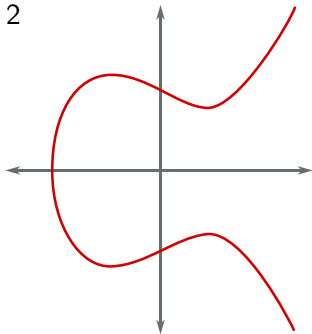
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Continuous Elliptic Curve

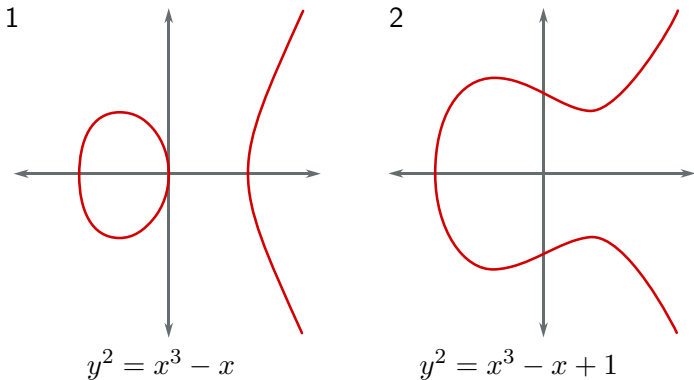
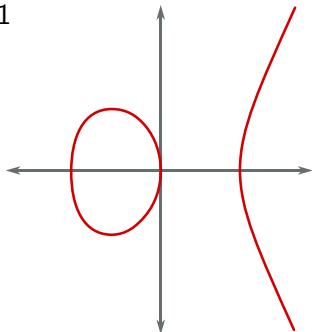


Figure: Two elliptic curves over \mathbb{R} [6]

- Element: point on the curve
- $+$: 3rd intersection of a line, reflect over the x-axis
- Neutral element: "infinity", O

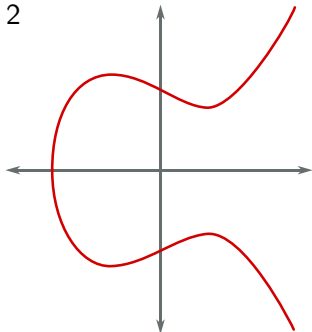
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Figure: Two elliptic curves over \mathbb{R} [6]

- Rounding errors
- Not suitable for cryptography

Elliptic curves over finite fields

- Make it discrete!
- "Random" jumps through a set of points

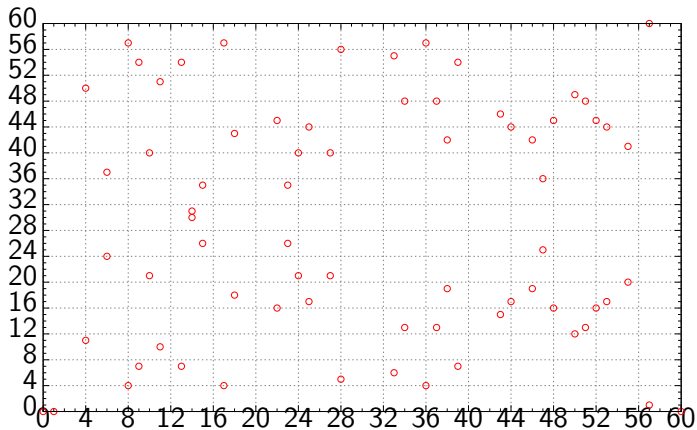


Figure: Set of affine points of elliptic curve $y^2 = x^3 - x$ over finite field \mathbb{F}_{61} .

ECC key sizes

Security strength	IFC(RSA)	ECC
≤ 80	$k = 1024$	$f = 160 - 223$
112	$k = 2048$	$f = 224 - 255$
128	$k = 3072$	$f = 256 - 383$
192	$k = 7680$	$f = 384 - 511$
256	$k = 15360$	$f = 512+$

Table: Security Strength of ECC compared to RSA[2]

Handshake size of ECC compared to RSA

RSA key (bits)	X.509 certificate (bytes)	Handshake, no chain (bytes)	Handshake, chain (bytes)
1024	589	1225	2073
2048	845	1481	2585
3072	1101	1737	3097
4096	1357	1993	

ECC key (bits)	X.509 certificate (bytes)	Handshake, no chain (bytes)	Handshake, chain (bytes)
160	291	959	1277
224	315	983	1317
256	331	999	1349
288	347	1015	

Table: Sizes of handshakes and certificates with ECC and RSA[5]

NIST-Curves

The National Institute for Standards and Technology (NIST) proposed some cryptographic curves in 1999.

- Special characteristics for efficiency
- Chosen "randomly"

ECC is not as hard as ECDLP!

Some attacks can be performed on special classes of curves.

- Surprise! Attacks on NIST-Curves have been found
 - NIST-Curves were probably not chosen at random
- [4]

Alternatives: Curve25519, M-511, M-383,
...

Curve25519:

- Proposed by Daniel Bernstein [3]
- No security flaws found until today
- De facto standard implemented in most libraries

M-511, M-383, M-221, E-521, E-382, E-222:

- Proposed by Diego F. Aranha et. al. [1]
- No security flaws found until today

Term Project

Goals for the semester:

- understand the maths behind ECC
- implement a library with M-511 with:
 - key generation
 - en-/decryption
 - signature and verification

"Never implement your own crypto"

We will not:

- Implement a library for real-world use
- Care about side-channel attacks

References I



Diego F. Aranha et al. *A note on high-security general-purpose elliptic curves*. Cryptology ePrint Archive, Report 2013/647.

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Elaine Barker et al. “Recommendation for key management part 1: General (revision 4)”. In: *NIST special publication 800.57* (2016), pp. 1–147.



Daniel J Bernstein. “Curve25519: new Diffie-Hellman speed records”. In: *International Workshop on Public Key Cryptography*. Springer. 2006, pp. 207–228.



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