LG-Beam Amplitude Prediction

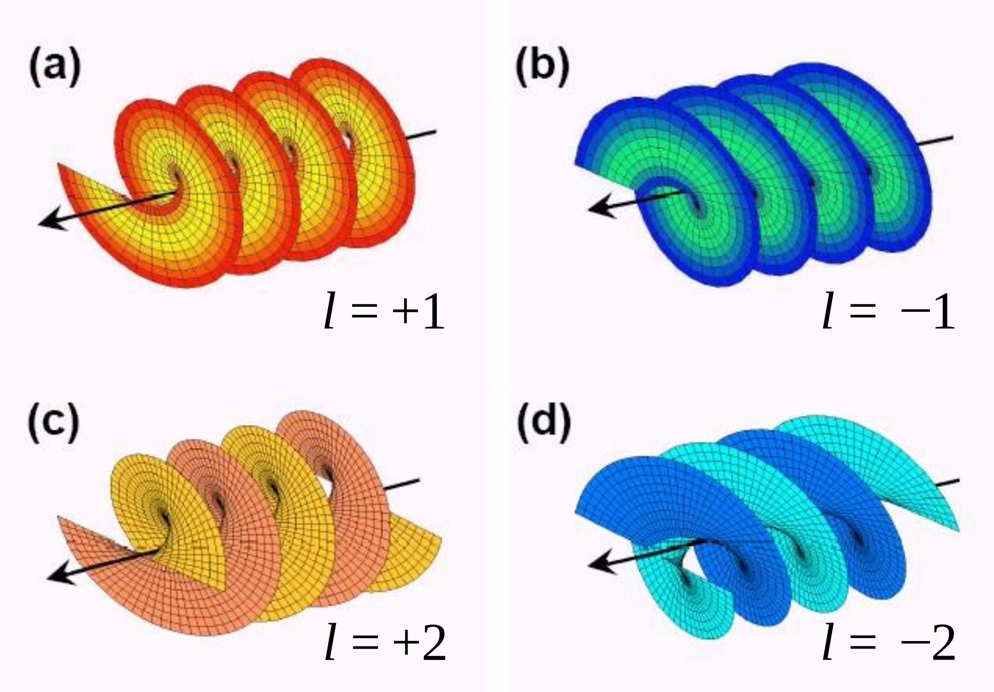
PARAG SHARMA

Outline

- Orbital angular momentum of light
- The Mathematics of OAM
- Representation of L-G modes
- States in OAM Basis
- Superposition of LG Beams: Vortex Beams
- Machine learning for predicting complex amplitudes of superposition of two LG beams

Orbital angular momentum of light (OAM)

- The simplest example of a light beam carrying OAM is one with a phase in the transverse plane of φ(r, φ) = exp(ilφ)
- helically phased beams carried an OAM equivalent to a value of $l\hbar$ per photon



Helical modes: (using cylindrical coordinates r, φ, z)

$$\mathbf{E}(\mathbf{r},t) = \mathbf{E}_0(r,z)e^{im\varphi}e^{i(kz-\omega t)}$$

helical phase factor:

$$e^{il\varphi}$$

$$l = 0, \pm1, \pm2, \pm3 \ldots$$

Angular momentum (OAM):

$$L_z = l\hbar \quad \text{per photon}$$

[L. Allen, M.W. Beijersbergen, R. J. C. Spreeuw, and J. P. Woerdman, *Phys. Rev. A* **45**, 8185 (1992)]

The Mathematics of OAM

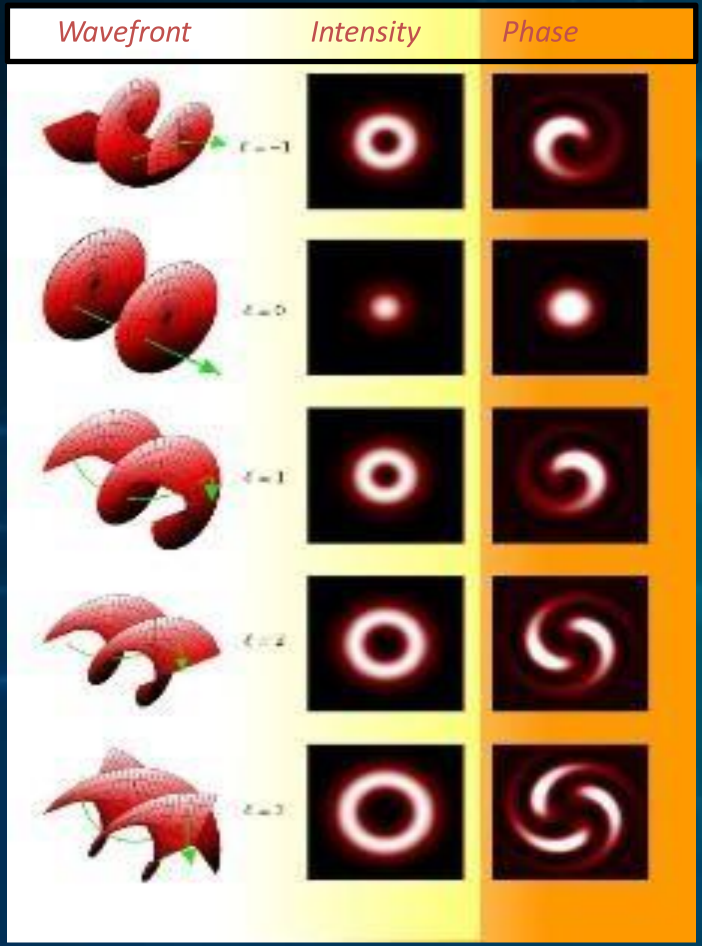
- Mathematically we can represent OAM with laguerre guassian modes
- Laguerre-Gaussian (LG) modes are solutions to paraxial Helmholtz Equation
- these modes have amplitude distributions, given by

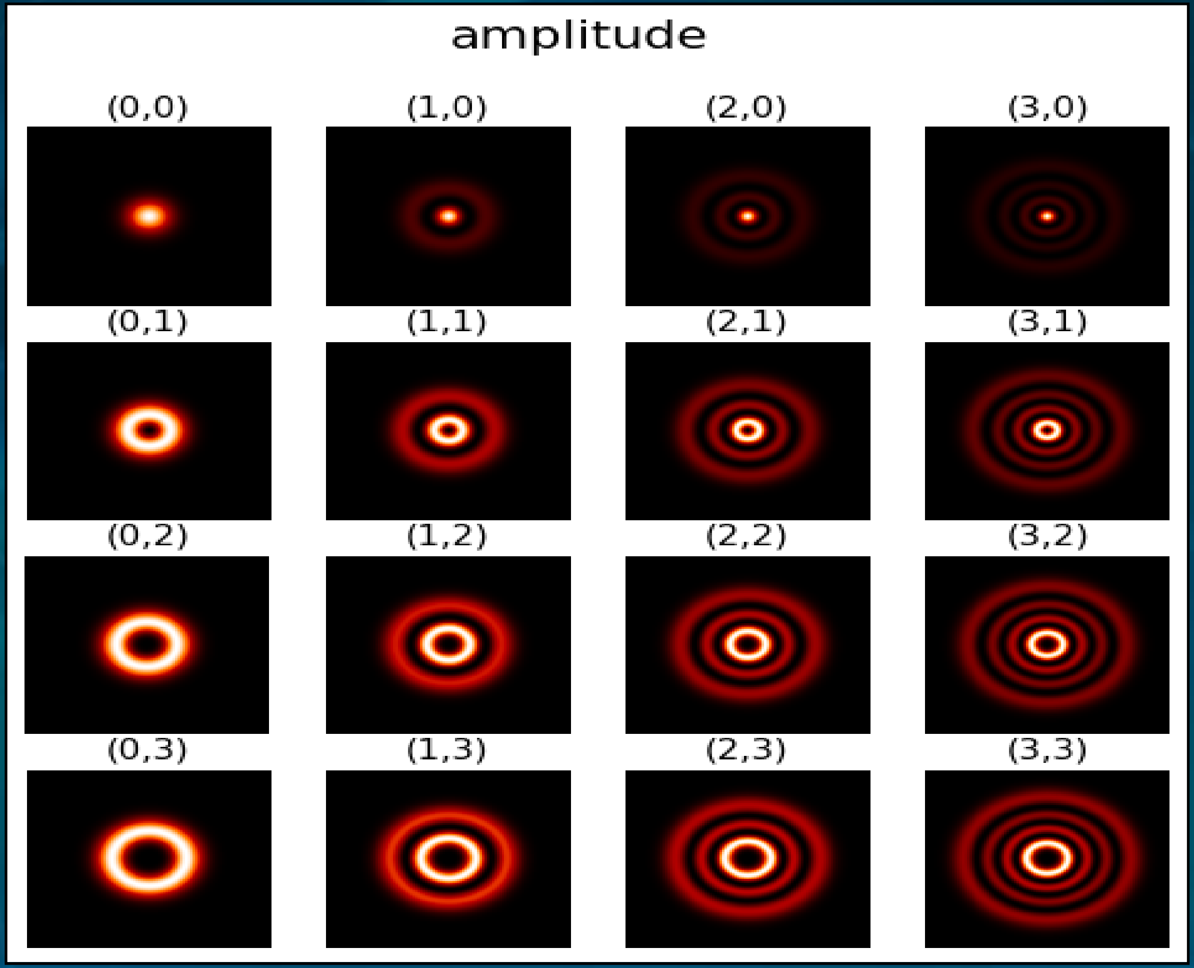
$$u_{pl}(r,\phi,z) = \frac{C}{(1+z^2/z_R^2)^{1/2}}[\frac{r\sqrt{2}}{w(z)}]^l L_p^l[\frac{2r^2}{w^2(z)}]exp[\frac{-r^2}{w^2(z)}]\times$$
$$\times\, exp[\frac{-ikr^2 z}{2(z^2+z_R^2)}]exp(-il\phi)exp[i(2p+l+1)tan^{-1}\frac{z}{z_R}]$$

- The mathematical representation in terms of Laguerre - Gauss modes contains two integer numbers: l is the azimuthal index giving an OAM of $l\hbar$ per photon, and p is the number of radial nodes in the intensity distribution

Graphical representation of L-G modes

- The wavefront has a *helical shape* composed by ℓ lobes disposed around the propagation axis z.





States in OAM Basis

- it is useful to express most beams in a complete basis set of orthogonal modes. For OAM carrying beams this is most usually the Laguerre–Gaussian (LG) mode set
- Any image can be synthesized from the appropriate superposition of modes that form a complete basis set
- Laguerre-Gaussian (LG) modes form a complete set
- A photon that is in an LG eigenmode can be represented as $|\, l, \mathrm{p}\rangle$ .We can prepare an OAM state in the space spanned by $|\, l, \mathrm{p}\rangle$ basis and measure it using the polar coordinate basis

$$\sum_{p=0}^{\infty}\sum_{l=-\infty}^{\infty} \rho LG^{*}{}_{p}{}^{l}(\rho,\phi,z) LG\,{}_{p}{}^{l}(\rho',\phi',z) = \delta(\phi-\phi')\delta(\rho-\rho')$$

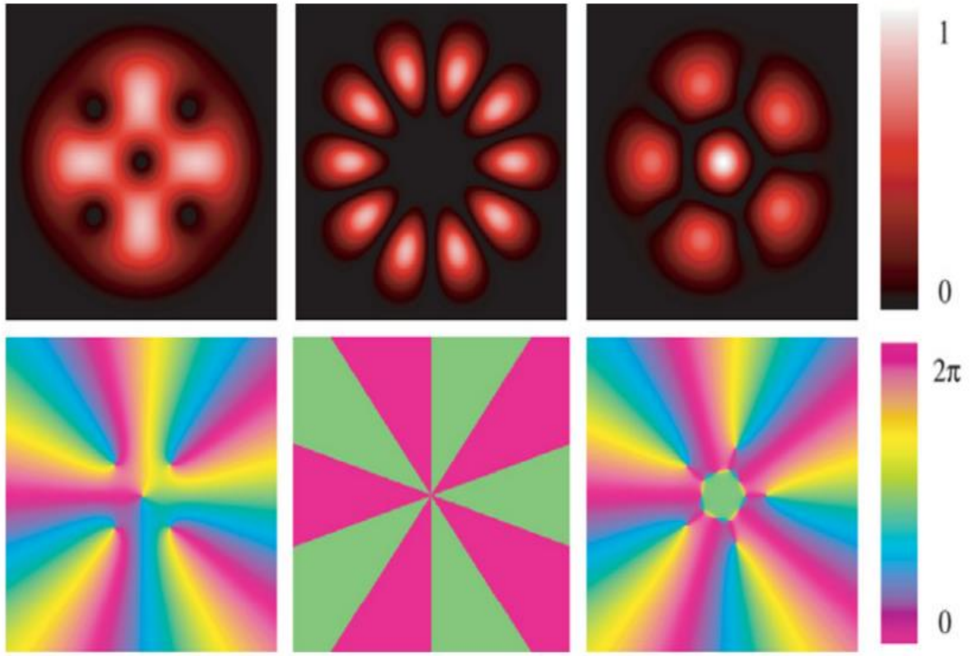
$$\int_{0}^{\infty}\int_{0}^{2\pi} \rho d\rho d\phi LG^{*}{}_{p}{}^{l}(\rho,\phi,z) LG\,{}_{p'}{}^{l'}(\rho,\phi,z) = \delta_{ll'}\delta_{pp'}$$

$$LG_{p}{}^{l}(\rho,\phi,z) \quad \rightarrow \quad \psi_{l}(\phi) = \frac{1}{\sqrt{2\pi}} e^{-il\phi} \quad \equiv \quad \langle\phi|l\rangle$$

$$\sum_{l=-\infty}^{\infty} |l\rangle\langle l'| = \delta(\phi-\phi') \qquad \text{with} \qquad \langle l|l'\rangle = \delta_{ll'}$$

Superposition of LG Beams: Vortex Beams

- When Laguerre–Gaussian beams interfere they produce vortex structures resulting in complex patterns of bright and dark regions
- By superposing two co-propagating LG beams with OAM l_1 and l_2 at z = 0, the beams interfere constructively at $|l_2 - l_1|$ azimuthal positions, separated by regions of destructive interference and constructive interference, resulting in a transverse intensity profile of $|l_2 - l_1|$ dark petals (zeroth intensity regions) or bright petals (high intensity regions

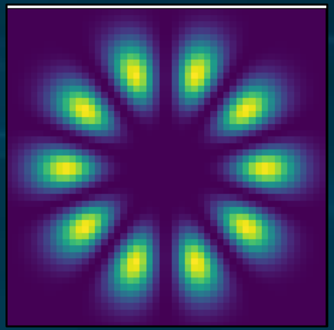


Normalized intensity (top) and phase (bottom) profiles of some superpositions of Laguerre–Gaussian modes: $LG_{01} + LG_{05}$, $LG_{0-5} + LG_{05}$, and $LG_{10} + LG_{05}$ (left to right).

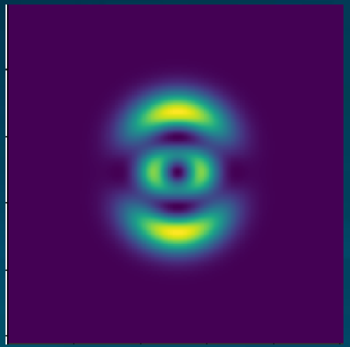
Yao and Padgett, Advances in optics and photonics, **3**, 161 (2011)

Machine learning for predicting complex amplitudes of superposition of two LG beams

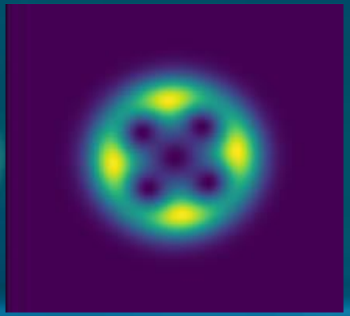
- We want to find the values of the complex coefficients for the constituent LG-beam from the superposed value $a|l_1, p_1\rangle + b|l_2, p_2\rangle$, here a and b can take any arbitrary complex value and l_1 and l_2 are the OAM values of two superposed LG beams



- This is a regression problem in which we need to find values of a and b using the matrix for superposed amplitude for two LG beams

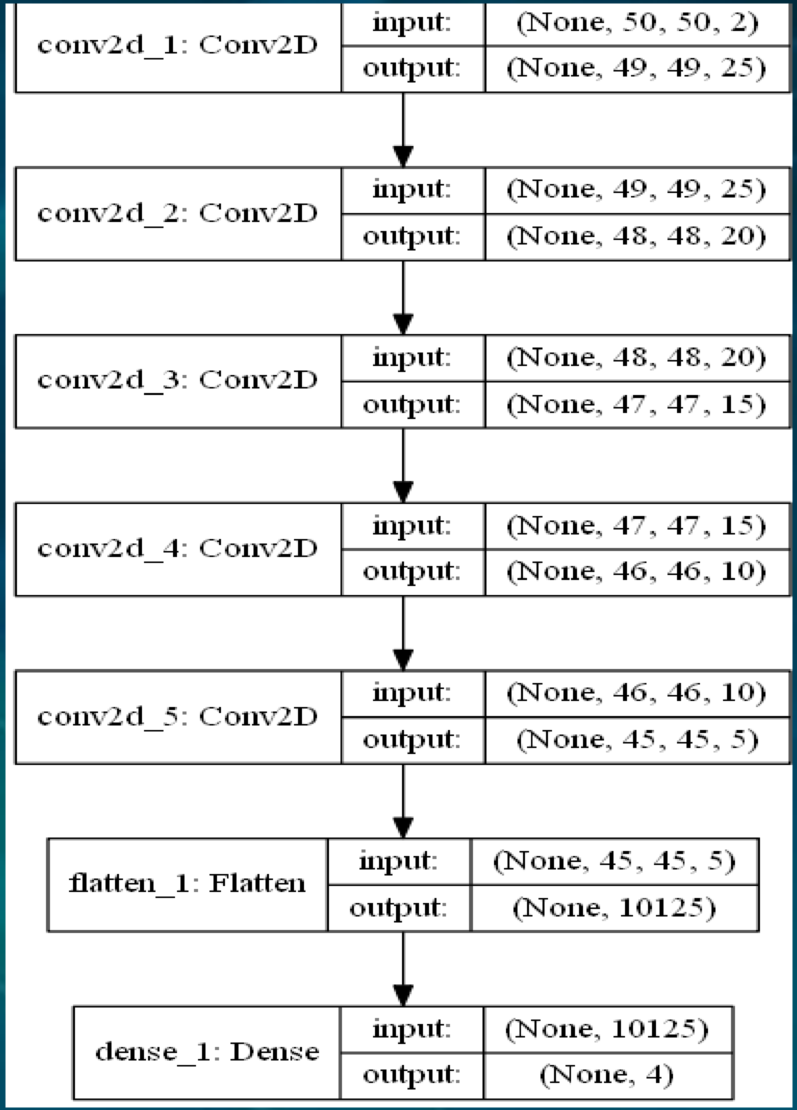


- We can use deep convolutional neural network for this purpose
- First step is to generate data by the simulation of LG beams



Model and training parameters

- To train the model we generate around 3000 superposed matrices with random complex coefficients a and b whose real and imaginary parts are chosen randomly from uniform distribution from -1 to 1
- The input consist of the matrix with dimensions 50x50x2 since each point is represented by a complex number
- The model consists of 5 2d convolution layers with filter size 25,20,15,10 and 5, and 1 densely connected layer with 4 neurons as the output
- The loss function is the mean squared error
- The model is trained on 70% data with learning rate of 0.01 for 30 epochs
- Different combinations for l and p were used to generate different dataset for training



Training Results for different combinations of l and p values