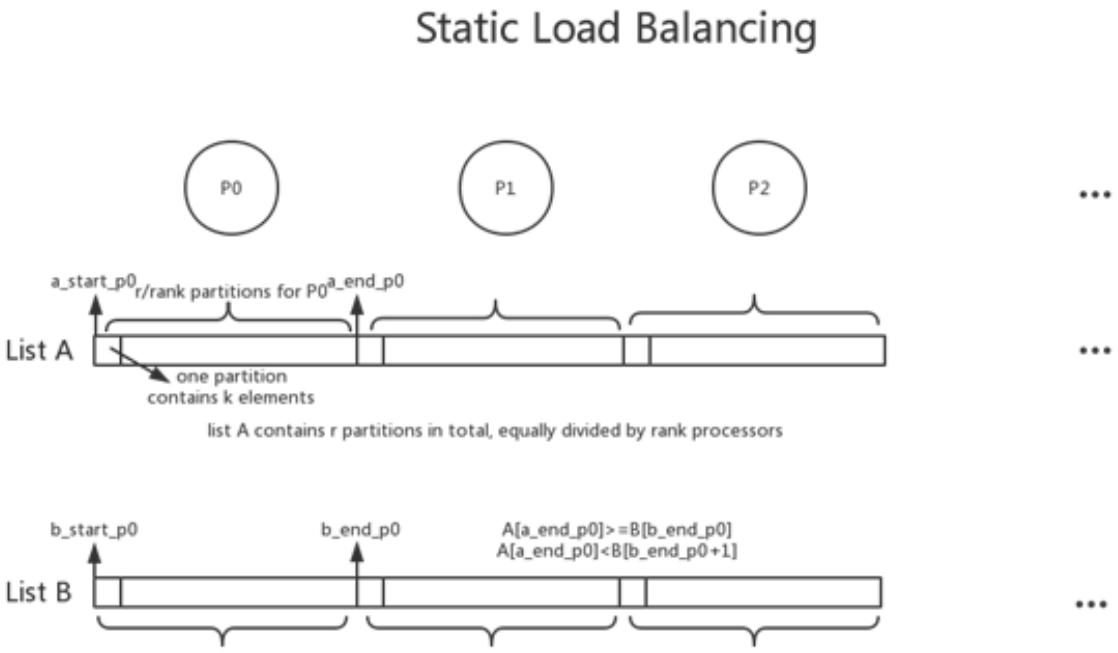
CP 431/631 Assignment 2

By group 2 (Omer Tal, Elizabeth Gorbonos, Tianran Wang)

1. Assignment description:

The assignment is to write the parallel merging algorithm to merge two large randomly generated sorted arrays *A* and *B*.
To accomplish this, we have written a MPI python 3 program (appendix 1).
We assume the arrays are of the same length which is a power of two ($n = 2^k$), *k* is a command line parameter.
First, we calculate the number of partitions $r = \left\lceil \frac{n}{k} \right\rceil$ to split both arrays into. Each partition of array A will consist of *k* elements, besides possibly the last one. We then generate the two random sorted arrays *A* and *B* of length *n* in process 0 and broadcast them to all other processes.
The work division is done by a method of static load balancing, each process calculates the range of partitions in array *A* to works on according to: $r_p = \left\lfloor \frac{r}{P} \right\rfloor + \begin{cases} 1 & if\ p < mod(r,P) \\ 0 & else \end{cases}$.
Each process's range starts from partition: $r_{start,p} = r * \left\lfloor \frac{r}{P} \right\rfloor + min(p, mod(r,P))$.



For each partition A_i we perform a binary search to find the index j_i in array *B* which stores the largest element (in B) that is smaller than the largest element in A_i. j_i is the upper bound of the corresponding partition B_i. The lower bound of B_i is equal to the upper bound of $B_{i-1} + 1$.

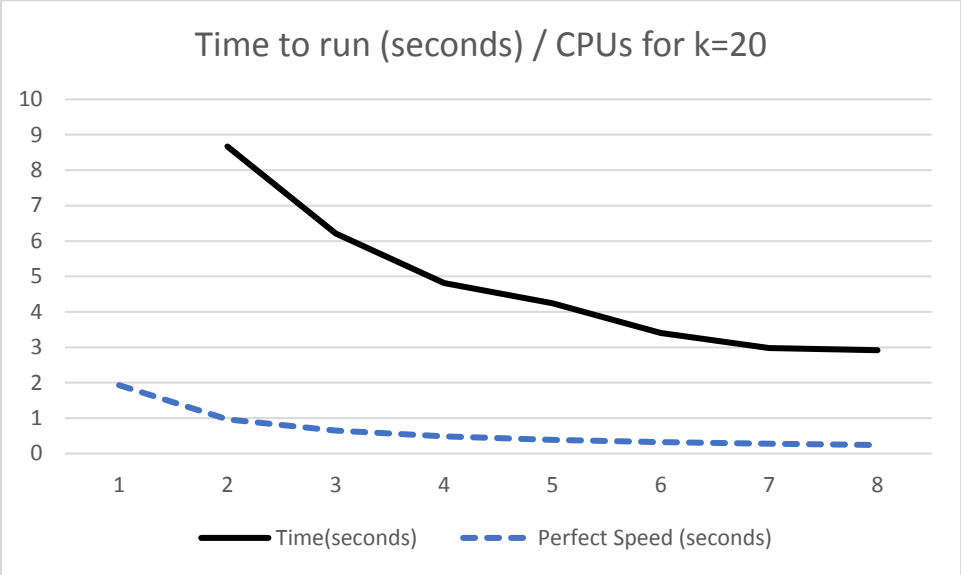
The lower bound of $r_{start,p}$ for all $p > 0$ is received from process p-1 (upper bound +1 of $r_{end,p-1}$).
Every partition is then merged by its process and added to a list in an ascending order. The merged lists are gathered by process 0, resulting in a sorted list of lists.
The final product is tested for correctness and written to a file by process 0.

2. Results:

We benchmarked the parallel section of the program, neglecting the array generation and output. The following benchmarks are based on merging arrays *A* and *B* of size 2^{20} each.

The serial benchmark was recorded for the "serial mode" of the program, no parallel overhead computations were done. We can see the serial program performs better for this task then our parallel version. We believe that a parallel merge algorithm will provide better results in a shared-memory architecture where no communication is required for this task.

Due to the large size of the output files we are only attaching the logs of Orca's sqsub (appendix 2) and a print screen of a sample output. The full output files are available in `/scratch/otwluq1/a2/`.



Data size (n)	Number of CPUs	Time (seconds)	Perfect Speed(seconds)
2^{20}	1	1.93	1.93
	2	8.67	0.97
	3	6.21	0.64
	4	4.82	0.48
	5	4.25	0.39
	6	3.41	0.32
	7	2.98	0.28
	8	2.92	0.24

Appendix 1 : Program code:

```
"""
-------------------------------------------------------------------------------
assingment2.py
A solution in python for CP431/CP631 assignment 2
The program generates 2 n-size  arrays where n is the power of input k in base 2
It then merges the two files using parallel merge and writes the output to a file
Tested for up to k=22 -> two arrays with 4,194,304 32 bit integers each
-------------------------------------------------------------------------------
Authors: Elizabeth Gorbonos, Omer Tal, Tianran Wang
-------------------------------------------------------------------------------
"""

import sys
import datetime
import math
import os
import random
import numpy as np
from mpi4py import MPI

def binary_search(array,i,j,value):
    """
    -------------------------------------------------------------------------------
    Performing a binary search for the biggest item smaller then value on array
    -------------------------------------------------------------------------------
    Preconditions:
        array - a list of data
        i - start index for the search
        j - end index for the search
        value - the item to find
    Postconditions:
        returning the index of the closest item to value, smaller or equal
    -------------------------------------------------------------------------------
    """
    # Stop condition for when the range is up to 2 values
    if (j-i<=1):
        # If the value is lower than the minimum value, return the index before it
        if (value<array[i]):
            return i-1;
        # If the value is higher than the small value but lower than the higher
        elif(value<=array[j]):
            return i;
        # The value is higher than the two indexes
        else:
            return j;
    # Comparing to the item found in the middle of the range
    k=i+math.floor((j-i)/2)
    if (value<=array[k]):
        # Recursion call for the lower half of the range
        return binary_search(array,i,k,value)
    else:
        # Recursion call for the higher half of the range
        return binary_search(array,k+1,j,value)
```

```
def get_input(rank,comm,n):
    """
    ------------------------------------------------------
    Generating two lists of random n numbers
    ------------------------------------------------------
    Preconditions:
        rank - the id of the current processsor
        comm - MPI communicator instance
    Postconditinos:
        a - sorted list of random items in size n
        b - sorted list of random items in size n
    ------------------------------------------------------------
    """
    # Maximum number to generate
    LIM=3999999999
    # Generate the two lists
    a = generate_randoms(n,LIM)
    b = generate_randoms(n,LIM)

    return a,b

def generate_randoms(n,lim):
    """
    ------------------------------------------------------------------
    Generating a list of n random numbers between 0 and lim
    ------------------------------------------------------------------
    Preconditions:
            n - number of items to generate
            lim - highest value to generate
    Postconditions:
            returns a sorted list of n random items between 0-lim
    ------------------------------------------------------------------
    """
    a=np.empty(n,dtype=np.uint32)
    increase=int(lim/n)
    last_value=1
    # Generate each new number as a random between the previous_value
    # and a relative increase to ensure a sorted order
    for i in range(n):
        a[i] = random.randint(0,increase) + last_value
        last_value = a[i]
    return a

def merge(a,b,a_start,a_end,b_start,b_end):
    """
    ------------------------------------------------------------------------
    Performing a merge of two local sorted lists
    ------------------------------------------------------------------------
    Preconditions:
        a,b - two sorted lists to be merged
        a_start,b_start - the indexes of a and b to start merging from
        a_end, b_end - the indexes of a and b to stop merging at
    Postconditions:
        merged and sortedlist c in size (a_end-a_start) + (b_end-b_start)
    ------------------------------------------------------------------------
    """
    c = []
    # As long as we haven't reached to the end of the range for either of the lists
    # we continue to merge the smallest item available
    while (a_end>=a_start and b_end>=b_start):
        if (a[a_start]<=b[b_start]):
            c.append(a[a_start])
```

```
            a_start+=1
        else:
            c.append(b[b_start])
            b_start+=1

    # Merging the tail of list a when list b is already merged
    while (a_end>=a_start):
        c.append(a[a_start])
        a_start+=1

    # Merging the tail of list b when list a is already merged
    while (b_end>=b_start):
        c.append(b[b_start])
        b_start+=1

    return c

def output_file(array,filename):
    """
    ------------------------------------------------------------------------------------
------
    Write the two generated lists and the merged list into a given output file name
    ------------------------------------------------------------------------------------
------
    Preconditions:
        array - merged list, given as an array of arrays of arrays
        (first array - each process, second array - each partition, third array - val-
ues)
        filename - the output file directory
    Postconditions:
        List in array are written to the file named filename
    ------------------------------------------------------------------------------------
------
    """
    # Wrtie to fle with  a buffer of 20MB
    file = open(filename,'w',20000)
    file.write("Merged list:")
    # Iterator over all processes
    for process in range(len(array)):
        # Iterate over all partitions
        for partition in range(len(array[process])):
            # Iterate over items
            for item in range(len(array[process][partition])):
                if (process==0 and partition==0 and item==0):
                    prefix=""
                else:
                    prefix=","
                file.write("{}{}".format(prefix,array[process][partition][item]))
    file.close()
    print("Sucessfully wrote results to file {}".format(filename))

def test(a,b,c):
    """
    ------------------------------------------------------------------
    Testing the function by comparing lists a and b to the merged list
    ------------------------------------------------------------------
    """
    index_a = 0
    index_b = 0
    count_c = 0
    for arr_process in c:
        for arr_partition in arr_process:
```

```
            for item in arr_partition:
                if (index_a<len(a) and a[index_a] == item):
                    index_a += 1
                elif (index_b<len(b) and b[index_b] == item):
                    index_b +=1
                else:
                    print("discrepancy found when comparing the two lists to the
third. a size={}, b size={}, c size={}, c_value={}"
                          .format(len(a),len(b),count_c,item))
                    return False
                count_c+=1
    if (index_a == len(a) and index_b==len(b) and count_c==len(a) + len(b)):
        print("Tested and found correct")
        return True
    else:
        print("Discrepancy found when comparing the lists length. a size={}, b
size={}, c size={}".format(len(a),len(b),count_c))
        return False

def main():
    # Get process size and rank
    comm = MPI.COMM_WORLD
    size = comm.Get_size()
    rank = comm.Get_rank()
    # Get the number k representing a power of 2 from the user
    if (len(sys.argv)<2 or not sys.argv[1].isdigit()):
        print("Please supply the program with the number of items")
        exit(1)
    k = int(sys.argv[1])
    n = 2**k
    r = int (n/k)
    # When n is not divided by k, increase the number of partitions by 1
    if (n%k>0):
        r+=1
    #Barrier to measure start time afte input
    comm.Barrier()
    if (rank==0):
        # Get the two random arrays
        a,b = get_input(rank,comm,n)
        print("Done with input stage")
        wt = MPI.Wtime()
    else:
        a = np.empty(n,dtype=np.uint32)
        b = np.empty(n,dtype=np.uint32)
    # Broadcasting the two random lists to size-1 processes
    comm.Bcast(a,root=0)
    comm.Bcast(b,root=0)
    # Dividing the partitions between the processes
    if(rank<(r%size)):
        partitions = int(math.floor(r/size))+1
    else:
        partitions = int(math.floor(r/size))

    # Calculate the range dealt by the current process by multiplying the process id
and it's k partition
    partitions_start = int(rank*math.floor(r/size) + min(rank,r%size))
    partitions_end = partitions_start+partitions-1

    #print("Process {} has {} partitions: {}-{}".format(rank,partitions,parti-
tions_start,partitions_end))
```

```
    if size > 1:
        a_start = []
        a_end = []
        b_start = []
        b_end = []

        # For each of the partitions, setting the values for the index in a and the
last item to partition in b
        for index in range(partitions):
            a_start.insert(index,int(partitions_start*k + k*index))
            # For the last partition, the highest item in b is the last
            if (a_start[index]+k<n):
                a_end.insert(index,int(a_start[index] + k-1))
                # Finding the local upper limit in list b, by using binary search to
find the minimum value bigger then the maximum local value in a
                index_b = binary_search(b,0,len(b)-1,a[a_end[index]])
                b_end.insert(index,index_b)
            else:
                a_end.insert(index,n-1)
                b_end.insert(index,n-1)
            # The index to start iterating over b is 1 after the last index of the
previous partition
            if (index>0):
                b_start.insert(index,b_end[index-1]+1)

        # Sending the process last found b_end to be the neighbor's first b_start, for
all but the last process
        if (rank<size-1):
            comm.send(b_end[partitions-1]+1,dest=(rank+1),tag=1)

        # Process 0 starts from index 0 in b
        if (rank==0):
            b_start.insert(0,0)
        # All other processes receive the first position from their left neighbor
        else:
            b_start.insert(0,comm.recv(source=(rank-1),tag=1))

        # Array of merged lists
        merged_lists=[]

        # Merging each partition using a loop in ascending order
        for index in range(partitions):
            # Merging lists a and b in relevant positions into a local list c

merged_lists.append(merge(a,b,a_start[index],a_end[index],b_start[index],b_end[index])
)

        # Process 0 gathers all merged lists into a new list merged, ordered by
ranking index
        merged = comm.gather(merged_lists,root=0)
    # Serial mode
    else:
        merged = [[merge(a, b, 0, n - 1, 0, n - 1)]]

    # Process 0 now has the merged list and needs to write it to input
    if (rank==0):
        # Print the total time the program took before writing to output
```

```
    print("Total time to compute: {:2f} seconds".format(MPI.Wtime() - wt))
        # Testing if the merged list is equal to the merged input lists serially
        test(a,b,merged)
        # Writing to output file
        output_file(merged,"/scratch/otwluq1/a2/output_{}.txt".format(os.getpid()))


main()
```

Appendix 2: Program logs

```
1 Processor (Serial):
Done with input stage
Total time to compute: 1.936928 seconds
Tested and found correct
Sucessfully wrote results to file /scratch/otwluq1/a2/output_28409.txt
 ---SharcNET Job Epilogue---
              job id: 10946869
         exit status: 0
            cpu time: 29s / 600s (4(%
        elapsed time: 30s / 600s (5(%
      virtual memory: 691.2M / 2.0G (33(%

Job completed successfully

--------------------------------------------------------------------------------
2 Processor:

Done with input stage
Total time to compute: 8.667797 seconds
Tested and found correct
Sucessfully wrote results to file /scratch/otwluq1/a2/output_12098.txt
--- SharcNET Job Epilogue ---
              job id: 10927619
         exit status: 0
            cpu time: 50s / 2.0h (0 %)
        elapsed time: 33s / 1.0h (0 %)
      virtual memory: 784.9M / 1.0G (76 %)

Job completed successfully

--------------------------------------------------------------------------------
3 Processor:

Done with input stage
Total time to compute: 6.214778 seconds
Tested and found correct
Sucessfully wrote results to file /scratch/otwluq1/a2/output_12165.txt
--- SharcNET Job Epilogue ---
              job id: 10927620
         exit status: 0
            cpu time: 64s / 3.0h (0 %)
        elapsed time: 32s / 1.0h (0 %)
      virtual memory: 263.3M / 1.0G (25 %)

Job completed successfully

--------------------------------------------------------------------------------
4 Processor:

Done with input stage
Total time to compute: 4.816678 seconds
Tested and found correct
Sucessfully wrote results to file /scratch/otwluq1/a2/output_11544.txt
--- SharcNET Job Epilogue ---
              job id: 10927621
         exit status: 0
            cpu time: 79s / 4.0h (0 %)
        elapsed time: 32s / 1.0h (0 %)
      virtual memory: 401.4M / 1.0G (39 %)

Job completed successfully

--------------------------------------------------------------------------------
5 Processor:
```

```
Done with input stage
Total time to compute: 4.247634 seconds
Tested and found correct
Sucessfully wrote results to file /scratch/otwluq1/a2/output_9748.txt
--- SharcNET Job Epilogue ---
              job id: 10927622
         exit status: 0
            cpu time: 104s / 5.0h (0 %)
        elapsed time: 31s / 1.0h (0 %)
      virtual memory: 520.8M / 1.0G (50 %)

Job completed successfully

--------------------------------------------------------------------------------
6 Processor:
Done with input stage
Total time to compute: 3.405965 seconds
Tested and found correct
Sucessfully wrote results to file /scratch/otwluq1/a2/output_27651.txt
--- SharcNET Job Epilogue ---
              job id: 10927623
         exit status: 0
            cpu time: 105s / 6.0h (0 %)
        elapsed time: 24s / 1.0h (0 %)

Job completed successfully


--------------------------------------------------------------------------------
7 Processor:

Done with input stage
Total time to compute: 2.980946 seconds
Tested and found correct
Sucessfully wrote results to file /scratch/otwluq1/a2/output_27727.txt
--- SharcNET Job Epilogue ---
              job id: 10927624
         exit status: 0
            cpu time: 121s / 7.0h (0 %)
        elapsed time: 28s / 1.0h (0 %)
      virtual memory: 613.4M / 1.0G (59 %)

Job completed successfully


--------------------------------------------------------------------------------
8 Processor:

Done with input stage
Total time to compute: 2.916491 seconds
Tested and found correct
Sucessfully wrote results to file /scratch/otwluq1/a2/output_27803.txt
--- SharcNET Job Epilogue ---
              job id: 10927625
         exit status: 0
            cpu time: 136s / 8.0h (0 %)
        elapsed time: 27s / 1.0h (0 %)
      virtual memory: 117.2M / 1.0G (11 %)

Job completed successfully
```

Output File Sample:

```
Merged
list:802,950,1012,3295,3430,4026,6657,7278,7891,10220,11359,11427,12965,1500
7,16044,16175,16789,18775,19774,19950,22158,22696,22951,23840,25272,25463,26
649,27313,27481,30381,30810,32423,33121,33669,34945,35002,35638,36621,38767,
39054,40158,41042,41404,41858,44120,45388,46622,48725,49932,50497,51247,5138
2,52547,52653,53323,53507,54911,55025,57623,58564,58713,60287,60590,61063,61
970,63203,64322,64374,64979,65016,66697,67909,68899,70219,72064,72555,72965,
73927,74114,77487,77514,79448,81103,82368,82939,82958,83078,84027,85199,8570
3,87684,88147,88683,88774,90869,90983,91295,92812,93939,94035,97167,97429,98
142,98233,98735,101301,101535,102170,103712,103731,106961,107126,107187,1074
28,109393,110753,111013,111577,114148,114382,114814,116542,116728,116730,118
864,119019,119872,120255,121903,122446,123211,124200,125326,126204,126366,12
7587,127667,128018,129509,131232,132822,133521,134442,137261,137856,138505,1
38844,139124,140151,141322,142720,143290,145254,146441,147870,148596,151281,
151399,154466,154852,156312,156415,158042,158344,160275,160710,162185,163767
,163854,164400,165129,166319,167047,167687,168170,168422,169701,170446,17253
8,172664,172723,173339,173761,174940,175118,177701,177769,179412,179505,1830
72,183190,185233,185971,186225,187542,188320,188424,191238,191414,192289,193
335,194709,195905,196638,197052,198268,199145,199848,200081,201727,202186,20
3668,204679,206324,208304,208329,208959,211463,212271,213845,214831,215124,2
```