Potsdam Institute For Climate Impact Research
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Introductory phase report

Optimal adjustment of the global trade system to local network disruption

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Contents

Chapter 1

Introduction

1.1 Introduction

Forecasts for the next century predict an increase of global extreme events like droughts, floods or tornados. The report of the *Intergovernmental Panel on Climate Change (IPCC)*[1] confirms the relevance of extreme events. Nature catastrophies cause destruction and misery in the affected region, inter alia whole economic sectors are struck leading to a reduction in production capabilites. As a result, economic sectors connected by the global trade network to the affected one could also be influenced causing the need to adapt their production, possibly leading to second order effects on further economic sectors. As a consequence of this effect a local perturbation spreads into the global trade network and therefore affects economic sectors all over the world - potentially even those far away from the source of catastrophy.
A representation of the global trade network can be obtained by using *multi regional input output tables (MRIOT)*, which contain information about all significant intermediate monetary flows between economic sectors of the world's regions and their final demands (consumption). A comprehensive MRIOT data set is the EORA data set[2], which is used in this work.
Imposing fundamental rules upon the global trade network, namely the produc-

[1] Intergovernmental Panel on Climate Change (IPCC). Managing the Risks of Extreme Events and Disasters to Advance Climate Change Adaptation. A Special Report of Working Groups I and II of the Intergovernmental Panel on Climate Change (SREX). [Field, C.B. et al.]. Cambridge, UK and New York, NY, USA : Cambridge University Press, 2012

[2] Lenzen M, Kanemoto K; Moran D, and Geschke A (2012) Mapping the structure of the world economy, Environmental Science & Technology 46(15) pp 8374-8381

tion output balance equation, the supply of required inputs and the necessity of satisfying the final demands of each region, defines the space of the network's reaction behaviours in answer to perturbations. Satisfying these equations, each economic sector adapts its input/output flows and produciton ratio, while the latter is kept as close to its initial (optimal) value as possible. Temporary over- or underproductions lead to additional global costs of adaptation. These rules lead to a linear optimisation problem (see chapter 3.1), which is solved by using the simplex method (see chapter 2.1).

Chapter 2

Theory

2.1 Simplex method

2.1.1 Mathematical derivation

The simplex optimisation method[1] is applied to linear programs

$$\min_x \{c^T x : x \in X\}, \tag{2.1}$$

$$X := \{x \in \mathbb{R}^n : Ax = b, x \geq 0\}, \tag{2.2}$$

with $A \in \mathbb{R}^{m \times n}, b \in \mathbb{R}^m$ and $\text{rank}(A) = m < n$. The solution space X is restricted by hyperplanes $Ax = b$ and $x \geq 0$, forming the shape of a convex polyhedron. For an under-determined system of equations ($m < n$) a special (not necessarily optimal) basic solution $\bar{x} \in X$ exists with m active components $\bar{x}_J \geq 0$, assigned by inidices $\{j\} = J \subseteq N := \{1, ..., n\}$. The remaining $n - m$ inactive components equal zero and are identified by indices $\{k\} = K = N \setminus J$, so that $\bar{x} = (\bar{x}_J, \bar{x}_K)$. Analogously, the constraint matrix A can be seperated into a submatrix $A_J = (a_{j1}, ..., a_{jm}) \in \mathbb{R}^{m \times m}$, containing the columns corresponding to the active components, and a matrix $A_K = (a_{k1}, ..., a_{km}) \in \mathbb{R}^{m \times n-m}$ containing the inactive columns of A, resulting in $A = (A_J, A_K)$. Solution $\bar{x}$ is called the basis solution with basis A_J. Using the above partitions

[1] This mathematical derivation is based on the script “Lineare Optimierung” of Prof. Dr. Bernhard Schmitt, University of Marburg

yields

$$b = A\bar{x} = A_J\bar{x}_J + A_K\bar{x}_K \;, \tag{2.3}$$

$$\xrightarrow{\bar{x}_K=0} b = A_J\bar{x}_J \;. \tag{2.4}$$

Moreover, with knowledge of a special solution $\bar{x} = (\bar{x}_J, \bar{x}_K)$, other points $x = (x_J, x_K) \in X$ (x_K not necessarily zero) on the edge of the polyhedron can be created, using Eq(2.4)

$$b = Ax = A_J x_J + A_K x_K \tag{2.5}$$

$$\leftrightarrow x_J = A_J^{-1}b - A_J^{-1}A_K x_K \tag{2.6}$$

$$= \bar{x}_J - A_J^{-1}A_K x_K \;. \tag{2.7}$$

Substituting x_K by the parameter λ_K leads to

$$x = \begin{pmatrix} x_J \\ x_K \end{pmatrix} = \begin{pmatrix} \bar{x}_J \\ 0_K \end{pmatrix} + \begin{pmatrix} -A_J^{-1}A_K \\ I_{n-m} \end{pmatrix} \lambda_K = \bar{x} - W_K\lambda_K \geq 0\,, \tag{2.8}$$

with

$$\begin{pmatrix} -A_J^{-1}A_K \\ I_{n-m} \end{pmatrix} = \begin{pmatrix} W_K^{(J)} \\ W_K^{(K)} \end{pmatrix} = W_K \in \mathbb{R}^{n\times(n-m)} \;. \tag{2.9}$$

By increasing the value of a component $x_l \in x, l \in K$ one moves along a ray inside the solution space X, originating in $\bar{x}$. All rays for each $l \in K$ create a cone with $\bar{x}$ being the peak and a corner of the convex polyhedron X. One elementary ray is

$$x(\lambda_l) := \bar{x} - \lambda_l w_l \leftrightarrow \begin{cases} x_J(\lambda_l) = \bar{x}_J - \lambda_l A_J^{-1}a_l \\ x_K(\lambda_l) = \lambda_l\delta_{kl}, k \in K \end{cases} \;, \tag{2.10}$$

with w_l being the l-th column of W_K, a_l the l-th column of A_K and δ_{kl} being the Kronecker Delta.

The fundamental concept of the Simplex method is to start from one basic solution $\bar{x}$ in the corner of the polyhedron and move along one selected ray $x(\lambda_l)$ towards another corner, reducing the target function meanwhile and repeating this procedure until the optimal solution corner x^* is found. To figure out which ray originating in $\bar{x}$ reduces the target function, we consider the cost function

$$c^T x = c_J^T x_J + c_K^T x_K = c_J^T(\bar{x}_J - A_J^{-1} A_K x_K) + c_K^T x_K \tag{2.11}$$

$$= \underbrace{c_J^T \bar{x}_J}_{c^T \bar{x}} + \underbrace{(x_K^T - c_J^T A_J^{-1} A_K)}_{\gamma_K^T} x_K = c^T \bar{x} + \gamma_K^T x_K \ . \tag{2.12}$$

The cost function in the area of $\bar{x}$ changes by increasing the non-basis variables $x_K = \lambda_K$, weighted by the vector of reduced costs γ_K. Moving along a ray $x(\lambda_l)$ with negative reduced costs γ_l leads towards the minimal solution of the linear program. The minimum is reached when each element of γ_K is non-negative $\gamma_l \geq 0 \,; \forall l$. In this case the corresponding special basic corner solution $\bar{x}$ is identified as the optimal solution x^*. While moving along the ray it is important to ensure that every component of x remains inside the solution area X. Consider ray $x(\lambda_l)$ in component form

$$x_j(\lambda_l) = \bar{x}_j - \lambda_l w_{jl} \ . \tag{2.13}$$

While increasing $\lambda_l \geq 0$ the components $x_j(\lambda_l)$ will increase if $w_{jl} \leq 0$. For executing a basis change λ_l will be set to a value, so that one component $x_p(\lambda_l)$ gets equal to zero and can therefore be removed from the active set. For this reason, only those components with $w_{jl} > 0$ are considered. To ensure that all components of x stay inside the solution area X, λ_l is increased until the first component x_p gets zero

$$\lambda_l := \min\{\frac{\bar{x}_i}{w_{il}} : i \in J, w_{il} > 0\} = \frac{\bar{x}_p}{w_{pl}} \geq 0 \ . \tag{2.14}$$

The index p is added to the inactive set K, while variable l is included into the new basis set J

$$J = J \setminus \{p\} \cup \{l\} \tag{2.15}$$

$$K = N \setminus J \ . \tag{2.16}$$

resulting in a new corner solution $\bar{x} = (\bar{x}_J, \bar{x}_K)$ moving the target function $F1$ closer to [1] the minimum.

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2.1.2 Simplex application scheme

Input	Initial feasible basis A_J, $J \subseteq N$, $N := \{1, ..., n\}$
Step 1	$\bar{x}_J := A_J^{-1} b$, $K := N \setminus J$
Step 2	search for $\gamma_l < 0$ out of $\gamma_j := c_j - c_J^T A_J^{-1} a_j; j \in K$
Step 3	if $\gamma_j \geq 0 \; \forall j \in K$: **STOP, optimum found**
Step 4	$w_l^{(J)} := A_J^{-1} a_l$
Step 5	if $w_{il} \leq 0 \; \forall i \in J$: **STOP, unbounded solution**
Step 6	Identify $p \in J$ with: $\frac{\bar{x}_p}{w_{pl}} := \min\{\frac{\bar{x}_i}{w_{il}} : w_{il} > 0, i \in J\} = \lambda_l$
Step 7	$J := J \setminus \{p\} \cup \{l\}$; **jump to step 1**

Table 2.1: Simplex algorithm

Chapter 3

Model setup

3.1 problem specific linear optimisation problem

The data of a MRIOT can be interpreted as a network representation with n_r global regions, each containing n_i economic sectors, so that $n_{ir} = n_i \cdot n_r$ yields the total amount of regional sectors. Each node $ir \in V := \{1, ..., n_{ir}\}$ represents a regional sector with a production ratio $p_{ir} = \frac{X_{ir}^{new}}{X_{ir}^{old}}$, X_{ir}^{old} indicating the initial total production of ir and X_{ir}^{new} the adapted total production. The nodes $ir \in V$ and $js \in V$ are connected by n_{qZ} intermediate flows $Z_{ir\to js}$, weighted with the flow ratios $q_{ir\to js}$. Similarly, the n_{qY} final demand flows $Y_{ir\to s}$ are weighted with the flow ratios $q_{ir\to s}$.

A linear optimisation problem is formulated, using $\{p_{ir}\}$, $\{q_{ir\to js}\}$ and $\{q_{ir\to s}\}$ as free (non-leading) variables, which are minimised according to the target function $T1(\{p_{ir}\}$, fulfilling the constraints $Ax = b$. The constraint matrix A consists of three different type of constraints $C1$, $C2$ and $C3$, which are described in the following. Two different linear problems LP1 and LP2 are formulated which both will be object of research.

3.1.1 Target function

The optimisation procedure is executed by using the following target function

$$F(\{p_{ir}\}) = \sum_{ir} |p_{ir} - 1| \tag{T1}$$

The vector of costs c describes the weighted costs for each variable used. Since no flow variables appear in the target function, their corresponding costs are set to zero, while each production ratio variable p_{ir} is weighted evenly with a value of one

$$c = (\underbrace{1, ..., 1}_{n_{ir}}\ ,\ \underbrace{0, ..., 0}_{n_{qY}+n_{qZ}}\ )\ . \tag{3.1}$$

3.1.2 Final demand constraint

$$\sum_r q_{ir\to s} Y_{ir\to s} = \sum_r Y_{ir\to s}\ ; \forall s; \forall i\ . \tag{C1}$$

The equation's left-hand side sums up all adaptable final demand flows $q_{ir\to s}Y_{ir\to s}$ of a sector i into a region s. The result has to equal the sum of all final demand flows $Y_{ir\to s}$ of the initial unperturbed network.
Each region receives at most n_i different input goods, therefore the maximum number of final demand constraints (C1) of the linear problem amounts to $n_{C1} = n_r \cdot n_i$ constraints.

3.1.3 Supply scaling constraint

$$\sum_r q_{ir\to js} Z_{ir\to js} = p_{js} \sum_r Z_{ir\to js}\ ; \forall js; \forall i\ . \tag{C2}$$

The sum of all adaptable intermediate flows $q_{ir\to js}Z_{ir\to js}$ of a sector i into a node js has to match the sum of its original inputs $Z_{ir\to js}$, scaled by its adaptable production ratio p_{js}.
At most, each node js requires an input of each type of good, hence the maximum number of supply scaling constraints (C2) of the linear program amounts to $n_{C2} = n_{ir} \cdot n_i = n_r \cdot n_i^2$.

3.1.4 Production output balance constraint

$$p_{ir} X_{ir} = \sum_{js} q_{ir\to js} Z_{ir\to js} + \sum_s q_{ir\to s} Y_{ir\to s}\ ; \forall ir\ . \tag{C3}$$

The adaptable production $p_{ir} X_{ir}$ of a node ir has to equal the sum of its outgoing adaptable intermediate flows $q_{ir \to js} Z_{ir \to js}$ plus the sum of its outgoing adaptable final demand flows $q_{ir \to s} Y_{ir \to s}$.
This constraint has to be fulfilled by each node, therefore the number of production output balance constraints (C3) amounts to $n_{C3} = n_{ir} = n_r \cdot n_i$.

3.1.5 Linear problem 1 (LP1): Maximal adaptation

The linear problem with maximal adaptation (LP1) is composed of the target function (3.1.1) and the constraints (3.1.2) (3.1.3) (3.1.4):

$$\min_{p_{ir}} F = \sum_{ir} |p_{ir} - 1| \tag{T1}$$

$$s.t. \quad \sum_r q_{ir \to s} Y_{ir \to s} = \sum_r Y_{ir \to s} \ ; \forall s; \forall i \tag{C1}$$

$$\sum_r q_{ir \to js} Z_{ir \to js} = p_{js} \sum_r Z_{ir \to js} \ ; \forall js; \forall i \tag{C2}$$

$$p_{ir} X_{ir} = \sum_{js} q_{ir \to js} Z_{ir \to js} + \sum_s q_{ir \to s} Y_{ir \to s} \ ; \forall ir \ . \tag{C3}$$

All production ratio variables p_{ir} and flow ratio variables $q_{ir \to js}, q_{ir \to s}$ are treated as a free variables under the optimisation procedure, so the solution vector x yields

$$x = (\{p_{ir}\}, \{q_{ir \to js}\}, \{q_{ir \to s}\}); \forall ir, \forall js, \forall s \ . \tag{3.2}$$

The maximal amount of free variables n_{var} consists of the amount of production ratio variables $n_{ir} = n_r \cdot n_i$, the amount of intermediate flows $n_{qZ} = n_{ir} \cdot n_{ir} = n_{ir}^2$ plus the amount of final demand flows $n_{qY} = n_r \cdot n_{ir} = n_r^2 \cdot n_i$

$$n_{var} = n_{ir}^2 + n_r^2 \cdot n_i + n_r \cdot n_i \ . \tag{3.3}$$

This represents a fully connected network, with a maximal amount of constraints

$$n_C = n_{C1} + n_{C2} + n_{C3} = n_r \cdot n_i^2 + 2 \ n_r \cdot n_i \ . \tag{3.4}$$

In the case of a drop out of node $i^* r^*$, its production ratio $p_{i^* r^*}$ together with all intermediate input flows from other nodes js into $i^* r^*$ $q_{js \to i^* r^*}$ and all outgoing flows $q_{i^* r^* \to js}, q_{i^* r^* \to s}$ are set to zero and removed from the set of free variables

x:

$$p_{i^*r^*} = 0 \, , \tag{3.5}$$
$$q_{i^*r^* \to js} = 0; \forall js \, , \tag{3.6}$$
$$q_{i^*r^* \to s} = 0; \forall s \, , \tag{3.7}$$
$$q_{js \to i^*r^*} = 0; \forall js \, . \tag{3.8}$$

3.1.6 Linear problem 2 (LP2): Reduced adaptation

By reducing the amount of free variables x of the optimisation problem by transforming some of them into leading variables (which are dependent on the free variables), one can influence the resulting network properties and save computing time.
In the case of the linear problem 2 (LP2) the focus of adaptation is set on flows transporting goods i^*, which shall remain freely adaptible and therfore part of the free variables x, while all other flows with $i \neq i^*$ are omitted

$$x = (\{p_{ir}\}, \{q_{i^*r \to js}\}, \{q_{i^*r \to s}\}); \forall i; \forall r; \forall js; \forall s \, . \tag{3.9}$$

The flows with $i \neq i^*$ are determined by predefined equations

$$q_{ir \to js} = p_{js} \; ; \forall i \neq i^*; \forall r; \forall js \, , \tag{3.10}$$
$$q_{ir \to s} = 1 \; ; \forall i \neq i^*; \forall r; \forall s \, . \tag{3.11}$$

This formulation of a linear problem keeps all final demand flows $Y_{ir \to s}$ with goods $i \neq i^*$ constant under perturbation. In addition, all intermediate input flows $Z_{ir \to js}$ with $i \neq i^*$ are assumed to be equal to the production ratio p_{js} of the input node js. This means that a change of production p_{js} and therefore a change of inputs $Z_{ir \to js}$ is evenly distributed over all suppliers of node js.
Using the above changes, the linear problem 1 (LP1) transforms into the new

linear problem 2 (LP2)

$$\min_{p_{ir}} F = \sum_{ir} |p_{ir} - 1| \tag{T1}$$

$$s.t. \quad \sum_{r} q_{i^*r\to s} Y_{i^*r\to s} = \sum_{r} Y_{i^*r\to s} \;;\forall s \tag{C1}$$

$$\sum_{r} q_{i^*r\to js} Z_{i^*r\to js} = p_{js} \sum_{r} Z_{i^*r\to js} \;;\forall js \tag{C2}$$

$$p_{i^*r} X_{i^*r} = \sum_{js} q_{i^*r\to js} Z_{i^*r\to js} + \sum_{s} q_{i^*r\to s} Y_{i^*r\to s} \;;\forall r \tag{C3a}$$

$$p_{ir} X_{ir} = \sum_{js} p_{js} Z_{ir\to js} + \sum_{s} Y_{ir\to s} \;;\forall i \neq i^*;\; \forall r \tag{C3b}$$

The final demand constraints (C1) only need to be formulated for flows with good i^*, because the predefined equations (3.10) determine all final demand flows with $i \neq i^*$, making the corresponding constraints dispensable. The maximal amount of final demand constraints (C1) reduces to $n_{C1} = n_r$. Similarly the supply scaling constraint (C2) is reduced to those equations using flows with good i^*. All other intermediate flows are determined, leading to omittance of the corresponding equations and to a maximal amount of supply scaling constraints of $n_{C2} = n_{ir} = n_r \cdot n_i$. The production output balance constraint (C3) is split into two constraints (C3a) and (C3b). The first is valid for nodes which produce the good i^* and keeps the form of LP1 with adaptability of intermediate flows as well as final demand flows. Contrarily, for nodes with $i \neq i^*$ constraint (C3b) shows with a reduced adaptability, because both flow variables are substituted by the predefined equations, only leaving the production ratio p_{ir} as a free variable. Still there is exactly one production output balance constraint for each node $n_{C3} = n_r \cdot n_i$. Altogether, the maximal amount of constraints for LP2 adds up to

$$n_C = n_{C1} + n_{C2} + n_{C3} = n_r + 2\, n_r \cdot n_i \;. \tag{3.12}$$

The maximal amount of intermediate flow variables is reduced to $n_{qZ} = n_{ir} \cdot n_r = n_r^2 \cdot n_i$, because each node gets one input with i^* from each regional sector i^*r. Moreover, the amount of final demand flow free variables equals $n_{qY} = n_r^2$. Adding the amount of production ratios $n_{ir} = n_r \cdot n_i$, the maximal amount of variables for LP2 amounts to

$$n_{var} = n_r^2 \cdot n_i + n_r^2 + n_r \cdot n_i \ . \tag{3.13}$$

Analogously to LP1, the drop out of node $i^* r^*$ leads to the removal of following variables out of the set of x

$$p_{i^* r^*} = 0 \ , \tag{3.14}$$

$$q_{i^* r^* \to js} = 0; \forall js \ , \tag{3.15}$$

$$q_{i^* r^* \to s} = 0; \forall s \ , \tag{3.16}$$

$$q_{js \to i^* r^*} = 0; \forall js \ . \tag{3.17}$$

3.2 Analysis of the EORA network

3.3 Aggregated network

Chapter 4

Results

4.1 statistics

4.1.1 F(ir)

4.1.2 comp_ir(ir)

4.2 comparison LPS/LPG

4.3 absorption potential

4.4 linear response

4.5 phase transition

4.6 case studies incl. forward/backward effects

4.6.1 Japan machinery drops out

4.6.2 other forward effect example

4.7 ??? time evolution ???

Chapter 5

Final

5.1 discussion

5.2 Ausblick

5.3 Appendix

Bibliography

[1] D. Witthaut and M. Timme. Braess' paradox in models of supply networks. *New Journal of Physics*, 14:083036, 2012.

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