Computer Class Model Identification and Data Fitting Recursive Least Squares Algorithm

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Exercise 1 Consider the scalar (stochastic) ARX model of order n with Gaussian white noise residuals e_t, described by the difference equation:

$$y_t = -a_1 y_{t-1} - \ldots - a_n y_{t-n} + b_1 u_{t-1} + \ldots + b_n u_{t-n} + e_t$$

(a) Rewrite this model in the form of a linear regression model, by defining a suitable parameter vector θ and a corresponding regression vector ϕ_t.

(b) For your choices under (a) implement a computer program to estimate the model parameters of such an ARX model, using the recursive identification scheme below (discussed in Section 5 of the lecture notes):

$$\begin{aligned} g_N &= H(N-1)\phi_N \\ d_N &= 1 + \phi_N^T g_N \\ H(N) &= H(N-1) - \frac{1}{d_N} g_N g_N^T \\ \hat{e}_N &= y_N - \phi_N^T \hat{\theta}_{LS}(N-1) \\ \hat{\theta}_{LS}(N) &= \hat{\theta}_{LS}(N-1) + \frac{\hat{e}_N}{d_N} g_N \end{aligned}$$

with the following recursion for the sum of squares of the residuals:

$$\Lambda(N) = \Lambda(N-1) + \frac{\hat{e}_N^2}{d_N}$$

Specify the default initial values for the variables that appear in this recursion.

(c) Simulate the following stable ARX system of order 3:

$$y_t = -0.5y_{t-1} - 0.6y_{t-2} - 0.3y_{t-3} + 0.2u_{t-1} + 1.4u_{t-2} - 0.5u_{t-3} + e_t$$

in which e_t is white noise having a standard normal distribution. Your are free to choose initial values and an input sequence u_t, which should be sufficiently exciting. In this way, generate a sequence of about 1000 measurement values for u_t and y_t.

(d) Apply the program developed under (b) to the simulated data obtained under (c). Store the entire sequence of parameter estimates. Plot the values of these estimates as a function of the number N of measurements and compare them to the true parameter values used to generate the measurement data.

(e) Determine the corresponding covariance matrix for the final parameter vector estimate. Which information does it provide with respect to the uncertainty in the estimated parameter values and how does this relate to the plots obtained under (d)?