PGI Quant Lecture 1: Basic Options Theory

An option is a derivative in which there is some sort of choice for the owner of the derivative.

The two most comman types of options are the call option and the put option.

Like futures, these type of derivatives are traded on many exchanges around the world.

Definition

A call option is the right, but not the obligation, of the owner of the option to buy an asset at a future time T for an amount K.

A put option is the right, but not the obligation, of the owner of the option to sell an asset at a future time T for an amount K.

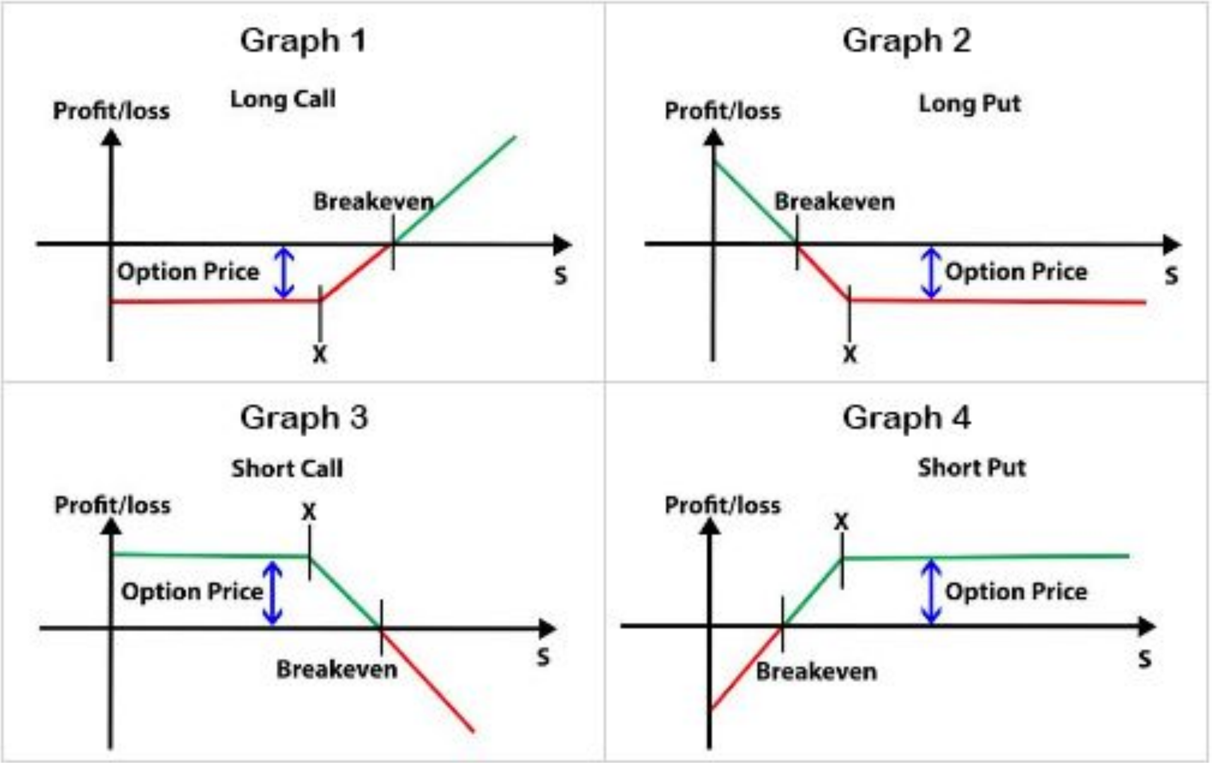
Here K is known as the strike price and T as the exercise time or maturity time.

The payoff at time T of a call option is given by

$$\max(S_T - K, 0).$$

The payoff at time T of a put option is given by

$$\max(K - S_T, 0).$$


Graph 1
Long Call
Profit/loss
Breakeven
Option Price
S
X
Graph 2
Long Put
Profit/loss
Breakeven
Option Price
S
X
Graph 3
Short Call
Profit/loss
X
Option Price
Breakeven
S
Graph 4
Short Put
Profit/loss
X
Option Price
Breakeven
S

There are different types of options with respect to the exercise time.

- European options
- American options
- Bermudan options
- Other: Parisian, Canadian, Russian,...

Call and put options are examples of simple derivatives

In this case the derivative's payoff X is a function F of the underlying S_T:

$$X = F(S_T).$$

There are also path-dependent derivatives:

$$X = G((S_t,\ 0 \leq t \leq T)).$$

In this case the whole (or part) of path of the underlying is needed to know on order to calculate the payoff X.

Example

An up-and-in contract is a derivative in which the value of the underlying must reach a given level L in order to be active.

An up-and-in call option has payoff

$$X = \max(S_T - K, 0)\mathbf{1}\big(S_t \geq L \text{ for some } t \in [0, T]\big).$$

There are also up-and-out, down-and-in and down-and-out contracts.

By combining basic options we can create new payoffs.

This can be used in order to help an investor to get the payoff he wants.

Assume that you belive that a stock is going to move from its current price, but you do not know in which direction.

If this is the case, you can buy a strangle. A strangle is a a sum of a put option with strike price K_1 and a call option with strike price $K_2 > K_1$.

Other examples of option combinations include

- A straddle: this is a strangle with $K_1 = K_2$
- A bull spread: buy a call option with strike price K_1 and sell a call option with strike price $K_2 > K_1$.
- A bear spread: sell a put option with strike price K_1 and buy a put with strike price $K_2 > K_1$.
- A butterfly: buy 1 call with strike price $K - a$, sell 2 calls with strike price K and buy 1 call with strike price $K + a$.

If we buy 1 call option and sell 1 put option both having the same strike price K and maturity time T, then the resulting payoff at T will be

$$S_T - K.$$

This is true since

$$\max(S_T - K, 0) - \max(K - S_T, 0) = S_T - K.$$

It follows from linear pricing that

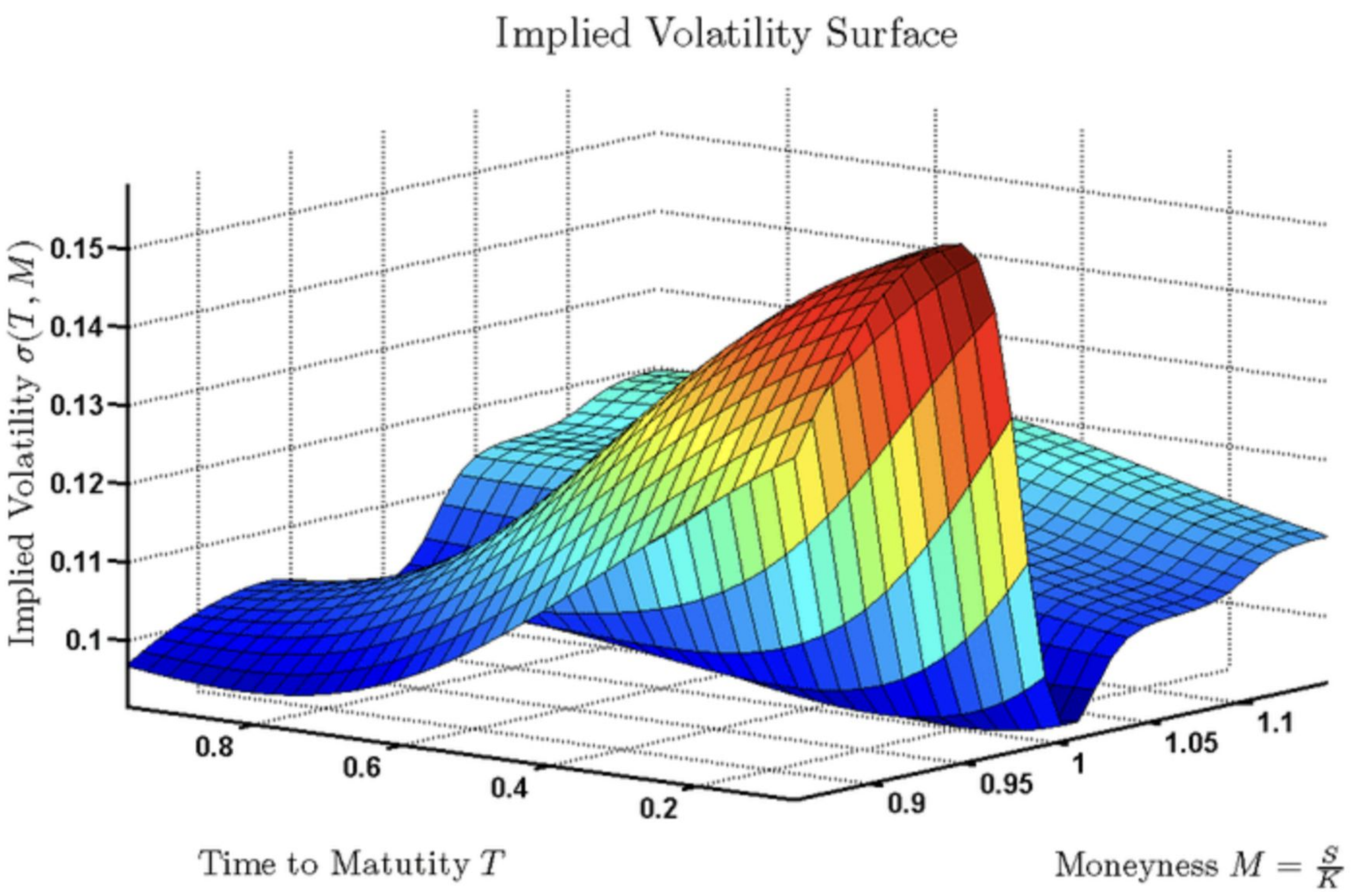
$$\text{Price of call at } 0 - \text{Price of put at } 0 = S_0 - K \cdot d(0, T).$$

This relation is called the put-call parity.

In order the find the price (or value) of an option we need to construct a stochastic model.

Two popular models are

- The binomial model
- The Black-Scholes(-Samuelson) model



Implied Volatility Surface
Implied Volatility $\sigma(T, M)$
0.15
0.14
0.13
0.12
0.11
0.1
0.8
0.6
0.4
0.2
Time to Matutity T
0.9
0.95
1
1.05
1.1
Moneyness $M = \frac{S}{K}$

Greek	*Symbol*	*Measures*	*Definition*
Delta	$\Delta = \frac{\partial V}{\partial S}$	Equity Exposure	Change in option price due to spot
Gamma	$\Gamma = \frac{\partial^2 V}{\partial S^2}$	Payout Convexity	Change in delta due to spot
Theta	$\Theta = \frac{\partial V}{\partial t}$	Time Decay	Change in option price due to time passing
Vega	$\upsilon = \frac{\partial V}{\partial \sigma}$	Volatility Exposure	Change in option price due to volatility
Rho	$\rho = \frac{\partial V}{\partial r}$	Interest Rate Exposure	Change in option price due to interest rates