Constraint satisfaction problems

Constraint satisfaction problems (CSPs)

- In a standard search problem:
  - State is a "black box“ to the search algorithm – it is not aware of the internal structure of the states
  - Internal data structure of states can only be accessed by problem-specific functions
    - Successor function, heuristic function and goal test
- CSP:
  - States and goal test of a CSP conforms to a standard, structured and a simple representation
    - This allows search algorithms to take advantage of the structure of states and use general-purpose heuristics instead of problem specific ones.

CSP Definition

- A constraint satisfaction problem (CSP) is defined by {V,D,C} where

V={V1,V2,V3,…..,Vn} is a set of variables

D={D1,D2,…..,Dn} is the set of domains, Di is the domain of possible values for variable Vi

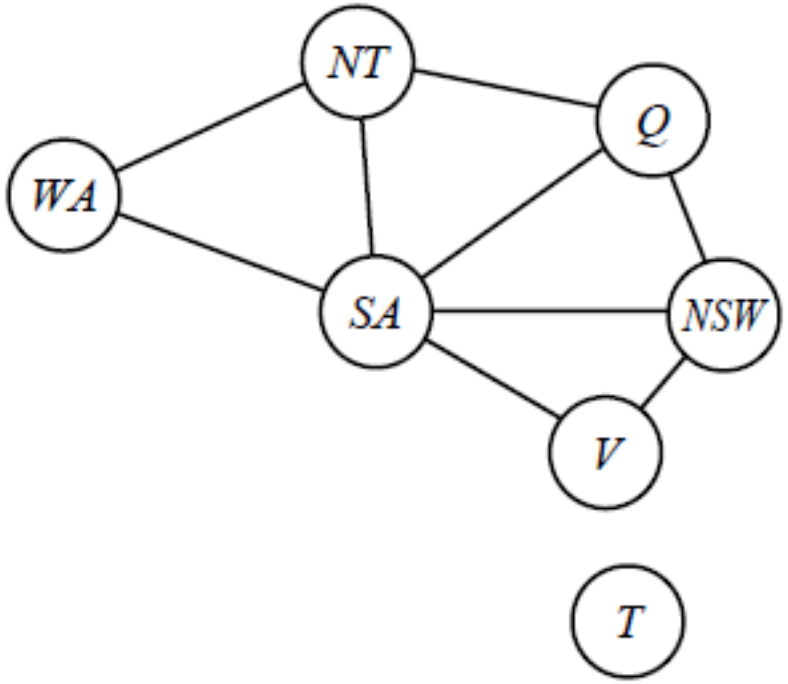
C={C1,C2,…,Cm} is the set of constraints

A solution to a CSP is a complete and consistent assignment, I.e. one that has all variables assigned with values, and satisfies all the constraints

Examples

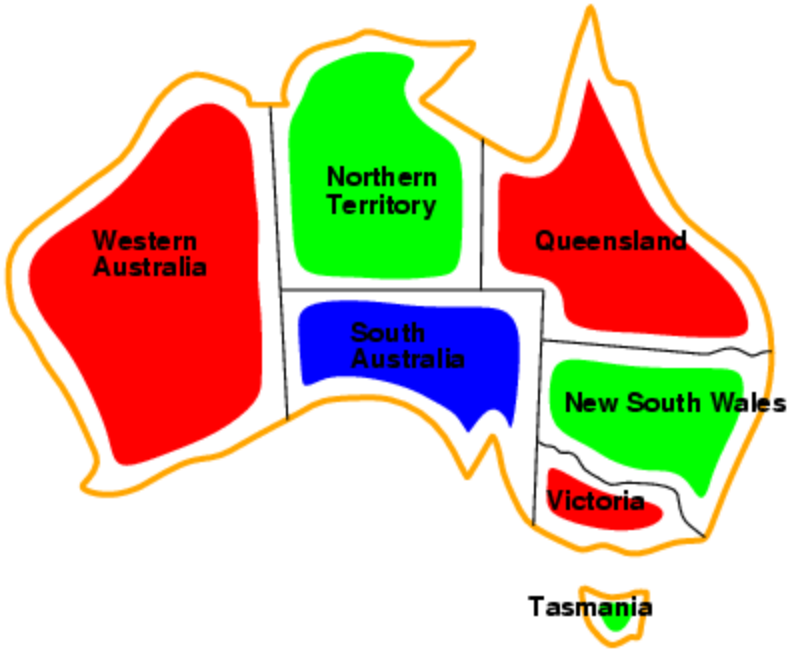
- N-Queens: Place N queens on an N×N board such that none can take any other
- Cryptarithmetic problems: SEND + MORE = MONEY
- Map coloring (no two neighboring countries the same color)

Example: Map-Coloring



- Variables *WA, NT, Q, NSW, V, SA, T*
- Domains D_i = {red, green, blue}
- Constraints: adjacent regions must have different colors
  - e.g., WA ≠ NT, Q ≠ NW, … etc.
  - legal values under the constraint WA ≠ NT are;
    (WT,NT) ϵ {(red,green), (red,blue), (green,red), (green,blue), (blue,red), (blue,green)}

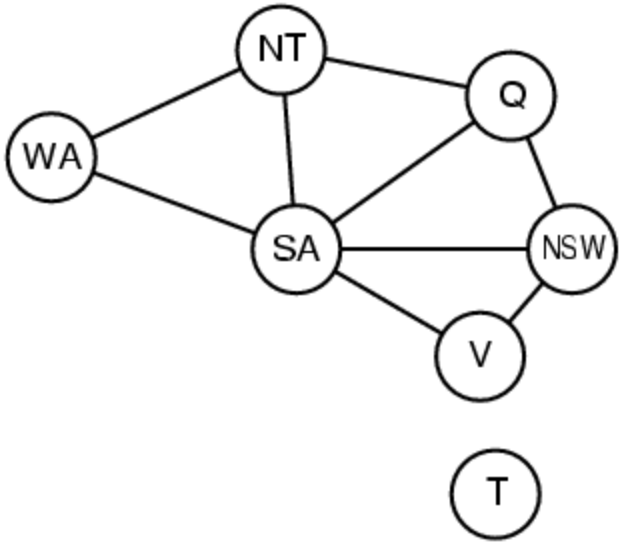
Map-Coloring



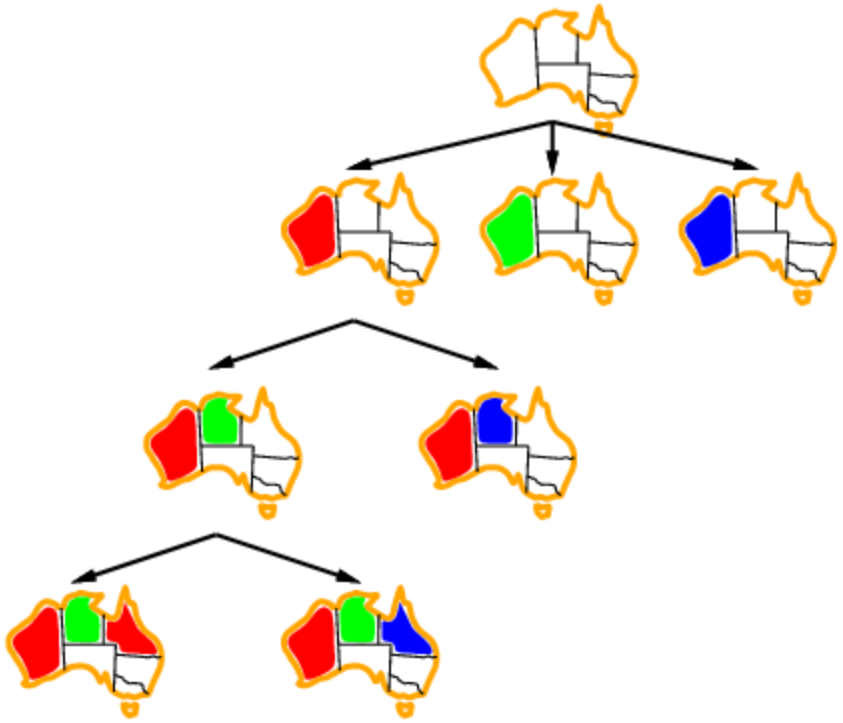
- Solutions are complete and consistent assignments, e.g., WA = red, NT = green,Q = red,NSW = green,V = red,SA = blue,T = green

Constraint graph

- Binary CSP: each constraint relates two variables
- Constraint graph: nodes are variables, arcs are constraints



Backtracking example



Varieties of constraints

- Unary constraints involve a single variable,
  - e.g., SA ≠ green
- Binary constraints involve pairs of variables,
  - e.g., SA ≠ WA
- Higher-order constraints involve 3 or more variables,
  - e.g., cryptarithmetic column constraints

Cryptarithmetic problems

SEND

MORE

MONEY

N-Queens