

# PyPinT

Towards a framework for rapid prototyping of iterative parallel-in-time algorithms

May 28, 2014 | Torbjörn Klatt, Dieter Moser, Robert Speck | 3rd Workshop on Parallel-in-Time Integration

# Overview

## 1 Recap of existing Parallel-in-Time Approaches

Parareal, (I)DC, (ML)SDC, PFASST

## 2 The *PyPinT* Framework Explained

concepts, design decisions, modules

## 3 Goals of *PyPinT*

creating a unicorn

## 4 Proof of Concept

coloured plots and pictures, yeah!



# PyPinT

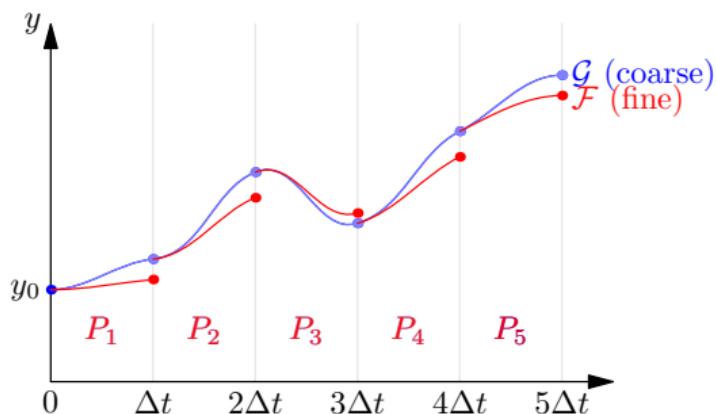
## Part I: Existing Parallel-in-Time Approaches

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# Parareal

Y. Maday. 2008. *The Parareal in Time Algorithm.*

- coarse  $\mathcal{G}$  and fine  $\mathcal{F}$  propagators make parareal flexible and modular
- an initial value is improved iteratively
- order is controllable through the fine propagator  $\mathcal{F}$



[Courtesy of M. Emmett, LBNL]

$$y_{m+1}^{k+1} = \mathcal{F}(y_m^k) + \mathcal{G}(y_m^{k+1}) - \mathcal{G}(y_m^k)$$

What is needed? Two Integrators

# Deferred Corrections

V. Pereyra. 1966. *On Improving an Approximate Solution of a Functional Equation by Deferred Corrections.* Numerische Mathematik. 8(4) 376–391.

$$y'(t) = f(t, y(t)), \quad y(0) = y_0$$

Using the residual

$$r(t) = f(t, \tilde{y}(t)) - \tilde{y}'(t)$$

to compute the error

$$e'(t) = f(t, \tilde{y} + e) - f(t, \tilde{y}) + r'(t)$$

$$e(0) = 0$$

for the next update

$$\tilde{y}_{j+1} = \tilde{y}_j + e_j$$

## Deferred Corrections

and

## Integral Deferred Corrections

A. Dutt, L. Greengard, V. Rokhlin. 2000. *Spectral Deferred Correction Methods for Ordinary Differential Equations*. BIT Numerical Mathematics 40(2) 241–266.

$$y'(t) = f(t, y(t)), \quad y(0) = y_0$$

$$y(t) = y_0 + \int_0^t f(\tau, y(\tau)) d\tau, \quad y(0) = y_0$$

Using the residual

$$r(t) = f(t, \tilde{y}(t)) - \tilde{y}'(t)$$

to compute the error

$$\begin{aligned} e'(t) &= f(t, \tilde{y} + e) - f(t, \tilde{y}) + r'(t) \\ e(0) &= 0 \end{aligned}$$

for the next update

$$\tilde{y}_{j+1} = \tilde{y}_j + e_j$$

Using the residual

$$r(t) = y_0 - \tilde{y}(t) + \int_0^t f(\tau, y(\tau)) d\tau$$

to compute the error

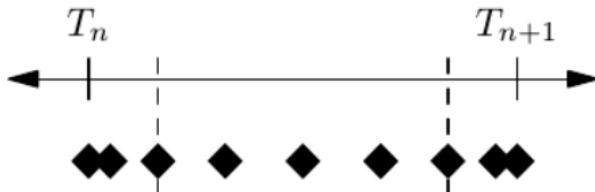
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# Spectral Deferred Corrections

A. Dutt, L. Greengard, V. Rokhlin. 2000. *Spectral Deferred Correction Methods for Ordinary Differential Equations*. BIT Numerical Mathematics 40(2) 241–266.



One sweep is performed by

$$u_{n+1}^{k+1} = u_n^{k+1} + \Delta t [F(u_{n+1}^{k+1}) - F(u_{n+1}^k)] + \mathcal{I}_n^{n+1}(F(\mathbf{u}^k))$$

new main constituent is the quadrature operator

$$\mathcal{I}_n^{n+1}\mathbf{y} = \sum_{i=1}^m \omega_i \tilde{y}_i \approx \int_{t_n}^{t_{n+1}} y(\tau) d\tau$$

What is needed? Quadrature, Evaluation of  $F$  (, Space Solver) → new Integrator

# Multilevel SDC

R. Speck, D. Ruprecht, M. Emmett, M. L. Minion, M. Bolten, R. Krause. 2013. *A Multi-Level Spectral Deferred Correction Method*. arXiv Math Numerical Analysis.

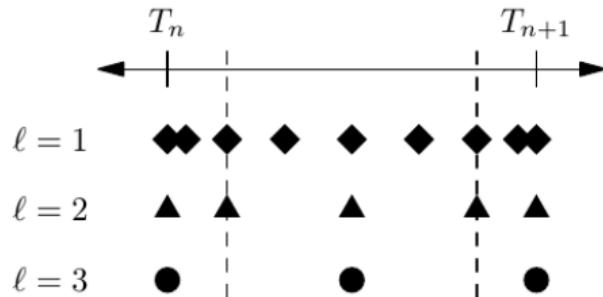
Slightly different sweep

$$u_{n+1}^{k+1} = u_n^{k+1} + \Delta t \left[ F(u_{n+1}^{k+1}) - F(u_{n+1}^k) \right] + \mathcal{I}_n^{n+1} \left( F(\mathbf{u}^k) \right) + \tau_n^k$$

and multiple level, correction  $\tau_n^k$  based on information from different levels

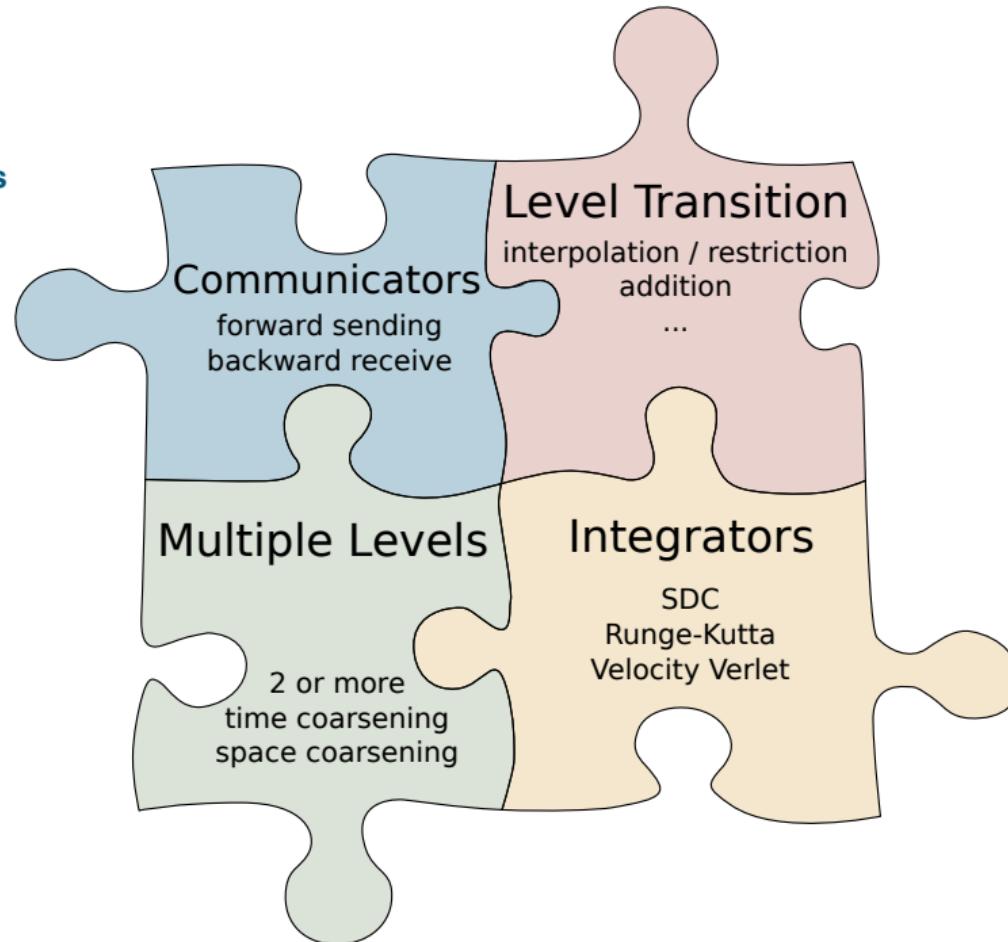
What is needed?

- same as SDC
- **Interpolation** (space and time)
- **Restriction** (space and time)



# PFASST

## Building Blocks





# PyPinT

## Part II: The PyPinT Framework Explained

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## Basic Concept

- *Python ≥3.2* as language of choice
  - for ease of use and extensibility (cf. *NumPy*, *SciPy*)
- modular building blocks
  - for fast exchange of algorithms' building blocks
- well-conceived and intuitive abstract interfaces
  - for reusable code ensuring DRY principle
- integrated analyzation tools
  - for introspection and plotting (cf. *matplotlib*)
- usage of a sophisticated unit-testing framework
  - nobody writes bug-free code

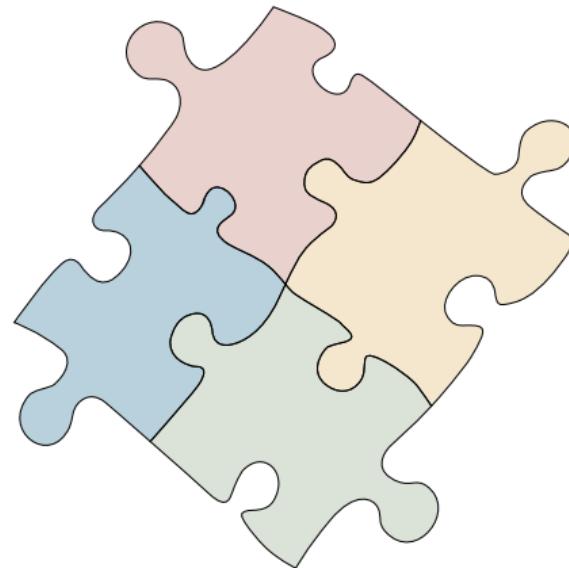


# Modules

## Abstract Modelling of PinT Algorithms

### pypint

- ~.problems
- ~.solvers
- ~.integrators
- ~.communicators
- ~.multi\_level\_providers
- ~.solutions
- ~.plugins

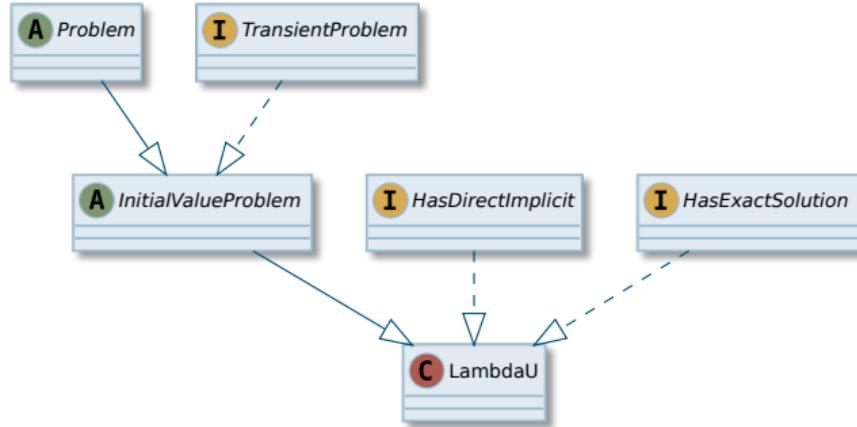


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```

- interfaces for problem setups
- generic problem with specializations via mixins



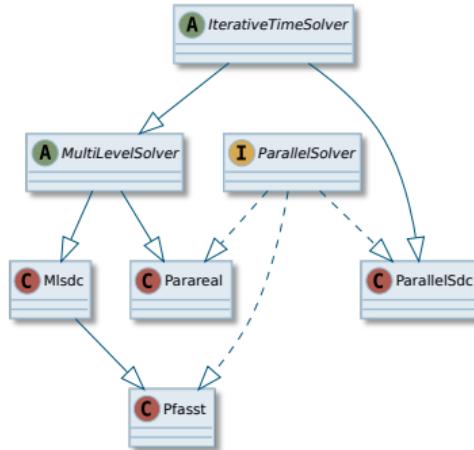
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```

- interfaces for iterative time solvers
- providing generic building blocks of solvers



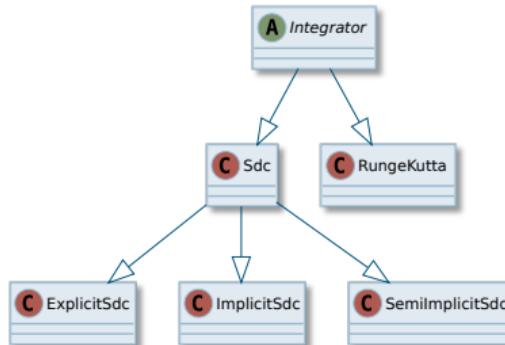
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- abstract interface for integrators of ODEs  
e.g. *Runge-Kutta*, *Expl./Impl. Euler*, *SDC* etc.
- unified generic interface for executing integrator  
via method `.apply(*args, **kwargs)`



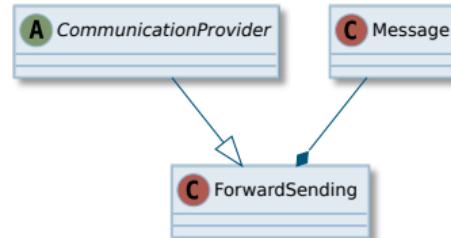
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- generic abstract interfaces for communication patterns  
e.g. implemented as *forward sending*, etc.
- extendable basic message buffer  
holding data, solver flags and other meta information

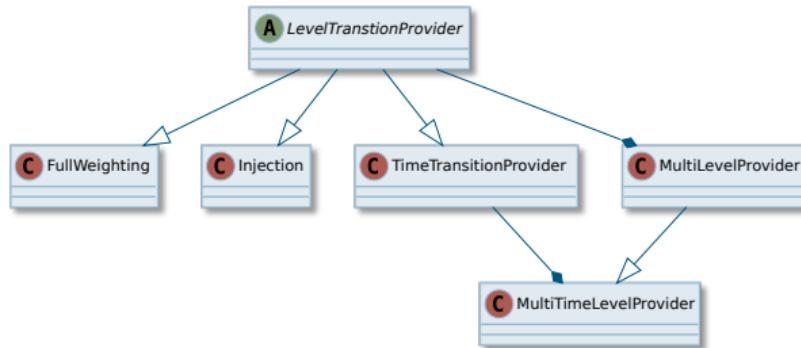


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- abstract interface for generic level transitions  
e.g. *full weighting, injection, Lagrange-polynomial integration* etc.
- containers for multiple levels  
providing access to integrators of each level
- unified generic interface for transitions between levels  
via methods `.restrict(data, fine_id, coarse_id)` and  
`.prolongate(data, coarse_id, fine_id)`



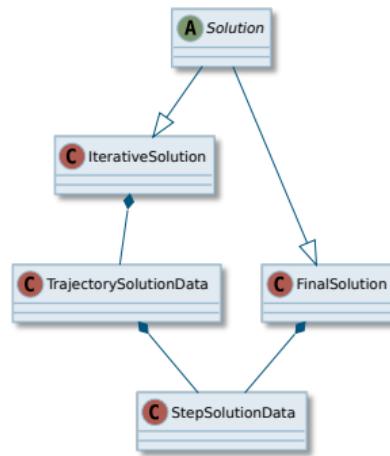
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- containers for solution data + meta information
  - e.g. time-node/step-wise, trajectory (consecutive time nodes)
- container interface for complete solutions
  - e.g. only last time node of last iteration or all nodes of all iterations



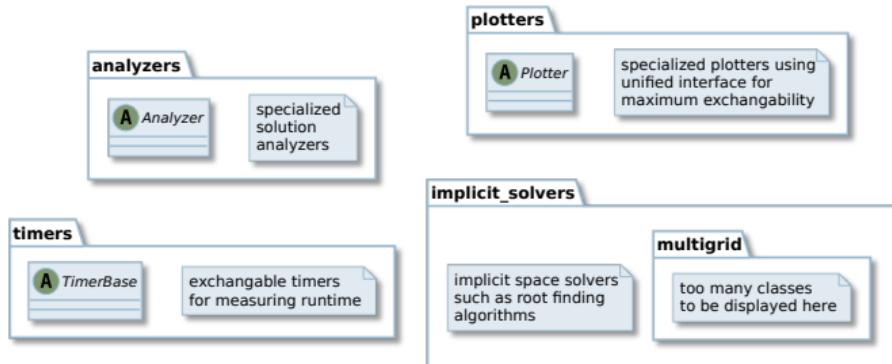
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# PyPinT

## Part III: Goals for PyPinT

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- well-tested and well-documented implementations of PinT algorithms

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⇒ reaching young & rising developers and scientists promoting PinT algorithms



# PyPinT

## Part IV: Proof of Concept

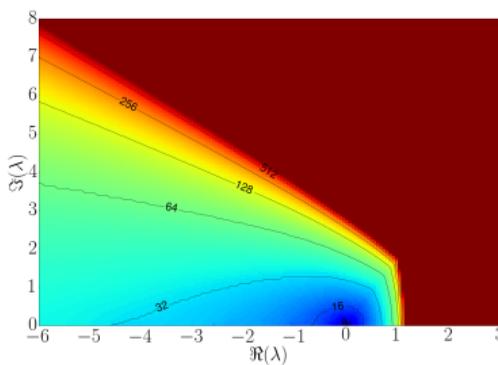
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# Implementation of SDC and MLSDC

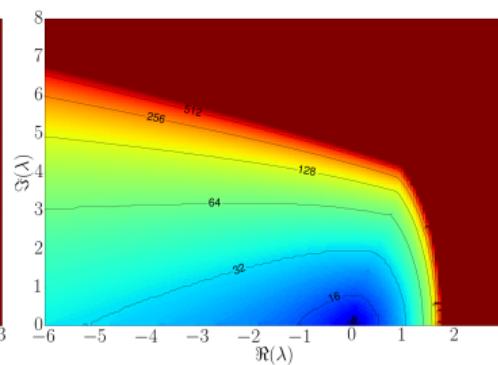
Convergence Regions (number iterations for residual  $\leq 10^{-14}$ ) of  $u'(x, t) = \lambda u(x, t)$ ,  $t \in [0, 1]$ ,  $\lambda \in \mathbb{C}$

SDC

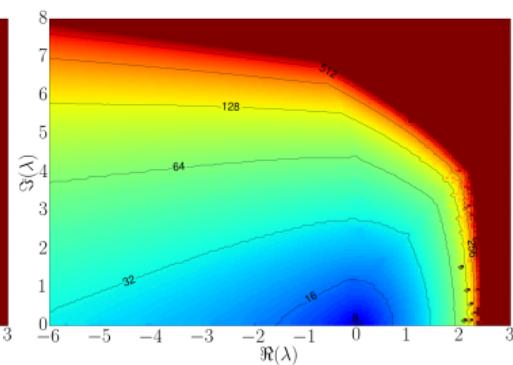
3 Gauss-Lobatto nodes



5 Gauss-Lobatto nodes



7 Gauss-Lobatto nodes



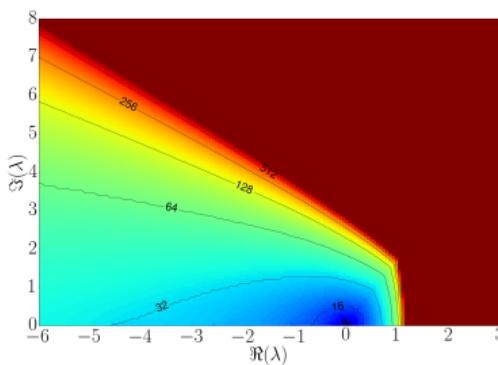
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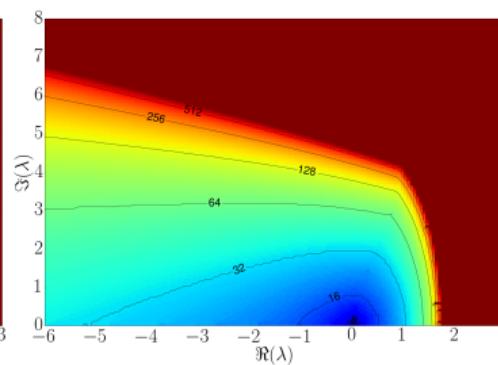
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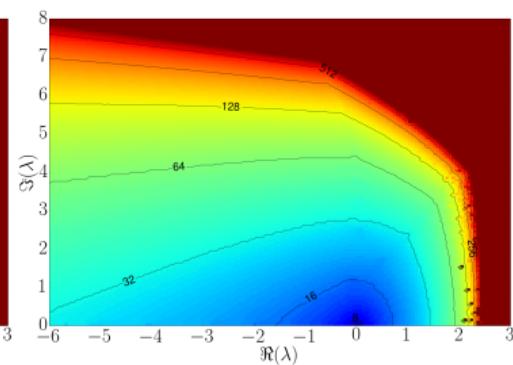
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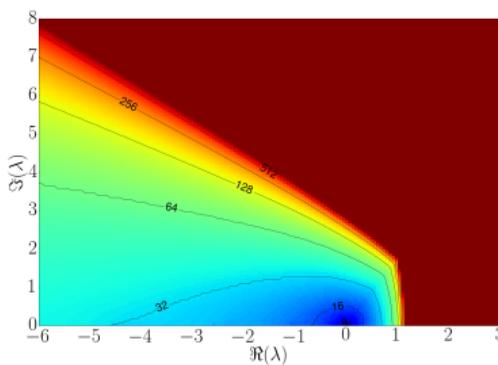


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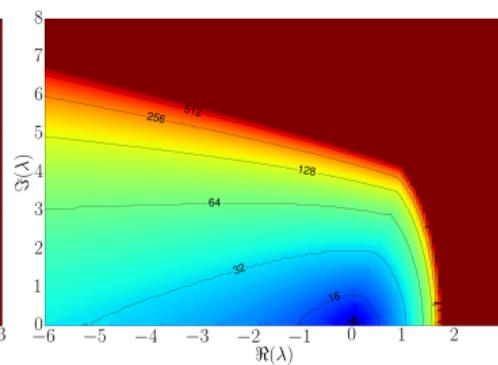
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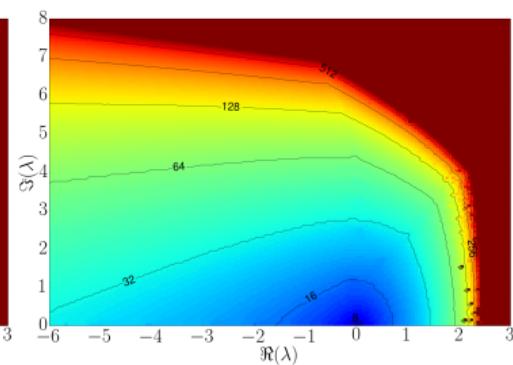
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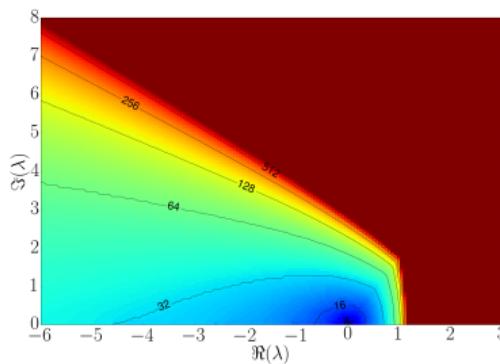


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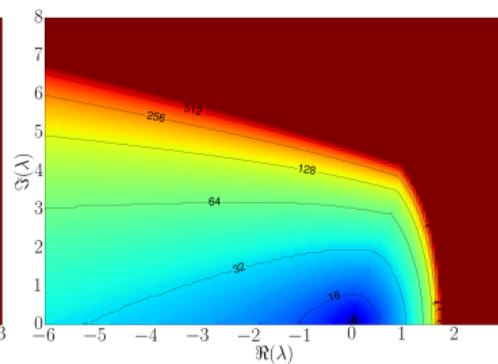
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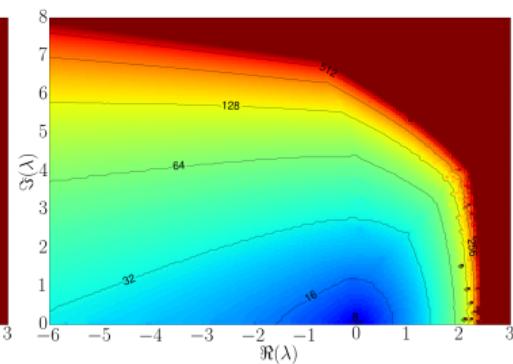
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MLSDC



# Implementation of SDC and MLSDC

Python Script for a SDC run with 3 Gauss-Lobatto nodes

```
1 prob = LambdaU(complex(-1.0, 1.0))
2 comm = ForwardSendingMessaging()
3 solver = Sdc(communicator=comm)
4 solver.init(problem=prob)
5 quadr = QuadratureBase(nodes=GaussLobatto(num_nodes=3),
6                           weights=Polynomial(coeffs=[1]))
7 integr = SemiImplicitSdc(quadrature=quadr)
8 solution = solver.run(integrator=integr)
9 # plotting via matplotlib.pyplot
```

# The Aviles-Giga Problem

An example from *real* life ...

$$u_t = \frac{1}{\varepsilon} \underbrace{\nabla \cdot \left( \left( |\nabla u|^2 - 1 \right) \nabla u \right)}_{\mathcal{N}} - \varepsilon \underbrace{\nabla^4 u}_{\mathcal{L}}$$

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- very stiff linear part  $\mathcal{L}$
- highly nonlinear part  $\mathcal{N}$
- computed on a periodic domain  $\mathbb{T} = (0, 2\pi)^2$

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But what is it good for?

It is a simple model for Pattern Forming Mechanisms in magnetic bulk

# The Aviles-Giga Problem

... for nice plots

# Advantages of *PyPinT*

“towards” ...

- playground for rapid prototyping of PinT algorithms
  - no need to implement e.g. matrix inversion or quadrature on your own  
⇒ saves your precious time
- use of Python's full feature set
  - easy parallelism  
via Python's own (e.g. `multiprocessing`) or 3rd-party modules (e.g. `mpi4py`)
  - built-in portability  
runs on Unix and MacOS (should run on Windows too)
  - interactive computing  
integrates well with IPython

IP[y]: IPython  
Interactive Computing



# Thank you for your attention!

## Questions?

(now or later)

PyPinT is on  GitHub: <https://github.com/Parallel-in-Time/PyPinT>

Torbjörn Klatt  
Juelich Supercomputing Centre  
Building 16.3, Room 022  
Tel: +49 2461 61 96452  
eMail: t.klatt@fz-juelich.de

Dieter Moser  
Juelich Supercomputing Centre  
Building 16.3, Room 022  
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eMail: d.moser@fz-juelich.de

Robert Speck \*  
Juelich Supercomputing Centre  
Building 16.3, Room 131  
Tel: +49 2461 61 1644  
eMail: r.speck@fz-juelich.de

\* corresponding author