Lab 2 – Mathematical Modeling

Joël M.H. Karel and Ralf Peeters

January 24, 2022

Introduction

The Matlab educational license of the Maastricht University contains the Matlab Control System Toolbox. This toolbox contains a large number of functions that allow easy implementation of the topics that are discussed in this course. To see a list of functions that are included in this toolbox along with a short description, type: `doc control` at the Matlab command prompt.

Look at the functions that are included in the Control System Toolbox and find the functions that are suitable to enter transfer functions and state-space systems.

The textbook of Williams II and Lawrence offers a bit of an introduction in section 1.5. It is a good starting point to go through that section. If you want to do the same for the simulation, you can also look at section 2.6, but then only look at the simulation part and ignore the coordinate transforms and canonical forms for now.

Exercise 1

Consider the following state-space system in continuous time, with input $u(t)$ and output $y(t)$:

$$\begin{aligned} x'(t) &= Ax(t) + Bu(t) \\ y(t) &= Cx(t) + Du(t) \end{aligned}$$

where the matrices A, B, C and D are given by:

$$\begin{aligned} A &= \begin{pmatrix} 4 & 1 & 0 \\ -7 & -1 & -1 \\ 119 & 32 & -8 \end{pmatrix} \\ B &= \begin{pmatrix} 0 \\ 0 \\ -1 \end{pmatrix} \\ C &= \begin{pmatrix} 227 & 55 & -13 \end{pmatrix} \\ D &= -2 \end{aligned}$$

Enter this system into Matlab. Call it “system1” and compute (using Matlab) its associated transfer function $H(s)$.

Exercise 2

In the file *dataset.mat* you will find a dataset in the form of a structure. `data.t` contains sampling times and `data.x` contains the corresponding values. Simulate `sys2` using `lsim` and use the dataset as the input. Also compute the transfer function, its poles and zeros and the eigenvalues of the A matrix.

Exercise 3

Solve the differential equation using Laplace transforms. Assume the system starts undeflected at rest:

$$\dddot{\theta}(t) + 40\ddot{\theta}(t) + 20\dot{\theta}(t) + 2\theta(t) = 4$$

You'll have to find a way to determine the roots of the denominator of the Laplace transforms output function. You therefore may use numerical approximations of the pole locations. In all the assignment is: find $\theta(t)$.

Exercise 4

Use the Matlab "residue" command to perform partial fraction expansion on the following transfer function.

$$H(s) = \frac{s^3 + 2s^2 + 3s + 4}{s^3 + 6s^2 + 11s + 6}.$$

Is this system stable? Check your results for the p.f.e.!

Exercise 5

With help of Matlab perform partial fraction expansion on the transfer functions of the following continuous-time systems (sometimes you have to find the transfer function first). Determine whether these systems are stable and find the impulse response and step response function. Plot the impulse response and step response function of each system in a single plot and try to determine the relationship between them. The definition of the step and impulse functions in the Laplace transform tables might proof useful here. Also cherish the Matlab `roots`, `poly`, `residue`, `pole`, `zero`, `impulse` and `step` functions. If you need the practice, you can also do this by hand and use Matlab for verification. (Perhaps you can let Matlab help you to find the poles, certainly for the 3rd one)

1. $$H(s) = \frac{(s+2)(s+10)}{(s-3)(s+5)s}$$

2. $$y^{(3)}(t) + 18y^{(2)}(t) + 95y^{(1)}(t) + 126y(t) = u^{(3)}(t) - 19u^{(1)}(t) + 30u(t)$$

3. $$H(s) = \frac{s^3 - 6s^2 + 5s + 12}{s^5 + 20s^4 + 156s^3 + 590s^2 + 1075s + 750}$$

4.

$$H(s) = \frac{s^3 - 7s + 6}{s^3 + 8s^2 + 25s + 26}$$