Bonus assignment 1 MM 2021/2022

February 10, 2022

Instructions

Hand in your solution as a single .m file. Do not use a livescript when you hand in. You are free to work with livescripts before handing in, but we will be running a plagiarism scan and will need .m files. Implement the tasks below in Matlab. Give comments where needed. You can optionally include a picture for challenge 1 and challenge 2a. The assignments should be uploaded as .m and not be compressed to facilitate automatic plagiarism checks. You have to upload your solution before 4pm (Maastricht time) on 23 February, 2022 through Canvas. This is an individual graded assignment. What you submit should be your work and your work alone, and by submitting it you testify that this is indeed the case. Obviously, what is already in the template can be used. You are not allowed to use any toolboxes (even not the control systems toolbox) unless it is explicitly mentioned in that specific question that you may.

Challenge 1

Consider the following pair of coupled nonlinear differential equations:

$$\begin{aligned} x' &= 2x - 3xy + y^2, \\ y' &= x + y + xy - 5. \end{aligned}$$

We are interested in the equilibrium points. Which of the following three points are equilibrium points?
(a) The point $(0, 0)$, (b) the point $(1, 2)$, (c) the point $(2, 1)$?

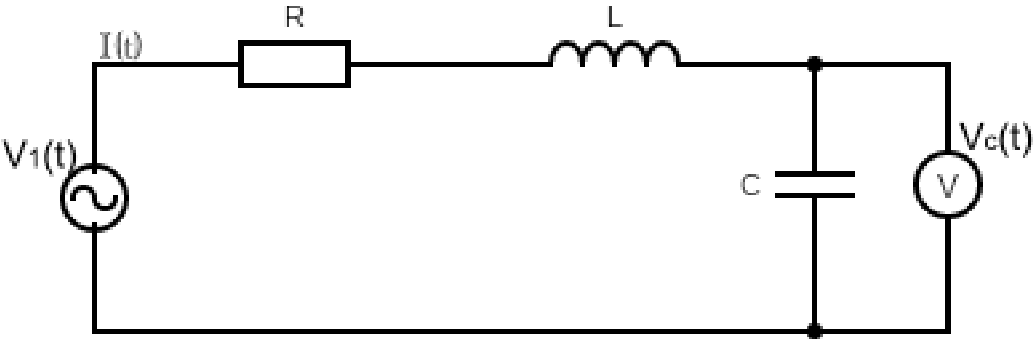
Select an equilibrium point from the three you just analyzed and call it (x_0, y_0). Next, compute the Jacobian and compute a linearisation around (x_0, y_0) in the form

$$\begin{pmatrix} \tilde{x}' \\ \tilde{y}' \end{pmatrix} = A \begin{pmatrix} \tilde{x} \\ \tilde{y} \end{pmatrix},$$

where A is a 2×2 matrix. [The notation we use here, is consistent with $x = x_0 + \tilde{x}$ and $y = y_0 + \tilde{y}$, so that $(\tilde{x}, \tilde{y})$ are local coordinates around the equilibrium point.]
You can either do this in Matlab in the .m file or also hand in a pdf in which you work this out by hand. If you choose to do this in Matlab, you are allowed to use the symbolic math toolbox for this task.

Challenge 2

Given is an RLC circuit:



Here R is an resistor, L and inductor and C a capacitor. We have a voltage source with voltage $V_1(t)$ and we will be measuring the voltage over the capacitor as $V_c(t)$. The current is $I(t)$. Since the R, L and C are connected in series we can use Kirhoff's voltage law to find the following differential equation:

$$V_1(t) - RCV_c'(t) - V_c(t) - LCV_c''(t) = 0$$

(I sincerely hope I have not made any electrical engineer cry) $V_1(t)$ is considered input and the voltage over the capacitor output.

2a

Construct a second-order continuous-time state-space representation based on the information provided. For this you can use the control systems toolbox.

2b

Given that $R = 120$, $L = 0.01$, $C = 0.002$ and $V_1(t)$ is a 50Hz sinusoid with amplitude 1, simulate the state-space system for 10 seconds. For the discretization in Matlab, you should use timesteps of $\delta t = 0.01$. For this you can use the control systems toolbox.

2c

Is this system stable or not? Argue why!

Challenge 3

Use the Matlab "residue" command to perform partial fraction expansion on the following discrete-time transfer function. Yeah I know"residuez" exists, but you're not allowed to use that. (Though you can use it to check things yourself, without putting it in the .m file that you hand in) The idea is that you show you understand what is fundamentally different here and how you can make it work with the residue function.

$$H(z) = \frac{z^2 + z + \frac{1}{4}}{z^3 + \frac{1}{4}z}.$$

Not only show the relevant output of the residue command, but in the comments of your .m file show the partial fraction expansion like this (obviously, this is a random one just for visualization)

```
% The partial fraction expansion found is:
%                    2z        -z
% H(z) = 1 + ---------- + -------
%            z + 0.3333   z - 0.5
```

Also indicate whether this system is stable or not and argue why. Finally provide the impulse response sequence.