Systematic Factor Portfolio Construction

Paragon Global Investments

Factors

- Factors are “attributes” that drive returns of assets
- You can use factors to predict the risk and return of a stock/other asset based on its relationship to a factor or group of factors
- Think of a factor as a portfolio that has its own return and variation statistics

Regression using Factors

- Factor investing uses regression models to analyze relationships
- In the regression model, the factor is the independent variable (regressor)

Different Types of Factors

- Style Factors
  - Relate to the characteristics of an asset/company
- Macroeconomic Factors
  - Relate to the effects of macroeconomic forces on the returns of an asset

Style Factors

- Value
  - Stocks that appear undervalued based on fundamental metrics such as low price-to-earnings or price-to-book ratios.
  - The value factor captures the tendency for undervalued stocks to eventually correct upward, offering potential for superior long-term returns.

Style Factors

- Momentum
  - Stocks exhibiting strong recent performance, with the expectation that these trends will continue.
  - The momentum factor leverages the persistence of price trends, suggesting that past winners may keep winning in the near term.
  - Often times you can look at windows of 3 months, 6 months, 12 months

Style Factors

- Yield
    - Stocks offering attractive income returns, typically measured by dividend yield.
    - The yield factor attracts investors seeking steady cash flows and stability, particularly in lower-volatility market environments.

Style Factors

- Size
  - A classification based on a company's market capitalization, often differentiating small-cap stocks from large-cap stocks
  - The size factor is predicated on the observation that smaller companies can offer higher growth potential and distinct risk-return characteristics compared to larger firms.

Macroeconomic Factors

- Interest Rates
- Inflation
- GDP growth
- Any other relevant macroeconomic influences on the economy

Part I: Factor Construction Models

Multi-Factor Portfolio Construction Decisions

- There are numerous variables that asset managers must consider when constructing multi-factor portfolios

Variable 1: Top-Down vs. Bottom Up

- Top-Down:
  - Capture exposure to multiple factors by combining multiple single-factor indices
  - Provides more diversification, but greater chance of factor dilution
- Bottom Up:
  - Capture exposure to multiple factors by selecting securities that score highly across all factors on average
  - Provides higher factor exposures

Variable 2: Sector-Neutral vs. Sector-Agnostic

- Sector-Neutral:
  - Stock selection is conducted independently within each sector to hit a sector weight target
  - reduce exposure to unintended risks
- Sector-Agnostic:
  - Stock selection is conducted purely by factor score without any sector constraint
  - may inadvertently overweight sectors that have performed well in the past
  - potentially leading to an imbalance that exposes you to higher concentration risk and increased vulnerability to sector-specific downturns

Variable 3: Rebalancing Frequency

- Reselect and reweight constituent securities on a regular basis
- There is a trade-off between turnover costs and factor decay
- Turnover costs are incurred every time a portfolio manager trades securities
- Factor decay occurs when factor exposure decreases over time

Variable 4: Factor Combinations

- The type and combination of factors
- Choosing which factors and weighing them accordingly

Part II: Construction Models for Security Selection

Heuristic Construction Parameters

- Heuristic Multi-factor approach could be used to derive a comprehensive factor score for a particular security
- Factor score: an assigned score based on an assets relationship to a factor or factors

Heuristic Construction Model Process

1. Identify the proper factor or factors you want to use in your model
2. Run a factor regression analysis of the companies you are looking at onto your set of factors
3. For each company, calculate a factor score (up to your discretion)
4. Select the top n assets based on their factor scores and construct a portfolio based on these results

Heuristic Construction Model Process

- Feed the information into a portfolio weighting scheme such as mean-variance to get the final portfolio allocation weights and develop a systematically adjusted portfolio of assets

Pros and Cons of Heuristic Construction Model

- It’s easy to implement
- Just run regression and calculate a score
- However, the heuristic approach might generate more tracking errors

Ways to Calculate Factor Score

$$\alpha_i = 0.2 * F_{1,i} + 0.2 * F_{2,i} + 0.2 * F_{3,i} + 0.2 * F_{4,i} + 0.2 * F_{5,i}$$

where α_i represents the comprehensive factor score for a security i, while $F_{i,j}$ represents the factor score of security i for a factor j.

Ways to Calculate Factor Score

- Normalize regression data
- This model is based on raw factor scores being normalized such that their distribution has a mean of 0 and a standard deviation of 1
    - Calculate the Z-scores

Optimized Multi-Factor Construction

- When constructing multi-factor portfolios, you can control the active risk of a portfolio through the introduction of a risk optimization framework
- Active risk: the risk characteristics of an actively managed portfolio relative to a benchmark)

Active Risk (Tracking Error) Minimization

- Tracking Error: measure of the divergence between the return profile of a portfolio and its corresponding benchmark

$$Tracking\ Error = \sqrt{var(r_p - r_b)}$$

- r_p and r_b represent the return of a portfolio and the return of a benchmark respectively.

Tracking Error

- Tracking error is indicative of a portfolio's performance divergence from the benchmark
- Positive TE suggests outperformance
- Negative TE suggests lagging performance

Tracking Error

- A tracking error of 1% implies an expected tracking portfolio return within ± 1% with a 67% probability and a tracking error of 2% implies an expected tracking portfolio return within ± 2% with a 95% probability
- You want to minimize TE

Let w_i be the weighting of stock i in the portfolio, such that $1 \leq i \leq N$, with N being an arbitrary constant. Three key variables are defined as follows:

1. w represents a vector of weightings of stocks in the preallocated portfolio;
2. x represents a vector of the index weightings of the same set of stocks (S&P500);
3. Q represents the positive definite covariance matrix of stock returns.

We suppose the tracking error function to be the following:

$$TE(w) = (w - x)' \, Q \, (w - x)$$

where $(w - x)'$ *is the transpose of* $(w - x)$

Tracking Error Minimization

- This is a convex function
- While optimizing this is possible, it can be mathematically difficult
- We can approximate the optimization by a sequence of “continuously differentiable non-convex piecewise quadratic functions”

Tracking Error Minimization

- We can view this as a problem subject to a constraint on the total number of assets, K
- This is a discontinuous optimization problem

$$\min TE(w), \; where \; w \in R^n$$

$$subject\ to\ \sum_{i=1}^{N} \Lambda(w_i) \leq K; \; \sum_{i=1}^{N} w_i = 1; \; w \geq 0$$

Tracking Error Minimization

$$\min \left(TE(w) + \mu \sum_{i=1}^{N} \Lambda(w_i)\right),\ where\ w \in R^n\ and\ \mu \geq 0$$

$$subject\ to\ \sum_{i=1}^{N} w_i = 1;\ w \geq 0$$

Steps to Minimize - Step 1

start by solving the simpler convex problem (no cardinality constraint)

$$\min TE(w), \ where\ w \in R^n$$

$$subject\ to\ \sum_{i=1}^{N} w_i = 1;\ w \geq 0$$

Steps to Minimize - Step 2

- then **incrementally** introduce the penalty approximation (with parameters λ and ρ_k)

$$min\ (TE(w)\ +\ \mu max(\sum_{i=1}^{N} g_{\lambda}(w_i;\ \rho_k)\ -\ K,\ 0)),\ where\ w \in R^n$$

$$subject\ to\ \sum_{i=1}^{N} w_i\ =\ 1;\ w\ \geq\ 0$$

Steps to Minimize - Step 3

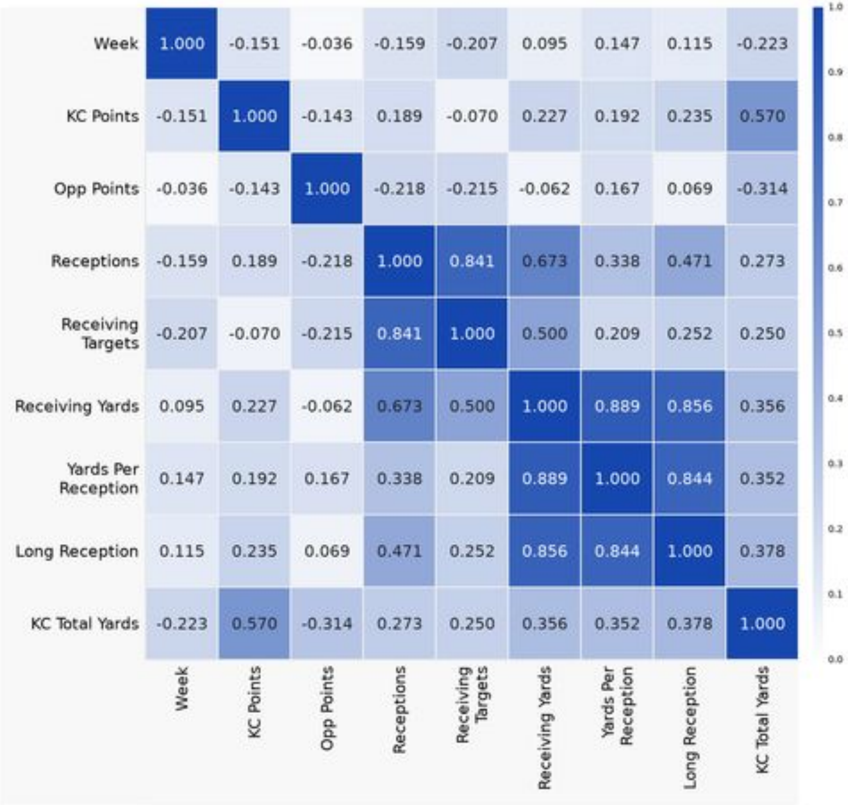
- Terminate the iteration if $\left(x_i\right)_k \le q_k$ or $\left(x_i\right)_k \le r_k$.
- Otherwise go back to step 1 and find the solution of the approximation at P_{k+1}

Principal Component Analysis (PCA)

- Five-factor models are not perfect ways to explain risk
- PCA (Principal Component Analysis) is a technique that automatically finds a set of new factors from your data
- These factors are chosen because they explain as much of the variation in your data as possible, and they don't overlap with each other (they're uncorrelated)
- This means that instead of starting with pre-assumed factors, PCA lets the data itself reveal the most important underlying patterns.

PCA

- Compute the correlation matrix between all the factors
- Example of a correlation matrix



PCA

- Do eigen decomposition to determine the eigenvalues
- Sort the eigenvalues from highest to lowest
- The largest eigenvalue corresponds to the first principal component (PC1), which explains the most variance. The second largest corresponds to PC2, and so on.
- Then decide on how many components to keep

PCA

- Each principal component is a linear combination of the original variables.
- The factor loadings (or eigenvectors) show you how each original variable contributes to each component.
- Project your original data onto the selected principal components.
- This gives you a new, reduced set of features (PC1, PC2, …).

Applying PCA

- The loadings (elements of the eigenvectors) reveal the exposure of each asset to the principal components.
- You can analyze these exposures to understand the risk sources in your portfolio, use them in risk management, or even construct factor-based portfolios by targeting specific principal components.