Q 1.1

Given the two cameras are at origin, we can write the matrices as

$$C_1 = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} \; C_2 = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}$$

Given two points X_1 and X_2, we can relate them using the fundamental matrix in the form:

$$x_2^T * F * x_1 = 0$$

If the coordinate origin coincides with the principle point, we can substitute the above equation as:

$$[0 \quad 0 \quad 1] * \begin{bmatrix} F_{11} & F_{12} & F_{13} \\ F_{21} & F_{22} & F_{23} \\ F_{31} & F_{32} & F_{33} \end{bmatrix} * \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} = 0$$

Upon simplifying the above equation, we get:

$$F_{33} = 0$$

Q 1.2

We are given that the only motion is translation in the X direction, hence the rotation and intrinsic parameters remain the same between the cameras. Therefore, the R matrix is Identity.

We know that the essential matrix (E) can be written as E = T x R. we can write this as

$$E = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & -x \\ 0 & x & 0 \end{bmatrix} * I$$

Where x is the translation in the x direction.

For a point X_1 in Camera 1 and its corresponding X_2 in Camera 2, the epipolar line L_2 in Camera 2 corresponding to X_1 in Camera 1 is given by $L_2 = E * X_1$. Similarly, the epipolar line L_1 in Camera 1 corresponding to X_2 in Camera 2 is given by $L_1 = E^T * X_2$

On multiplying the two equations, we can see that the slope of the line will remain the same since E is skew-symmetric, and hence we can say that the lines will be parallel.

Q 1.3

We can write the relative rotation as:

$$R_{relative} = R_{i+1} \times R_i^T$$

And the relative translation as:

$$T_{relative} = R_{i+1} \times R_i^T \times t_i + t_{i+1}$$

Given the intrinsic matrix of the camera K, we can write the essential matrix as:

$$E = T_{relative} \times R$$

We can hence define the fundamental matrix as:

$$F = K^{-T} \times E \times K^{-1}$$

Q 1.4

The mirror reflection essentially flips the image across the image plane, due to which, we can write the translation vector between the camera and the reflected image as:

$$T_{relative} = [t_x, t_y, t_z]$$

We know that

$$E = T_{relative} \times R$$

And

$$F = K^{-T} \times E \times K^{-1}$$

Therefore, we can write F as:

$$F = \begin{bmatrix} 0 & -t_z & t_y \\ t_z & 0 & -t_x \\ -t_y & t_x & 0 \end{bmatrix}$$

Q 2.1

The recovered F matrix is:

```
[[ 9.78833288e-10 -1.32135929e-07  1.12585666e-03]
 [-5.73843315e-08  2.96800276e-09 -1.17611996e-05]
 [-1.08269003e-03  3.04846703e-05 -4.47032655e-03]]
```

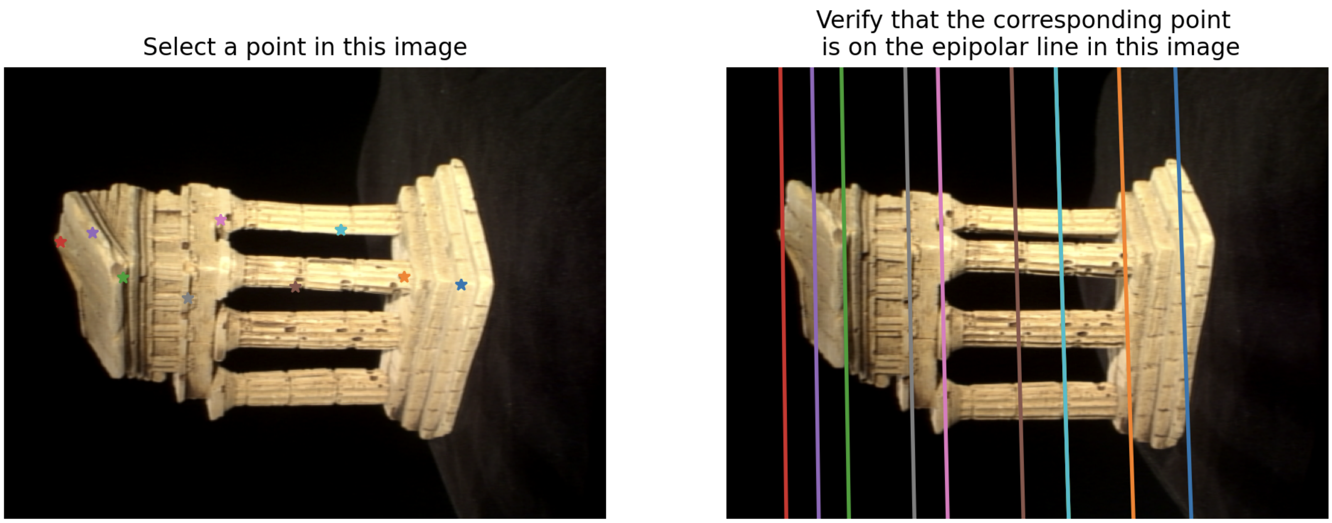

Figure: sample output from displayEpipolarF

Q 3.2

We know that C.W = λ x, Where C is a 3x4 camera matrix, X represents the point in 2D, and W represents the point in 3D. We can re-write this as:

$$\begin{bmatrix} C_{11} & C_{12} & C_{13} & C_{14} \end{bmatrix} * W = \lambda * X$$
$$\begin{bmatrix} C_{21} & C_{22} & C_{23} & C_{24} \end{bmatrix} * W = \lambda * Y$$
$$\begin{bmatrix} C_{31} & C_{32} & C_{33} & C_{34} \end{bmatrix} * W = \lambda * 1$$

Substituting the value of λ,

$$\begin{bmatrix} C_{11} & C_{12} & C_{13} & C_{14} \end{bmatrix} * W = \begin{bmatrix} C_{31} & C_{32} & C_{33} & C_{34} \end{bmatrix} * W * X$$
$$\begin{bmatrix} C_{21} & C_{22} & C_{23} & C_{24} \end{bmatrix} * W = \begin{bmatrix} C_{31} & C_{32} & C_{33} & C_{34} \end{bmatrix} * W * Y$$

Simplifying this further, we get:

$$\begin{bmatrix} \mathrm{C2}_{11} & \mathrm{C2}_{12} & \mathrm{C2}_{13} & \mathrm{C2}_{14} \end{bmatrix} * \mathrm{W2} = \begin{bmatrix} \mathrm{C2}_{31} & \mathrm{C2}_{32} & \mathrm{C2}_{33} & \mathrm{C2}_{34} \end{bmatrix} * \mathrm{W2} * \mathrm{X2}$$
$$\begin{bmatrix} \mathrm{C2}_{21} & \mathrm{C2}_{22} & \mathrm{C2}_{23} & \mathrm{C2}_{24} \end{bmatrix} * \mathrm{W2} = \begin{bmatrix} \mathrm{C2}_{31} & \mathrm{C2}_{32} & \mathrm{C2}_{33} & \mathrm{C2}_{34} \end{bmatrix} * \mathrm{W2} * \mathrm{Y2}$$

Similarly, for a different set of points W2 and the corresponding x2, we can write

$$\left(\begin{bmatrix} C_{11} & C_{12} & C_{13} & C_{14} \end{bmatrix} - \begin{bmatrix} C_{31} & C_{32} & C_{33} & C_{34} \end{bmatrix} * X\right) * W = 0$$
$$\left(\begin{bmatrix} C_{21} & C_{22} & C_{23} & C_{24} \end{bmatrix} - \begin{bmatrix} C_{31} & C_{32} & C_{33} & C_{34} \end{bmatrix} * Y\right) * W = 0$$

$$\left(\begin{bmatrix} \mathrm{C2}_{11} & \mathrm{C2}_{12} & \mathrm{C2}_{13} & \mathrm{C2}_{14} \end{bmatrix} - \begin{bmatrix} \mathrm{C2}_{31} & \mathrm{C2}_{32} & \mathrm{C2}_{33} & \mathrm{C2}_{34} \end{bmatrix} * \mathrm{X2}\right) * \mathrm{W2} = 0$$
$$\left(\begin{bmatrix} \mathrm{C2}_{21} & \mathrm{C2}_{22} & \mathrm{C2}_{23} & \mathrm{C2}_{24} \end{bmatrix} - \begin{bmatrix} \mathrm{C2}_{31} & \mathrm{C2}_{32} & \mathrm{C2}_{33} & \mathrm{C2}_{34} \end{bmatrix} * \mathrm{Y2}\right) * \mathrm{W2} = 0$$

Therefore, we can write our A matrix as:

A =

$$\begin{bmatrix} \left(\begin{bmatrix} C_{11} & C_{12} & C_{13} & C_{14} \end{bmatrix} - \begin{bmatrix} C_{31} & C_{32} & C_{33} & C_{34} \end{bmatrix} * X\right) \\ \left(\begin{bmatrix} C_{21} & C_{22} & C_{23} & C_{24} \end{bmatrix} - \begin{bmatrix} C_{31} & C_{32} & C_{33} & C_{34} \end{bmatrix} * Y\right) \\ \left(\begin{bmatrix} \mathrm{C2}_{11} & \mathrm{C2}_{12} & \mathrm{C2}_{13} & \mathrm{C2}_{14} \end{bmatrix} - \begin{bmatrix} \mathrm{C2}_{31} & \mathrm{C2}_{32} & \mathrm{C2}_{33} & \mathrm{C2}_{34} \end{bmatrix} * \mathrm{X2}\right) \\ \begin{bmatrix} \mathrm{C2}_{21} & \mathrm{C2}_{22} & \mathrm{C2}_{23} & \mathrm{C2}_{24} \end{bmatrix} - \begin{bmatrix} \mathrm{C2}_{31} & \mathrm{C2}_{32} & \mathrm{C2}_{33} & \mathrm{C2}_{34} \end{bmatrix} \end{bmatrix}$$

Q 4.1

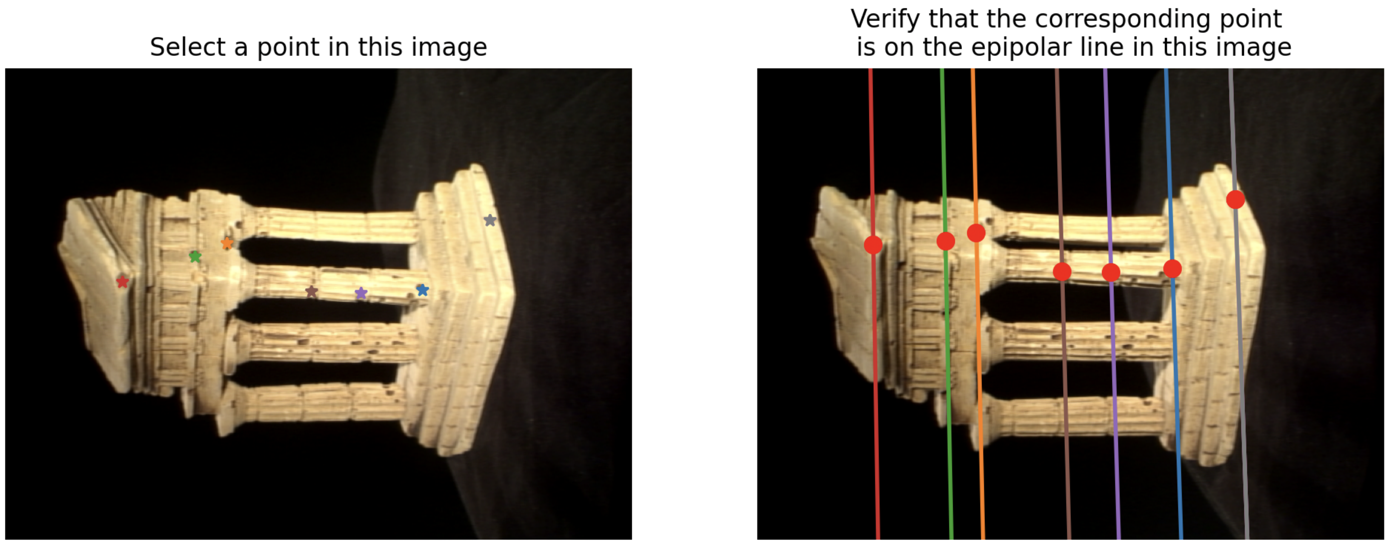

Figure: sample output from epipolarMatchGUI with some detected correspondences

Q4.2

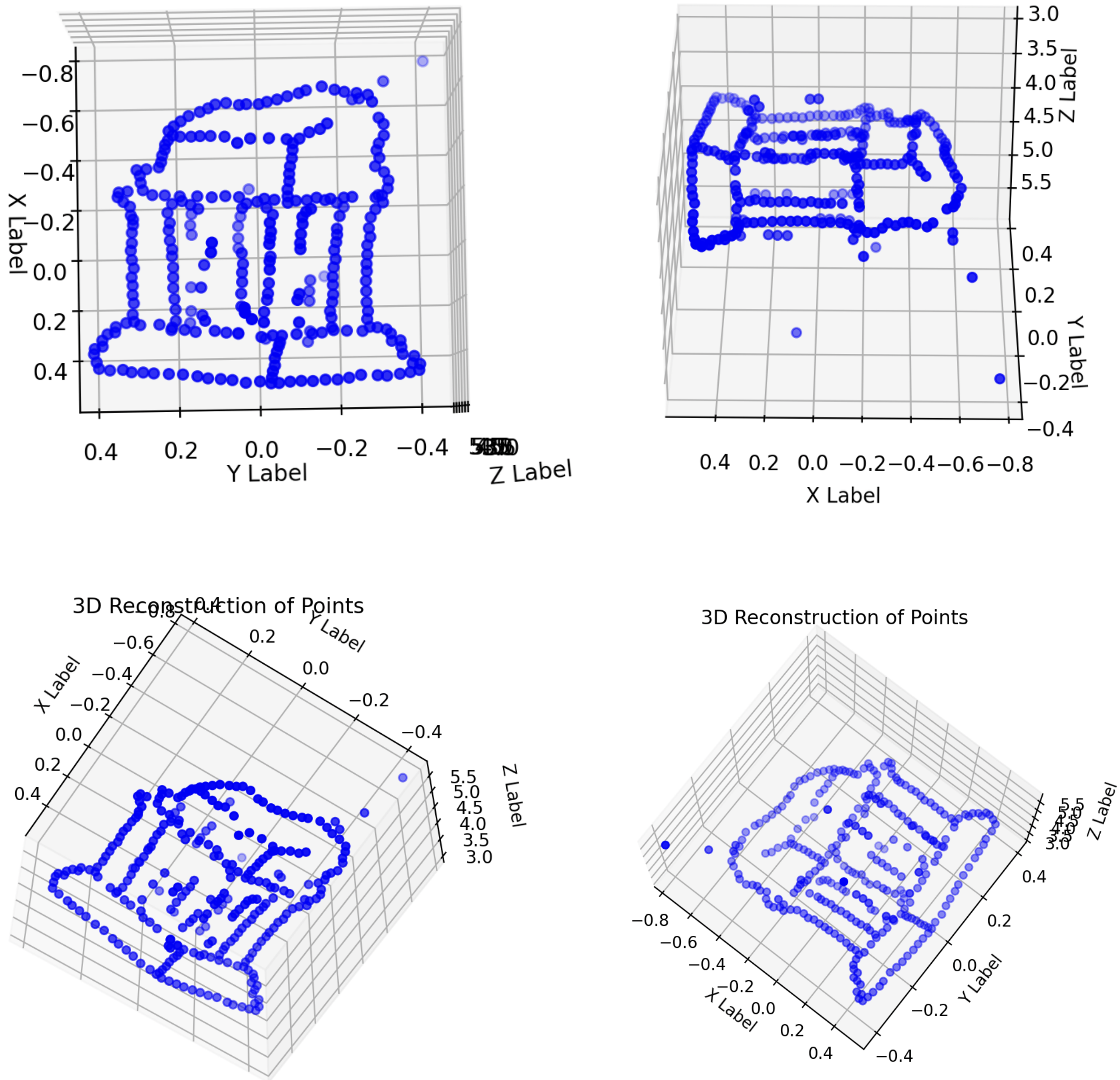

Figure: Output of the 3D reconstruction from different angles

Q5.1

To improve the performance of the code, we use RANSAC to fit the fundamental matrix better on the noisy data.

Without it, the results of the eightpoint algorithm look like:

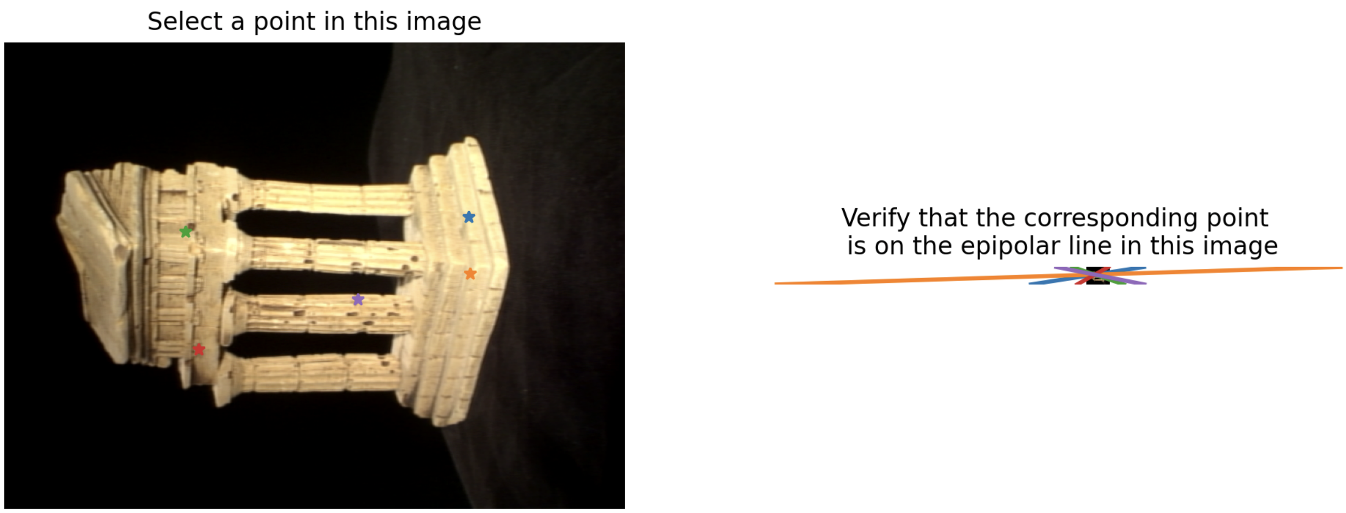

Figure: sample output from displayEpipolarF, with noisy data and no RANSAC

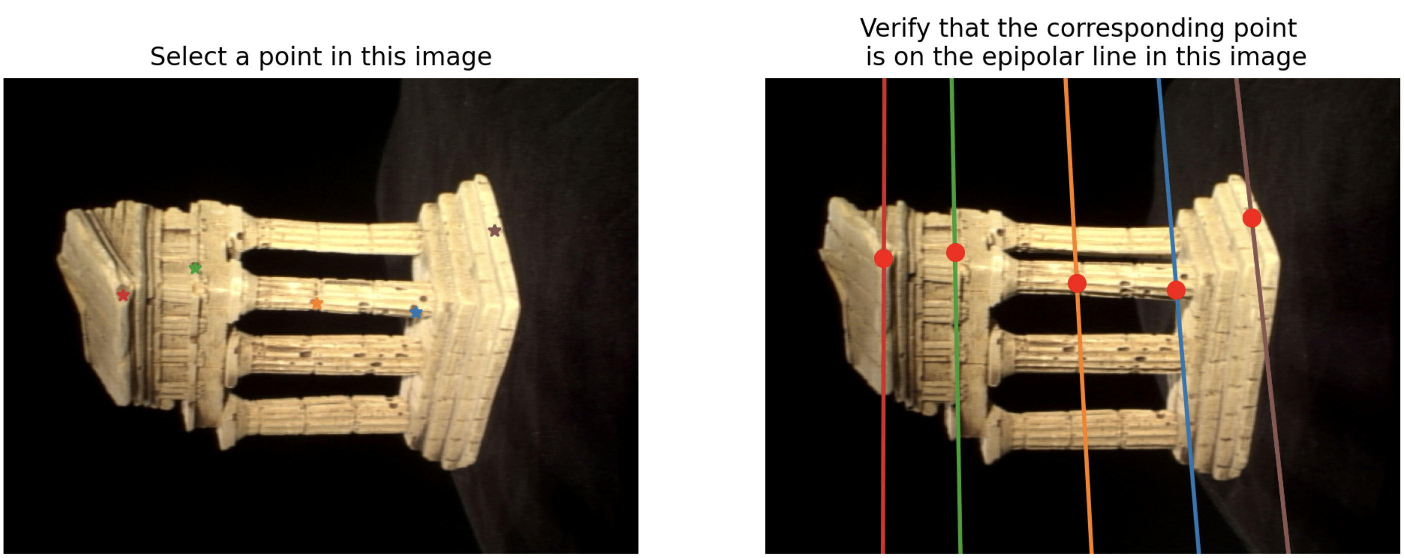

Figure: sample output from displayEpipolarF, with noisy data and RANSAC

While tuning the parameters of RANSAC, we see that if the number of iterations are too low, the algorithm wouldn't be able to find a proper fit for the fundamental matrix. If we decrease the tolerance, the number of outliers shoots up. The error metric used is the distance from the line.