Linear Factor Pricing Models

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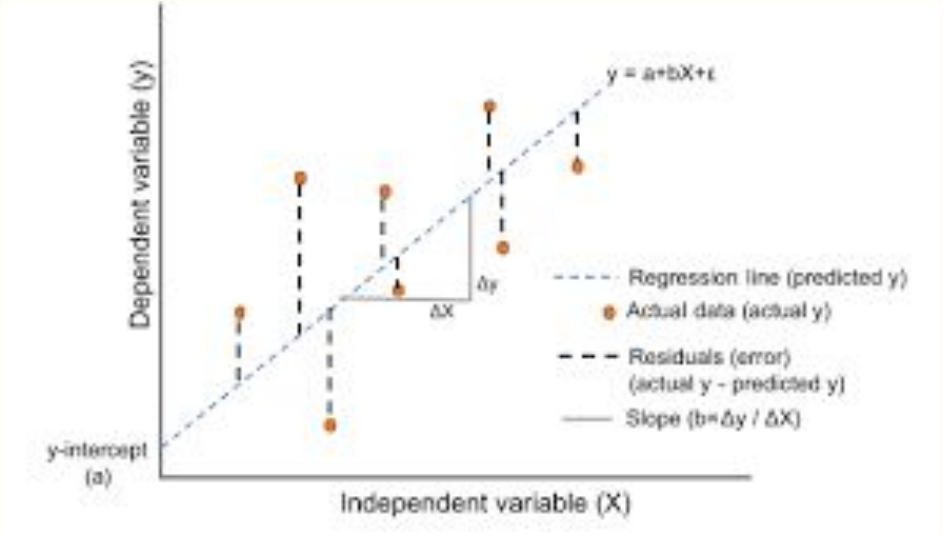
PNG Quant Education Week 2

Introduction

- Linear regression models the relationship between a dependent variable Y and independent variables X_1, X_2, ..., X_k
- $Y = \beta_0 + \beta_1 X_1 + \beta_2 X_2 + \ldots + \beta_k X_k + \epsilon$
- Key assumptions
  - Linear relationships
  - No residual autocorrelation
  - No multicollinearity
  - Zero mean of errors
  - Normality of errors



$$\text{Minimize } \sum_{i=1}^{n} (Y_i - \hat{Y}_i)^2$$

$$\hat{\beta} = (\mathbf{X}^\top \mathbf{X})^{-1} \mathbf{X}^\top \mathbf{y}$$

Measuring Linear Fit

- The effectiveness of a linear model is generally measured through the coefficient of determination, or R^2

$$R^2 = 1 - \frac{\text{Sum of Squared Residuals (SSR)}}{\text{Total Sum of Squares (TSS)}} \qquad SSR = \sum_{i=1}^{n}(Y_i - \hat{Y}_i)^2 \qquad TSS = \sum_{i=1}^{n}(Y_i - \bar{Y})^2$$

- R^2 quantifies the proportion of the variation in Y that is explained by the model
- Higher values indicate that the model fits the data better
- A strong fit on training data does not guarantee predictive accuracy
- Adding more predictors artificially inflates R^2

$$\text{Adjusted } R^2 = 1 - (1 - R^2)\frac{n-1}{n-p-1}$$

Where:

- n = Total number of observations.
- p = Number of predictors (independent variables).
- R^2 = Regular coefficient of determination.

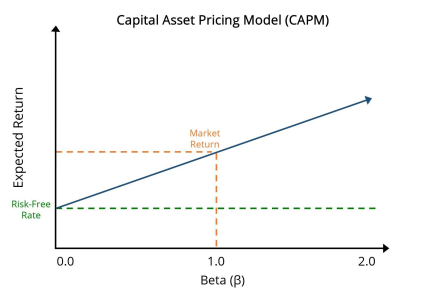
Forecasting Equity Returns

Importance of Forecasting Equity Returns

- Investment planning
  - Accurate models could reveal mispriced assets
  - Difficult to implement in practice due to alpha decay and complexity
- Portfolio construction
  - MPT relies on expectations of returns and risk to weight assets
  - Better diversification and risk-adjusted performance
- Market timing
  - Forecasts can help determine when to enter or exit a position
- Risk management
  - Understanding expected returns helps manage downside risk

CAPM (1964)

- Capital Asset Pricing Model posited that an asset's expected return is determined by its beta to the overall market
- Higher beta assets should offer higher expected returns
- Limitations
  - Efficient markets assumption
  - Unconstrained leverage at risk-free rate[1]
  - Beta instability
  - Market risk is the only driver of returns[2]



$$E(r_i) = r_f + \beta_i(E(r_m) - r_f)$$

where:

- $E(r_i)$ = Expected return of asset i
- r_f = Risk-free rate
- β_i = Beta of asset i, measuring market exposure
- $E(r_m)$ = Expected market return
- $E(r_m) - r_f$ = Market risk premium

Fama and French Three Factor Model (1992)

- Expands on the Capital Asset Pricing Model to better explain stock returns
- Motivation
  - CAPM's limitations in explaining stock returns
  - Observed patterns in historical stock performance not captured by CAPM
  - Need for a more comprehensive model to account for additional risk factors
- Three factors
  - Market risk from CAPM
  - Size factor (Small Minus Big)
  - Value factor (High Minus Low)

$$R = R_f + \beta_1(R_m - R_f) + \beta_2(SMB) + \beta_3(HML)$$

Evaluation of FF3FM

Improvements over CAPM

- Fama-French model explains over 90% of diversified portfolio returns compared to CAPM's average of 70%
- Incorporation of additional risk factors that account for observed patterns
- Lays out a framework that can be extended to include additional factors

Limitations

- Fama-French factors are derived from empirical verification rather than theoretical premises
- Relationships between factors could introduce multicollinearity[3]
- Betas can still vary over time[4]
- Predictive power of CAPM and Fama-French is quite low out of sample
- Factors are not necessarily applicable to other markets and international analyses of Fama-French yield poor results[5]

Conclusion

- Linear factor models continue to be an area of research
- Tradeoff between predictive power and generalizability
- Dynamic versions of models show promising results
- Potential for machine learning to play a role in analysis