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FINAL PROJECT

Desktop application supporting learning of stereometry

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Thesis title

Desktop application supporting learning of stereometry

Abstract

The aim of the work is to design and implement a desktop application that supports children in learning geometry. The user of the application will have several types of tasks to choose from, in which it will be necessary to calculate the areas and volumes of three-dimensional geometric figures. There will also be an educational component where it will be possible to visualize the changes in the areas and volumes of the figures as the parameters describing the figure are altered.

Key words

application, education, geometry, stereometry

Tytuł pracy

Applikacja na komputer stacjonarny wspomagająca naukę stereometrii

Streszczenie

Celem pracy jest zaprojektowanie i wdrożenie aplikacji desktopowej wspomagającej dzieci w nauce geometrii. Użytkownik aplikacji będzie miał do wyboru kilka typów zadań, w których będzie musiał obliczyć pola i objętości trójwymiarowych figur geometrycznych. Aplikacja będzie również zawierać część edukacyjną, umożliwiającą wizualizację zmian pól i objętości figur wraz ze zmianą parametrów opisujących figurę.

Słowa kluczowe

aplikacja, edukacja, geometria, stereometria

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Chapter 1

Introduction

1.1 Introduction into the problem domain

With the development of technology, education changes from classical approaches to ones using tools and methods previously unavailable. As computers become more accessible and widespread, schools introduce more digital ways of teaching students. One of the goals of educational applications is to aid teachers in conveying knowledge more effectively than using classical methods. A well-designed educational application should be able to provide all the tools necessary to teach students a certain subject. It may also be installed on the student's personal computer, allowing them to study and practice efficiently even outside of school.

The thesis is concerned with applications which allow the user to gain and improve their skills in the field of stereometry. Stereometry (or solid geometry) is geometry in three-dimensional space. Stereometry is concerned with the measurement of surface areas and volumes of solid figures, such as polyhedrons, spheres, cylinders and cones.

1.2 Objective and scope of the thesis

Learning stereometry may prove difficult to a student without adequate tools. There are several tools to learn stereometry without the assistance of a computer application. These include:

Blackboard and chalk Offer only static representations of figures which are difficult to draw precisely. The drawings are usually only wireframes and do not have shading. This might make it hard to visualize what the drawing is meant to represent. Drawing the figures is also time-consuming.

Printed textbooks Include shading, but are still limited to static images which can be viewed from a single angle.

Physical models Present a true three-dimensional representation which can be rotated.

The dimensions are constant and not every figure might be available as a physical model.

The objective of the thesis is to design and implement a desktop application which helps the user learn stereometry.

Nowadays, web applications are more popular than desktop applications. Despite that, desktop applications have some advantages:

- Desktop apps do not require an internet connection after installation, while web apps require constant connection to the server hosting the service.
- Desktop apps are faster and more responsive as no data needs to be transferred between the client computer and a remote server.
- Desktop apps give the user control over software updates - the user may use an older version if desired.

The application will allow the user to solve solid geometry tasks, consequently expanding their skills and knowledge in this field. Its target group is students aged 12 to 15. The application can be used as a didactic tool in the classroom to help the teacher in teaching stereometry. It can also be used at home by a student to practice stereometry outside the classroom. The application will feature tasks. A task will contain one or more three-dimensional figures. The user's goal is to calculate the total area and volume of said figures. After solving a task incorrectly, the user will receive a tip, to help them solve the task successfully. A database containing all mathematical formulas, as well as tips related to the tasks will be accessible. The user will have the possibility to change the task data and see how the shapes change dynamically. An important part of the application is its customizability - the user should be able to tweak the settings of the application as well as each task such that they get the best experience tailored to their needs.

The application will be built using the Godot game engine. While designed primarily with video games in mind, it can be used to create other types of software. The application requires a modern graphical user interface (GUI) and 3D rendering capabilities. Godot provides a suite of tools which simplify working with both of those systems. Other solutions like Flutter and Electron were considered but ultimately rejected. They provide a wide array of tools for developing desktop applications but do not have first-class 3D rendering support which would make developing the application more difficult.

1.3 Short description of chapters

- The second chapter analyses the problem and presents already existing solutions.

- The third chapter lists the functional and nonfunctional requirements, describes the use cases and presents the tools used in the project's development.
- The fourth chapter serves as the user manual. It specifies the system requirements and provides instructions on how to run and use the application.
- The fifth chapter is focused on the implementation of the project. It explains the architecture of the application.
- The sixth chapter **TODO**.
- The seventh chapter concludes the thesis. It compares the achieved results to the objectives of the thesis and explores prospects of the project's further development.

Chapter 2

Problem analysis

2.1 Mathematical description of selected solids

2.1.1 Prism

A prism is a polyhedron consisting of two congruent n -sided polygon bases and n joining faces. Every joining face is a parallelogram. A prism is called after its base, e.g. a prism with a hexagonal base is called a hexagonal prism. An n -gonal prism has $n + 2$ faces, $3n$ edges, and $2n$ vertices.

Special cases

An *oblique prism* is a prism whose joining faces are not perpendicular to the bases. A prism whose joining faces are perpendicular to the bases is a *right prism*.

A right prism with a rectangular base is also called a *cuboid*. A right prism with a square base is also called a *square cuboid*. A square cuboid whose height is equal to the side length of its base is a *cube*.

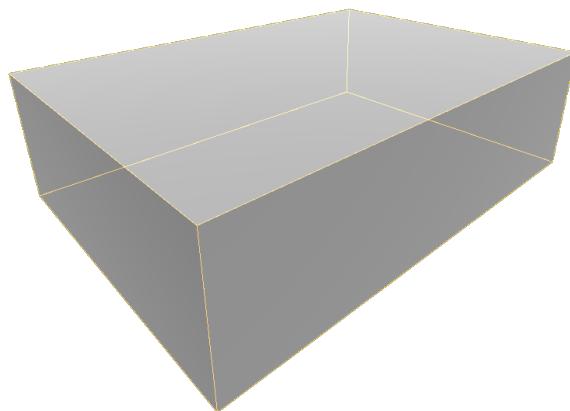


Figure 2.1: A right prism with a rectangular base.

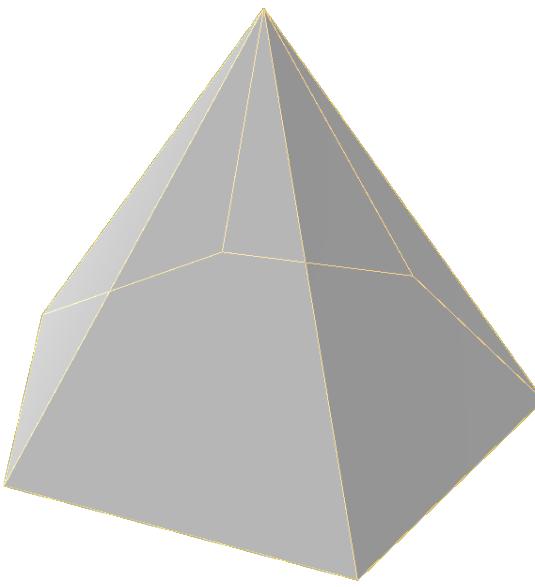


Figure 2.2: A right pyramid with a hexagonal base.

Properties

Volume The volume of a prism is the product of the base area and the height. The formula for the volume is:

$$V = Bh, \quad (2.1)$$

where B is the base area and h is the height.

Surface area The surface area of a prism is the sum of all its faces' areas. The formula for the surface area is:

$$A = 2B + Ph, \quad (2.2)$$

where B is the base area, P the perimeter, and h the height.

2.1.2 Pyramid

A pyramid is a polyhedron consisting of one polygonal base face connected to a point, called the apex. Each of the base's edges is connected to the apex, forming a triangle, called a lateral face. A pyramid with an n -gonal base has $n + 1$ faces, $2n$ edges, and $n + 1$ vertices.

Special cases

A pyramid is called a *regular pyramid* when its base is a regular polygon. The definition of a *right pyramid* varies between sources. Some sources define it as a regular pyramid whose height falls on the center of the base [3]. According to other sources, a right pyramid

is a pyramid, whose height falls on the center of a circle circumscribed about the pyramid's base [6]. A pyramid with a triangular base is also called a *tetrahedron*.

Properties

Volume The volume of a pyramid is one third of the product of the base area and the height. The formula for the volume is:

$$V = \frac{1}{3}Bh, \quad (2.3)$$

where B is the base area and h is the height.

Surface area The surface area of a pyramid is the sum of the area of the base and the area of the lateral faces. For a regular right pyramid with an n -gonal base and height h , the surface area can be calculated as follows:

Each regular n -gon can be divided into n congruent triangles. The base of the triangle is the side s of the pyramid. The triangle's height h_B is calculated as:

$$h_B = \frac{1}{2}s \tan\left(\frac{n-2}{n}90^\circ\right) \quad (2.4)$$

Then, the polygon base area can be calculated by multiplying the area of one triangle by the number of sides:

$$\begin{aligned} A_B &= \frac{sh_B}{2}n \\ &= \frac{1}{2}s\left(\frac{1}{2}s \tan\left(\frac{n-2}{n}90^\circ\right)\right)n \\ &= \frac{1}{4}s^2 \tan\left(\frac{n-2}{n}90^\circ\right)n \end{aligned} \quad (2.5)$$

Height of lateral triangles can be obtained by using the Pythagorean theorem:

$$h_L = \sqrt{h_B^2 + h^2} \quad (2.6)$$

The lateral area is calculated in a similar way to the base area:

$$\begin{aligned} A_L &= \frac{sh_L}{2}n \\ &= \frac{s\sqrt{h_B^2 + h^2}}{2}n \end{aligned} \quad (2.7)$$

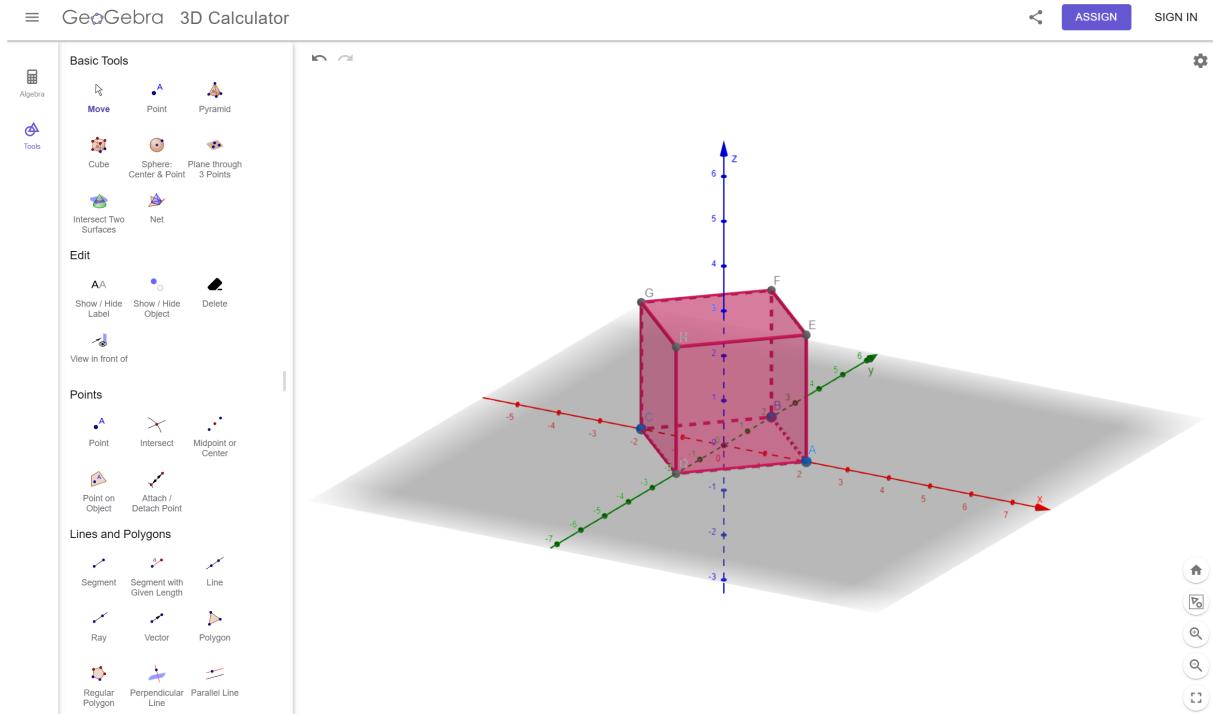


Figure 2.3: GeoGebra 3D Calculator.

Finally, the total area is the sum of the base area and the lateral area:

$$\begin{aligned}
 A &= A_B + A_L \\
 &= \frac{1}{4}s^2 \tan\left(\frac{n-2}{n}90^\circ\right)n + \frac{s\sqrt{h_B^2 + h^2}}{2}n \\
 &= \frac{1}{2}sn\left(\frac{1}{2}s \tan\left(\frac{n-2}{n}90^\circ\right) + \sqrt{h_B^2 + h^2}\right)
 \end{aligned} \tag{2.8}$$

2.2 Existing solutions

2.2.1 GeoGebra 3D Calculator

The 3D Calculator¹ by GeoGebra is an extensive application which provides many tools for exploring 3D geometry. It allows the user to create and manipulate solids, points, lines and polygons. It also provides tools for measuring angles, distances, areas and volumes. The 3D Calculator can be of great help when exploring stereometry, but it does not provide didactic features. It is not suited for structured, guided learning. The measuring tools allow the user to calculate certain characteristics but do not teach how to do so. GeoGebra 3D Calculator does not provide support for keyboard control, the mouse being the only allowed input method. The application is only available on web and mobile platforms with no downloadable version for desktop computers.

¹<https://www.geogebra.org/3d>

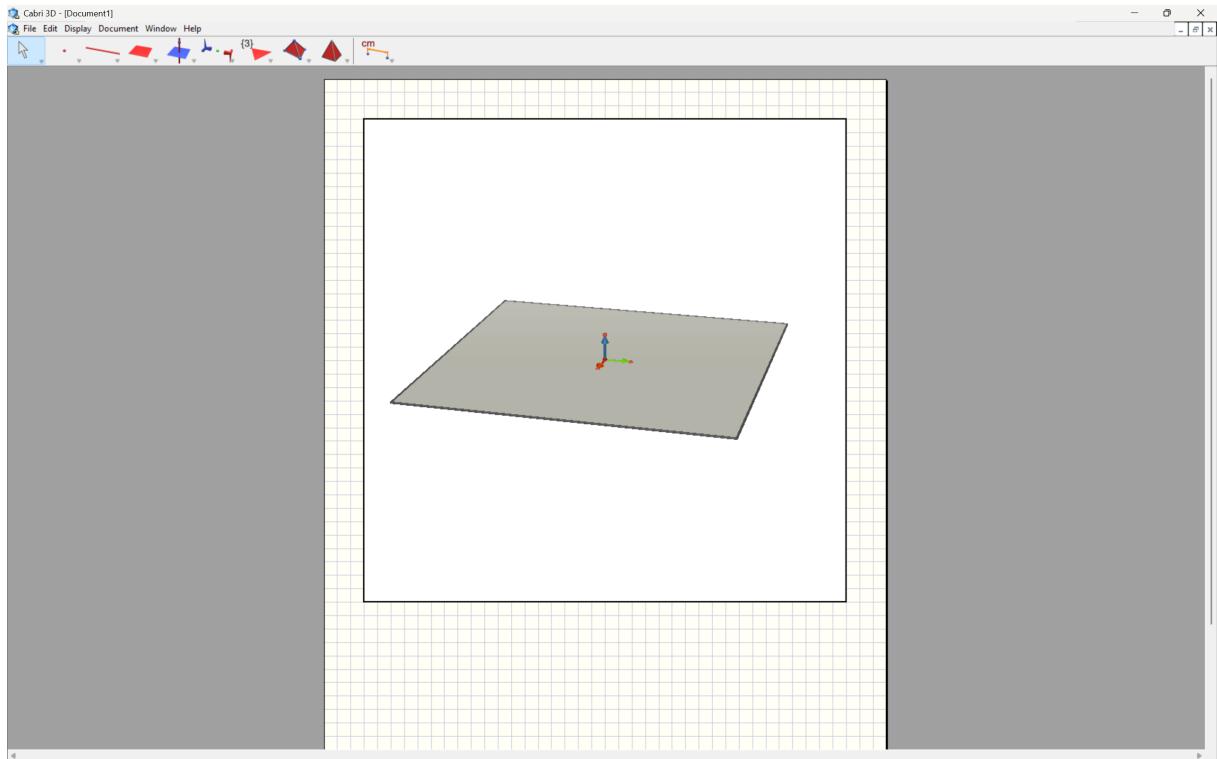


Figure 2.4: Cabri 3D.

2.2.2 Cabri 3D and Cabri Express

Cabri 3D² and Cabri Express³ are 2 applications for learning three-dimensional geometry developed by Cabri. Both applications are designed with teaching in mind. Cabri products are closed-source, which means that the software will no longer be able to receive updates after the original creators stop developing them.

Cabri 3D

Cabri 3D is an application for the Windows and Mac OS operating systems. It is commercial software which requires the purchase of a license. The application is used primarily to create documents consisting of text areas and 3D views. This makes it suitable to be used by teachers to prepare lesson materials, which can be printed on paper. It is not suited to be used by students to explore 3D geometry or solve tasks. The user interface has an outdated look and has some bugs when running on a system with multiple displays. When running on a system with two displays, the *Tool Help* window produces visual artifacts on the other display when it is moved.

²<https://cabri.com/en/enterprise/cabri-3d/index.html>

³<https://cabri.com/fr/enterprise/cabri-express/>

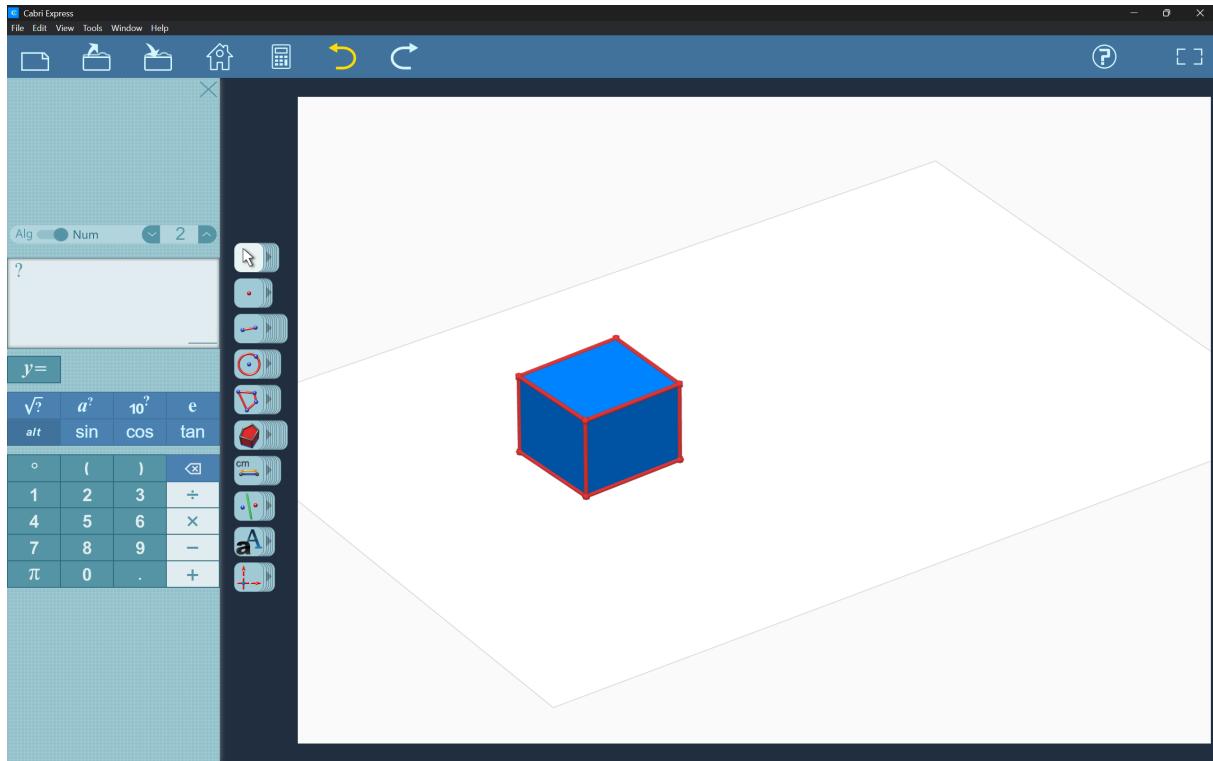


Figure 2.5: Cabri Express.

Cabri Express

Cabri Express is a general mathematical application and features tools not limited to 3D geometry. It has tools related to stereometry but it is not specialized in this field. This, combined with it being closed-source introduces a problem. If one doesn't find the desired 3D tool in Cabri Express, it may never be added by the developers. Learning how to use the applications may be problematic, as most of the official tutorials for the applications are in French. Moreover, there are few unofficial learning resources in other languages.

2.2.3 Shapes 3D Geometry Learning & Drawing

Shapes 3D Geometry Learning⁴ and Shapes 3D Geometry Drawing⁵ are two similar applications whose main feature is viewing solids. Both applications are available in a subscription model. One may also buy a perpetual license for the Shapes 3D Geometry Drawing application on the iOS platform. A free demo accessible from the browser exists for both applications. Unfortunately, the author of the thesis was unable to run the Shapes 3D Geometry Drawing demo due to an error present on the website.

⁴<https://shapes.learnteachexplore.com/shapes-3d-geometry-learning/>

⁵<https://shapes.learnteachexplore.com/shapes-3d-geometry-drawing/>

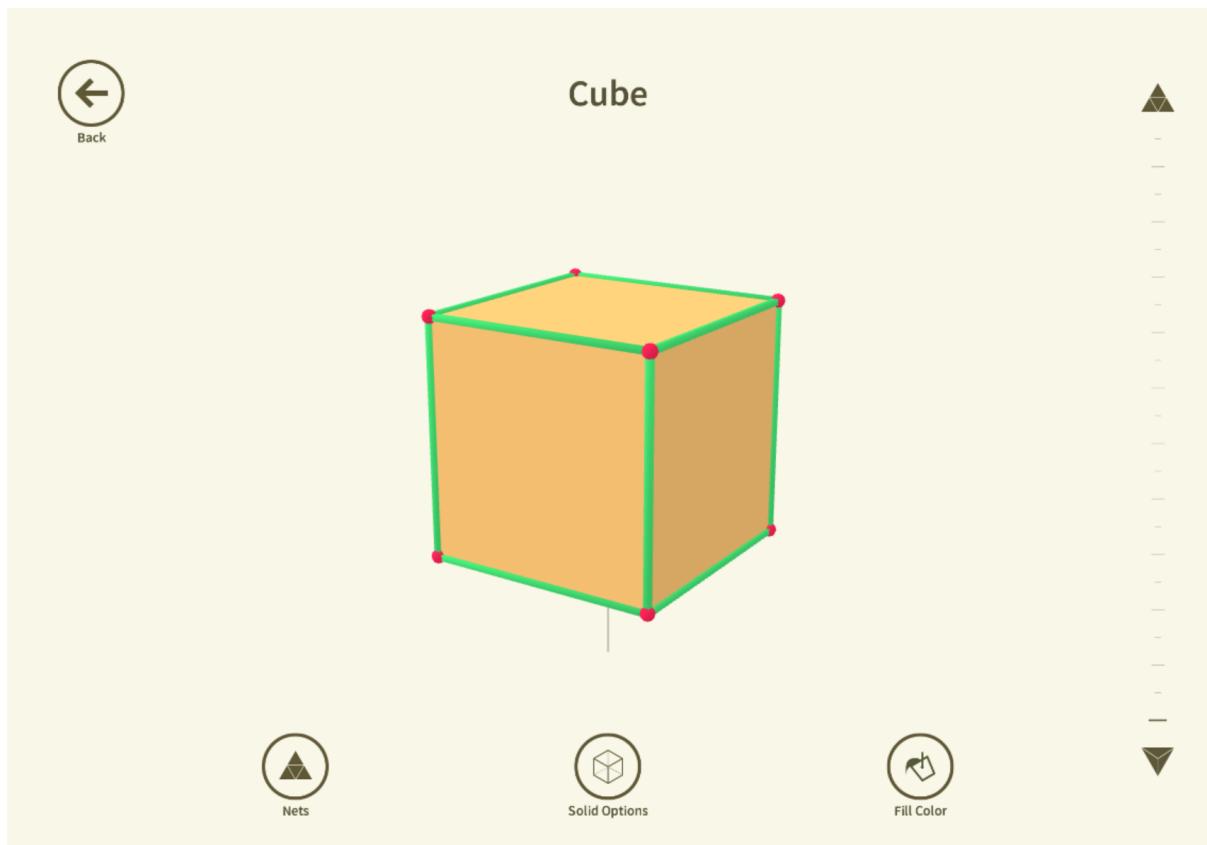


Figure 2.6: Shapes 3D Geometry Learning.

Shapes 3D Geometry Learning

Shapes 3D Geometry Learning is available on all major desktop and mobile operating systems. The target group of the application is students of grades 4-6. The application allows the user to select from a set of predefined solids. When a solid is selected, the user can view its edges, faces and vertices. The solid can be unfolded it into a net. The user may also build the net themselves, using the solid's faces. The net can be printed and subsequently cut out of the paper, to create a paper net. The paper net can be glued to obtain a paper version of the solid displayed in the application.

Shapes 3D Geometry Drawing

Shapes 3D Geometry Drawing is available only on the iOS operating system. The target group of the application is students of grades 7-8. It has a user interface and application structure similar to Shapes 3D Geometry Learning. The main differences is the tools available when a solid is selected. Shapes 3D Geometry Drawing allows the user to place line segments and cross sections between points on the solid's faces.

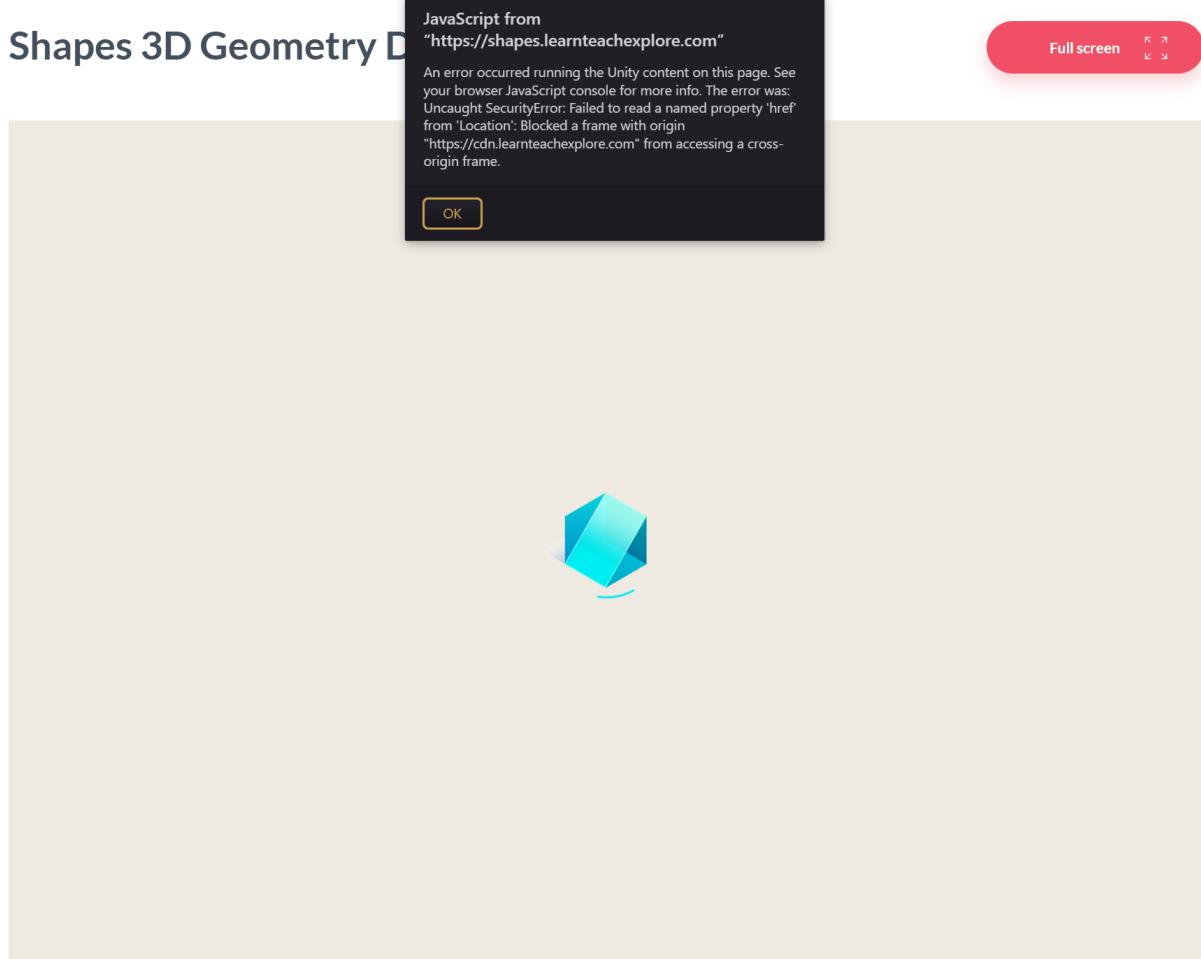


Figure 2.7: Shapes 3D Geometry Drawing. An error occurs when trying to run the demo.

Summary of Shapes 3D applications

Both applications contain many features which help visualize and explore 3D geometry. The applications lack features related to calculation of solids' characteristics like area and volume. They also do not allow the user to create their own solid and only give access to predefined solids. The predefined solids cannot be manipulated by the user, e.g the scale of the solid cannot be changed.

Chapter 3

Requirements and tools

3.1 Functional requirements

Visualization of 3D objects The application will be able to display three-dimensional solids (e.g. cube, prism, sphere, cylinder). The camera will allow for rotation and zooming.

Inspection of solids When hovering on a solid, its vertices or edges, the object will be highlighted to visually indicate that it is selected. Upon clicking a selected object, an informational label will appear. The informational label will contain information related to the object, e.g. the position of a vertex or length of an edge.

Manipulation of solids The application will allow the user to change the parameters of tasks. The changes in parameters will be reflected in the 3D view. The application will contain a module which will allow for creation and manipulation of custom polyhedrons.

Educational support The application will contain a task module. There will be a number of predefined tasks from the field of stereometry with varying degrees of difficulty. Each task will be divided into steps. Each step will involve calculation of one portion of the problem described by the task.

User interface The application will provide an intuitive graphical user interface. The application will support mouse and keyboard interaction. User interface elements will include tooltips to help the user understand how the application works.

3.2 Non-functional requirements

Usability The user interface will be straightforward to use and understand. It will not be required for the user to be familiar with other 3D software to use the application

effectively. Language within the application will be simple and use terms consistent with those used in mathematics education.

Performance The application will perform well on hardware capable of smoothly running other similar applications. There will be no noticeable stutters and delays when interacting with the application.

Reliability The application will not crash during runtime and gracefully handle invalid input.

Correctness The application will correctly calculate all characteristics of 3D objects.

Maintainability The application code will be open-source. The application will be structured in a way allowing easy future extension in the form of new tasks and features.

Security The application will not require internet access after installation. Manual editing of files used by the application will not compromise security.

Educational suitability The application will be suited for use in the classroom and at home. The provided tasks will be comparable to tasks from a school stereometry curriculum.

3.3 Used tools

3.3.1 Godot Engine

Godot¹ is a cross-platform, free and open-source game engine. It is designed to create 2D and 3D games mainly for the PC (personal computer), mobile and web platforms. Despite its main focus being the creation of games, it can be used to develop other types of software.

The engine itself is written mostly in C++. The primary language used in Godot to create software is GDScript. Other than GDScript, C# and C++ are also officially supported. Aside from the officially supported languages, support for other languages is provided by community extensions. Some of the community-supported languages are Rust, Swift and Java.

Godot introduces the concept of *nodes*. A node is the basic building block of the application. A node encapsulates a specific functionality related to 2D or 3D graphics, user interface, etc. Nodes are organized in a tree structure. Each node can have child nodes. This allows for the creation of complex behaviors through the composition of simple elements.

¹<https://godotengine.org>

A *scene* is a reusable composition of nodes. Scenes are typically used to represent logical parts of the application, such as user interface screens or 3D objects. A scene can be instantiated multiple times and can be composed of other scenes.

Through the use of nodes and scenes, Godot enables modularity and maintainability. This model is well suited for application which require interactive user interfaces and dynamic scene management.

3.3.2 Neovim

Neovim² is a modern and open-source text editor that runs in the terminal. Neovim was used as the primary editor for the application's source code.

Neovim provides features which include syntax highlighting and code completion. Plugins add additional functionality which increases the speed of editing, refactoring and navigation in the codebase.

The support for Godot projects in Neovim is enabled by the Language Server Protocol (LSP) [4]. The Godot editor runs an LSP server process to which Neovim connects. This provides features like syntax highlighting and code completion for GDScript in Neovim.

3.3.3 Git and GitHub

Git³ is a free and open-source distributed version control system [2]. It is used to track changes in files during software development. It allows developers to track the complete history of changes, revert to previous versions and manage development in a team.

In this project, Git was used to manage project files during the development. It allowed for tracking of changes and easy rollback in situations where new changes introduced issues. Access to the history of file modifications made locating the sources of bugs in code easier.

Github⁴ is an online platform which hosts Git repositories and provides additional tools for project management. It was used to provide a remote backup of the project and allow access to the project from multiple devices.

²<https://neovim.io>

³<https://git-scm.com>

⁴<https://github.com>

Chapter 4

External specification

4.1 System requirements

A desktop computer or a laptop is required the application. The application can run on the Windows, macOS and Linux operating systems. The application requires approximately 100 MB of storage space. The application was mainly tested on a laptop device with the following software and hardware configuration:

| | |
|------------------|-------------------------|
| CPU | AMD Ryzen 7 5800H |
| GPU | NVIDIA GeForce RTX 3050 |
| RAM | 16 GB |
| Operating system | Windows 11 |

The application performed smoothly on the device. Due to limited access to other hardware, the application could not be tested on low-end devices to establish minimum system requirements accurately. The Godot Engine documentation lists the following as the minimum system requirements [7]:

| | |
|------------------|---|
| CPU | x86_32 CPU with SSE2 support, x86_64 CPU with SSE4.2 support, ARMv8 CPU |
| GPU | Integrated graphics with full Vulkan 1.0 support, Metal 3 support (macOS) or Direct3D 12 (12_0 feature level) support (Windows) |
| RAM | 2 GB |
| Operating system | Windows 10, macOS 10.15, Linux distribution released after 2018 |

4.2 Installation

The application does not require installation on any supported platform. It is launched directly from the appropriate executable.

The application was tested only on the Windows operating system. As such, the instructions for other operating systems may contain errors.

4.2.1 Windows

For the Windows operating system, the application is provided as a single executable file. To start the application, the user should run the **GeoApp.exe** executable.

Depending on system security settings, Windows may display a warning about an unknown publisher. In this case, the user can confirm the warning to start the application. The executable is compiled for the x86-64 architecture.

4.2.2 Linux

For the Linux operating system, the application is provided as a single executable file. The user should ensure that the file has execution permissions. This can be done through the file manager or by executing the following command in the terminal:

```
chmod +x GeoApp.x86_64
```

When the file has execution permissions, the application can be started by running the **GeoApp.x86_64** executable. The executable is compiled for the x86-64 architecture.

4.2.3 macOS

On the macOS operating system, the application is provided in a **.zip** archive containing an application bundle. First, the user should extract the provided **GeoApp.zip** file. The user may then optionally move the extracted **GeoApp.app** file to the **Applications** folder. To start the application, the user should run the **GeoApp.app** application.

Since the application is not digitally signed, a security warning may appear when the application is started for the first time. When this happens, the user can start the application by right-clicking the **GeoApp.app** file, selecting **Open** and confirming the dialog. Alternatively, the application can be allowed in the system settings under **Privacy & Security**.

4.3 User manual

The application consists of several modules, namely *Task exploration*, *Task solving* and the *Playground*. Each module provides different ways for the user to learn stereometry.

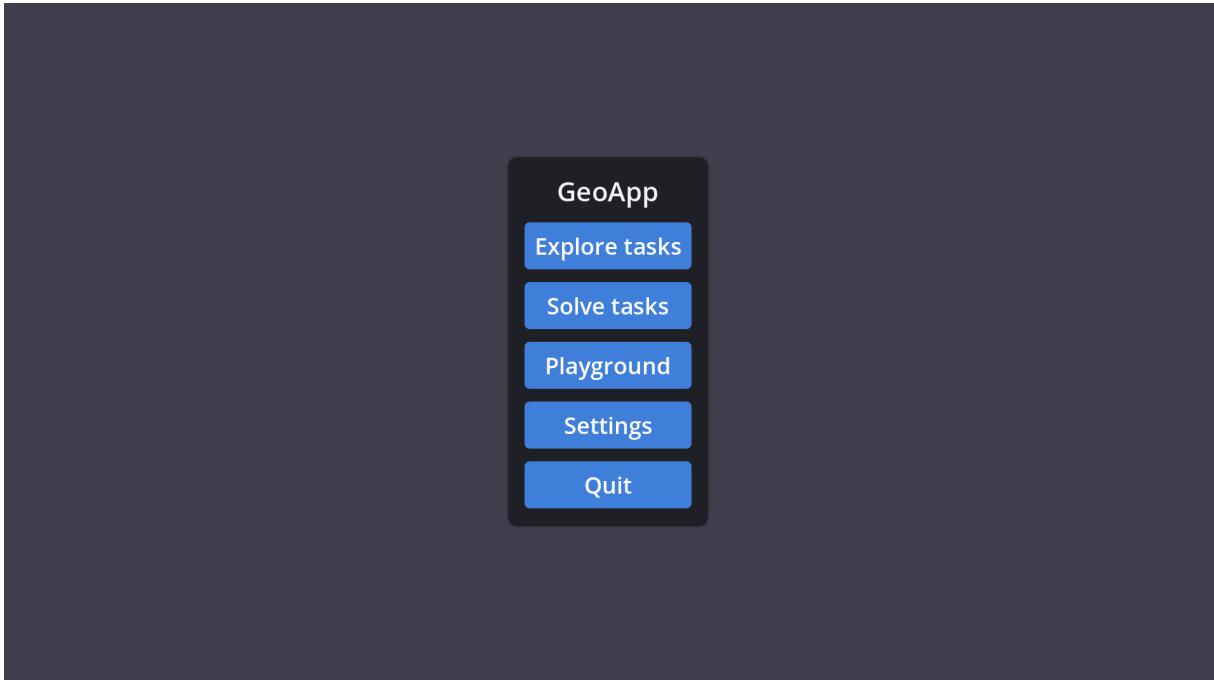


Figure 4.1: The main menu of the application.

4.3.1 Main menu

Upon starting the application, the user sees the main menu. The main menu is a central hub from which the rest of the application can be accessed. The main menu contains several button used to navigate the application.

- The **Explore tasks** button takes the user to the *Task exploration* module,
- The **Solve tasks** button takes the user to the *Task solving* module,
- The **Playground** button takes the user to the *Playground* module,
- The **Settings** button takes the user to the settings,
- The **Quit** button closes the application.

4.3.2 The 3D view

The 3D view is a part of the *Task exploration*, *Task solving* and *Playground* modules. When the 3D view is visible, the user can interact with it.

The camera of the 3D view can be moved. The camera orbits the center of the coordinate system and always looks in its direction. The camera can be rotated up, down, left and right with the **W**, **S**, **A** and **D** keys respectively. The camera can be zoomed in with the **E** key and zoomed out with the **Q** key.

When a solid is loaded in the 3D view, the user can inspect some of its properties. When an object (the solid, one of its vertices or one of its edges) is hovered over by the

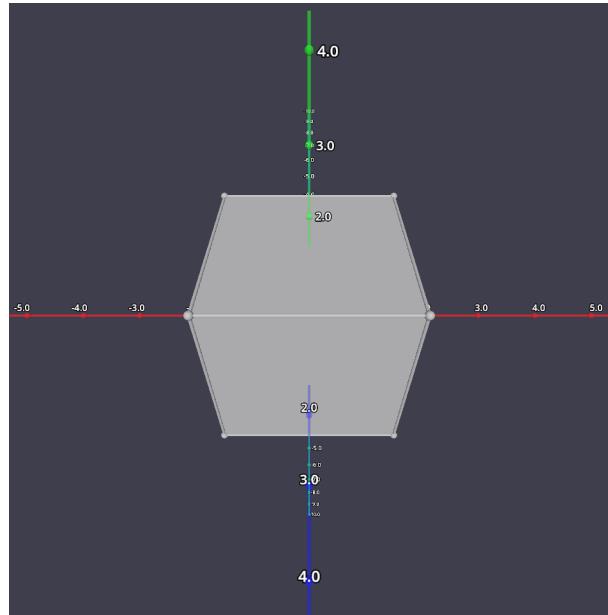


Figure 4.2: The 3D view as seen in the *Task exploration* and *Task solving* modules.

mouse cursor, it becomes red. When a hovered vertex or edge is clicked, an informational label appears. The informational label for a vertex contains its position. In the *Playground* module, the label also contains the name of the vertex. The informational label for an edge contains its length. When an object with a visible informational label is clicked, the informational label disappears.

4.3.3 The navigation bar

Each screen, except for the main menu, features a navigation bar. The navigation bar appears above the main content of the current screen. The navigation bar contains five buttons:

- The **Main menu** button transfers the user to the main menu,
- The **Explore tasks** button transfers the user to the task exploration screen,
- The **Solve tasks** button transfers the user to the task selection screen,
- The **Playground** button transfers the user to the playground screen,
- The button with the gear icon transfers the user to the settings screen.

The button corresponding to the current screen is disabled.

4.3.4 Task exploration

The purpose of the *Task exploration* module is to teach the user about the tasks available in the application. Ordered from left to right, the user interface features:

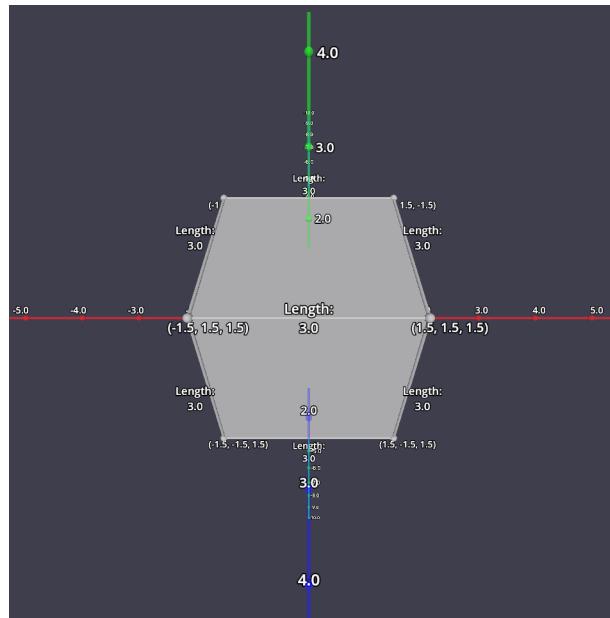


Figure 4.3: The 3D view as seen in the *Task exploration* and *Task solving* modules. The informational labels are visible

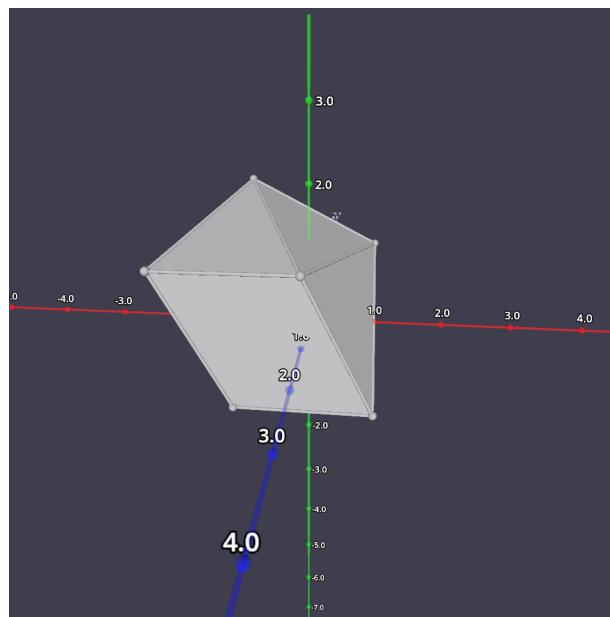


Figure 4.4: The 3D view as seen in the *Playground* module.

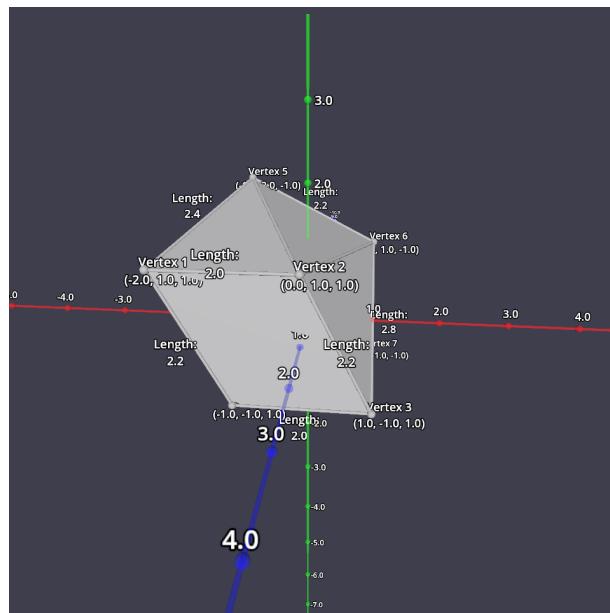


Figure 4.5: The 3D view as seen in the *Playground* module. The informational labels are visible



Figure 4.6: The navigation bar as seen in the task exploration screen.



Figure 4.7: The navigation bar as seen in the settings screen.

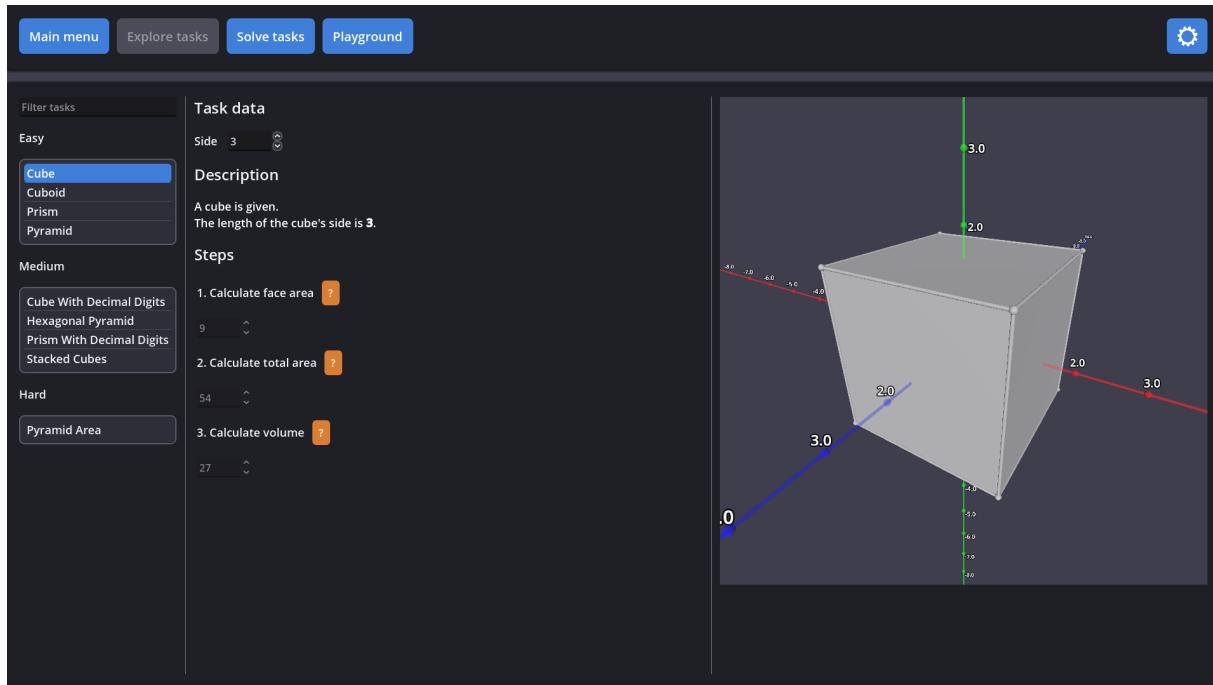


Figure 4.8: The *Task exploration* module.

- A list of all task, grouped by difficulty. The tasks can also be filtered by their name.
- All data describing the task. Each parameter can be changed. All UI (user interface) elements will update dynamically when a parameter is changed.
- The description of the task.
- A list of all of the steps required to solve the tasks. For each step, there is:
 - The description of what should be calculated for the step.
 - A help button. The button shows a hint for its respective step when pressed.
 - A number control which displays the correct answer for its respective step.
- A view of the selected task.

4.3.5 Task solving

The purpose of the *Task solving* module is to test the user's knowledge and skills gained in the *Task exploration* module. The *Task solving* module is split into two parts.

The first part is task selection. Here, the user selects the task they wish to solve. To select a task, the user must first select a difficulty. When a difficulty is selected, all available tasks with this difficulty are shown. Then, the user should select their desired task and press the **Start** button. The **Start** button can be pressed only when a task is selected.

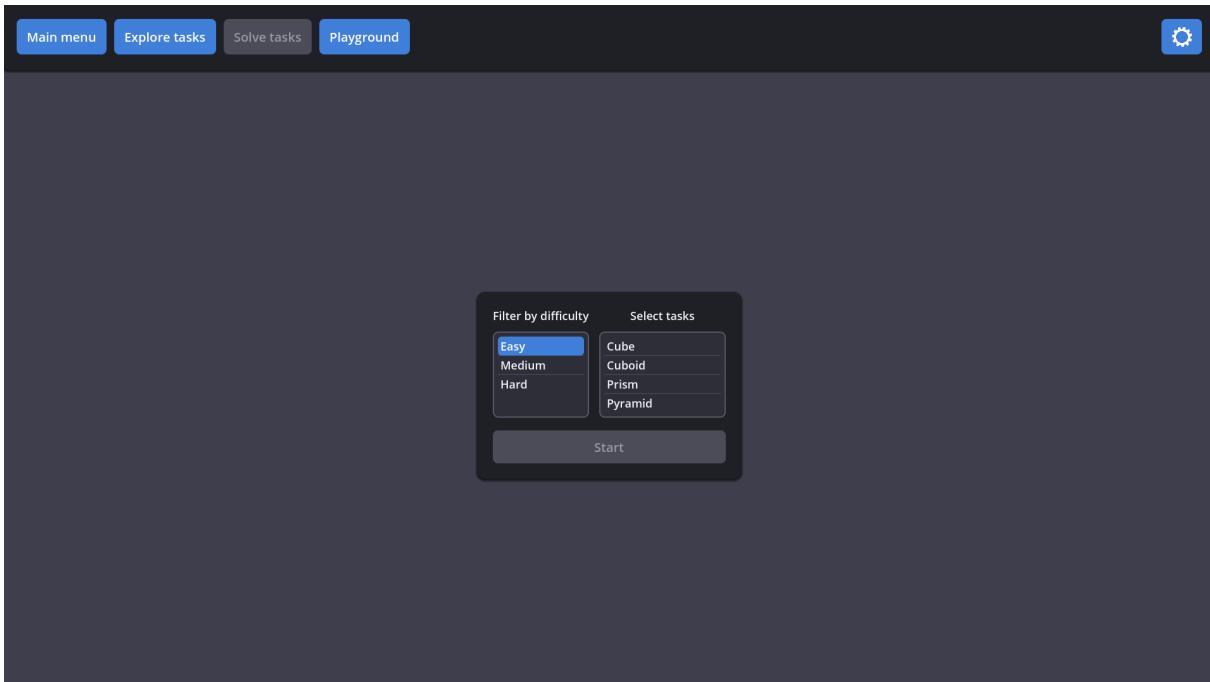


Figure 4.9: Task selection.

After pressing the **Start** button, the user is transported to the main part of the *Task solving* module.

The parameter values of the task are randomly picked from a predefined range. The range is defined on a per-parameter basis. The precision (the number of decimal digits of the value) is also defined on a per-parameter basis. Inside the main part of the *Task solving* module, the user can find:

- The view of the current task,
- The description of the task,
- The step navigation buttons,
- The *step container*,
- The **Check answer** button,
- The **Get new task** button.

The *step container* is the part of the UI responsible for displaying the task's steps. The step container looks similar to the one present in the *Task exploration* module. Despite that, the two differ in several aspects.

First, the user can interact with the number controls. The number controls are where the user puts their solutions to the task. The user may edit the value only of the current step's number control. If the user enters the correct value and presses the **Check answer**

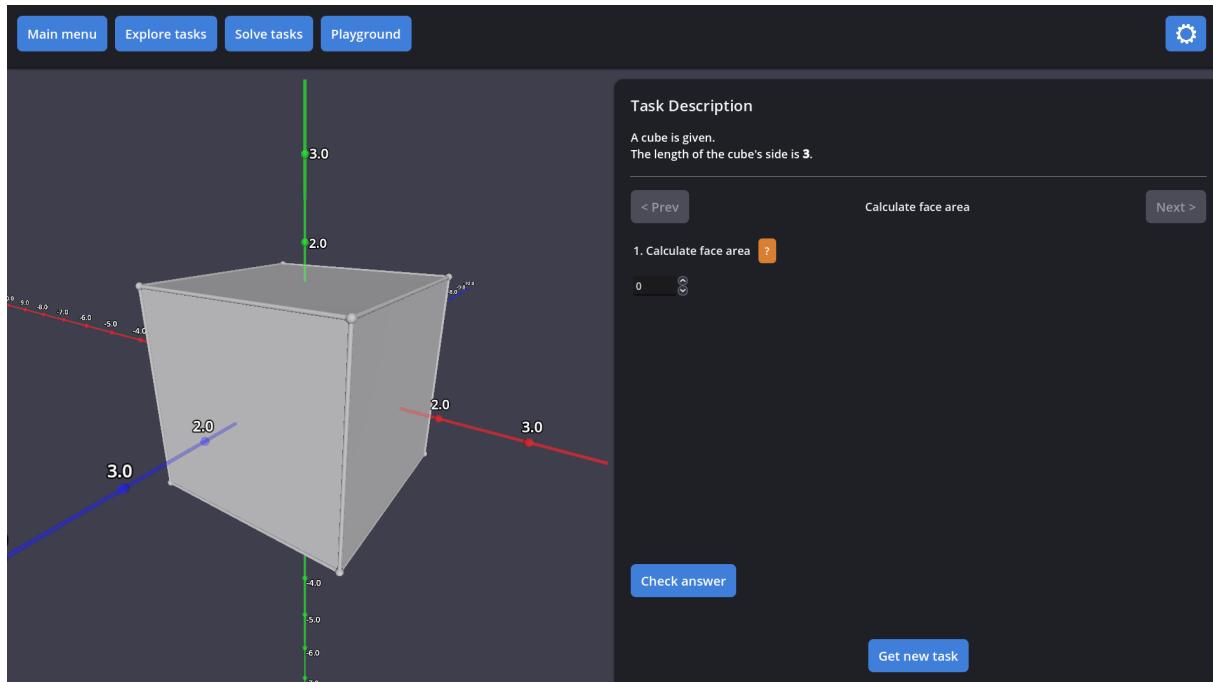


Figure 4.10: The default state of the task solving screen.

button, a green checkmark symbol appears to the right side of the number control. If the user enters an incorrect value, a red X symbol appears instead.

Second, only steps up to the current step are visible. The navigation buttons are used to move between steps. The **< Prev** button moves to the previous step. The **Next >** button moves to the next step. The buttons are active only when the user is able to change the current step. The **< Prev** button is inactive when the user is on the first step of the task. The **Next >** button is inactive when the user is on the last step of the task or the answer for the current step is incorrect.

When the user enters the correct answer for the last step of the task, the task is completed. When a task is completed, a label with the text "Task complete!" appears under the **Check answer** button.

The **Get new task** button resets the current task and provides the user with a new one with randomized values. Returning to the task selection screen and selecting the same task achieves the same goal.

4.3.6 Playground

The playground allows the user to create an arbitrary polyhedron and inspect its properties. The playground starts empty and the user can add vertices from which the polyhedron is created. The vertices can be added in several ways.

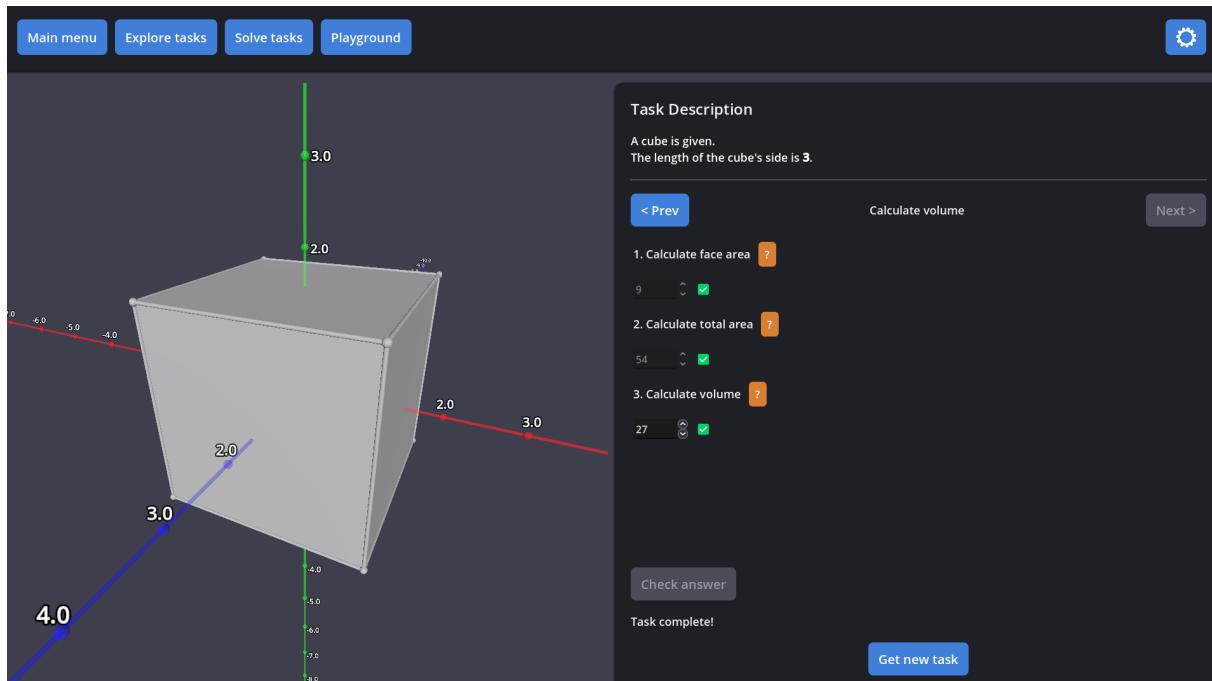


Figure 4.11: The task solving screen after the current task is completed.

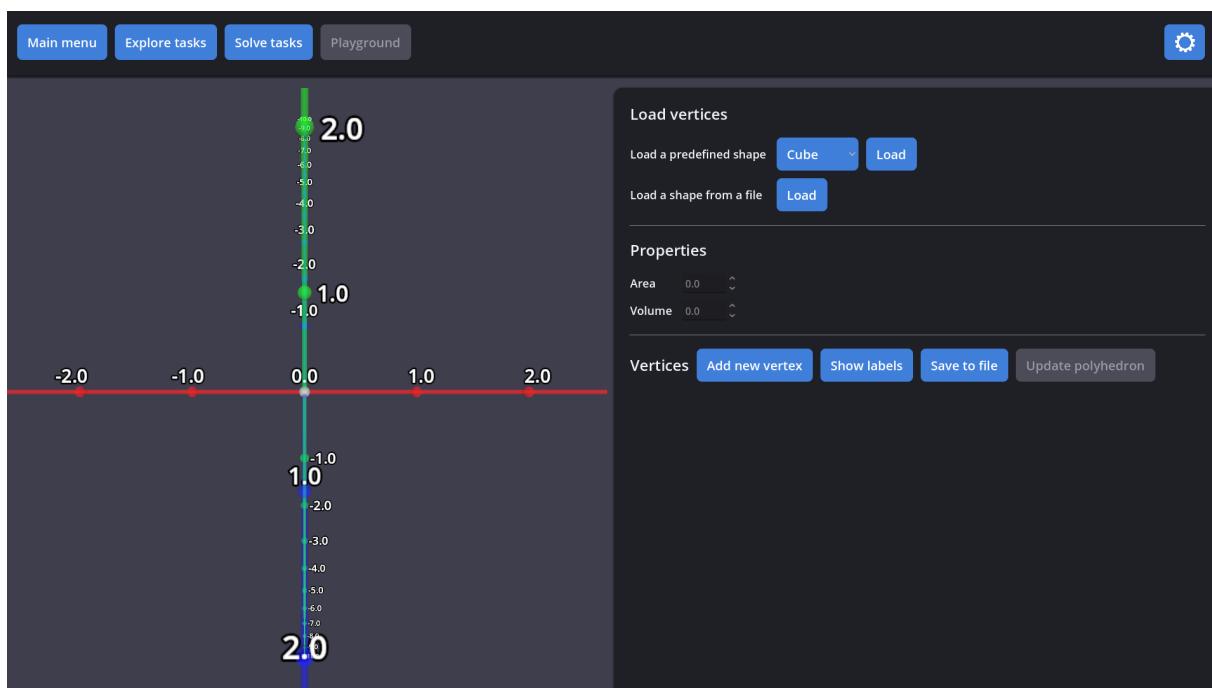


Figure 4.12: The default state of the playground module.

Adding vertices

The easiest way to add vertices is to load a predefined polyhedron. To do this, the user should select one of the predefined polyhedrons next to the "Load a polyhedron" label. Next, they should press the **Load** button to the right. This loads a predefined set of vertices.

Another way to load a set of vertices is to load them from a file. To do this, the user should press the **Load** button next to the "Load a polyhedron from a file" label. A system file dialog should appear, expecting the user to select a file with the `.poly` extension. No example `.poly` files are provided by default. A detailed explanation of `.poly` files is provided in section 4.3.6.

The last way to add a vertex is to use the **Add new vertex** button next to the "Vertices" label. This adds a new vertex in the origin of the coordinate system. When adding vertices this way, the polyhedron may become invalid. Polyhedron validity is explained in section 4.3.6.

The vertex list

The vertex list is the part of the UI where all of the added vertices are shown. Each polyhedron vertex corresponds to one entry in the vertex list. Each entry consists of:

- The editable name of the vertex. It is used for display only. It appears in the vertex informational label alongside the coordinates.
- Three number controls corresponding to the coordinates of the vertex. The first, second and third number controls correspond to the X, Y and Z coordinates respectively.
- The remove button. The vertex is removed when this button is pressed.
- A red "Duplicate vertex" warning. This warning appears only when there are multiple vertices with the same coordinates.

Changes in the vertex list are immediately reflected in the 3D view.

Polyhedron validity

When modifying vertices, the polyhedron may become invalid. The polyhedron is invalid when it cannot be created from the provided vertices. A polyhedron which contains concave faces is considered invalid. Otherwise, it is valid. When the polyhedron is valid:

- The polyhedron in the 3D view is filled, its vertices and edges are shown,
- The properties UI displays the values of the polyhedron's area and volume,

- No warning is shown.

When the polyhedron is invalid:

- Only the vertices are shown in the 3D view,
- The properties UI displays 0 for both area and volume,
- A warning in the form of a red exclamation mark is shown next to the "Vertices" label.

It may happen that the user adds two or more vertices with equal coordinates. All such vertices except the first one are marked as duplicate. Duplicate vertices show a red "Duplicate vertex" warning in the vertex list. When there are any duplicate vertices, the polyhedron is considered invalid.

Polyhedron files

A polyhedron file is a file that stores all of the data of one polyhedron. Polyhedron files have the `.poly` extension. They can be created in the *Playground* module. A polyhedron file is created with the use of the **Save to file** button near the "Vertices" label. When the **Save to file** button is pressed, a system file dialog appears. The user may save the polyhedron to a file even if the polyhedron is invalid. In the dialog, the user should specify the name of the file and where to save it. After the file is saved, it is ready to be loaded. The instructions on how to load a polyhedron file are provided in section 4.3.6.

4.3.7 Settings

The settings screen allows the user to change certain behaviors of the application. Settings are split into two sections. The *Camera* section contains two settings related to the camera of the 3D view:

- The *Move speed* setting determines the rotation speed. It is expressed in degrees per second.
- The *Zoom speed* setting determines the zoom speed. It is expressed in the 3D view world units per second.

The *Playground* section contains one setting related to the playground module. The *Coordinate precision* setting determines how precisely the vertices can be placed. It is expressed in the number of decimal digits.

The settings are stored in a **settings.cfg** file. The location of this file is platform-dependent:

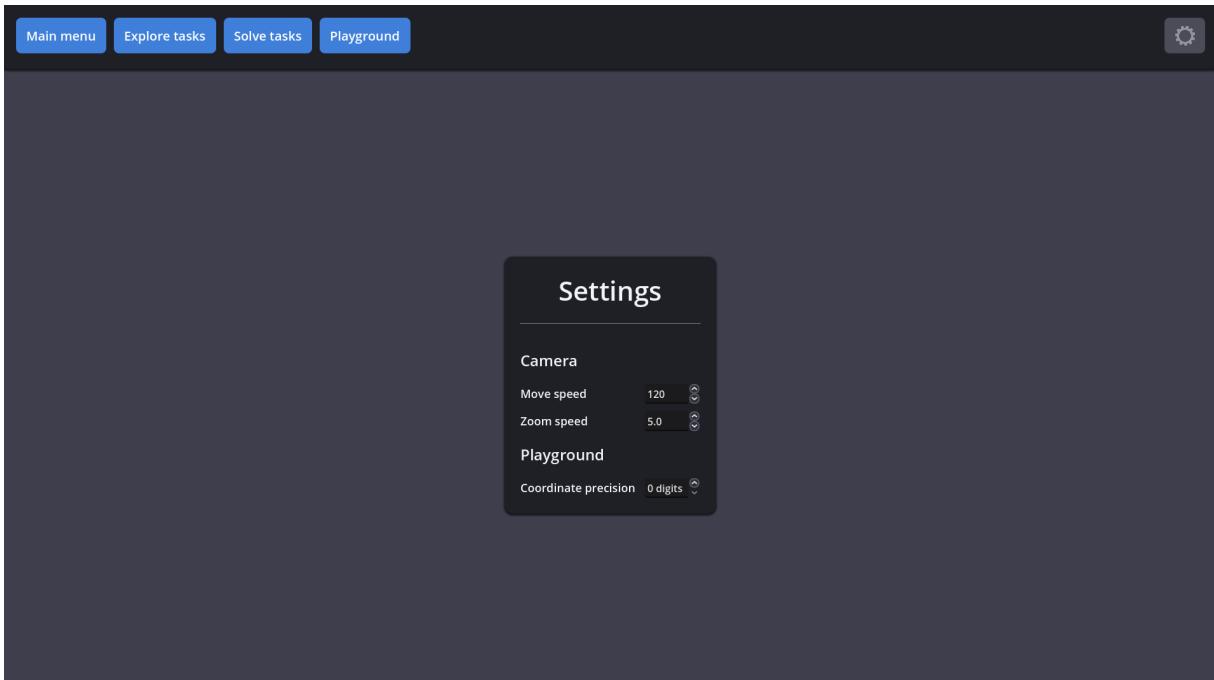


Figure 4.13: The settings screen.

| Platform | Location |
|----------|--------------------------------------|
| Windows | %APPDATA%\GeoApp |
| Linux | ~/.local/share/GeoApp |
| macOS | ~/Library/Application Support/GeoApp |

The user should not modify this file manually. If the file is modified in a way that the application cannot read it correctly, the settings are set to the default values. The file is then saved in the correct format. The same happens when the file is deleted.

If the user manually enters a value outside of the range defined by the setting, the value is constrained to the defined range. The file is not changed and contains the value entered manually.

With all settings set to default values, the **settings.cfg** has the following content:

```
[settings]

"Move speed"=120.0
"Zoom speed"=5.0
"Coordinate precision"=0
```


Chapter 5

Internal specification

5.1 Nodes and scenes

To understand the structure of a Godot project, one must understand the concept of *nodes* and *scenes*. A node represents a single logical part of the application. For example, a node of the `MeshInstance3D` type (class) represents the mesh of a 3D object. A node may have a script attached to extend its functionality. To be attached to a node, the script must extend the right class. A script attached to a `MeshInstance3D` node should extend the `MeshInstance3D` class or any class that is more general. For `MeshInstance3D`, this could be the general `Node3D` class. A script may be given a class name to create a user-defined class. A node can have multiple children, creating a tree structure.

A scene is a tree of nodes which can be instantiated multiple times. For instance, a simple scene representing a 3D object may be composed of an `Area3D` as the root node with a `CollisionShape3D` and a `MeshInstance3D` as its children. Such a scene would represent a 3D object which has a mesh and a collider.



Figure 5.1: A single `MeshInstance3D` node.

5.2 GeoApp structure

As with any Godot project, the `root` node is the top-level node which contains the entirety of the application. In this project, the root node contains two logical parts. The first and most important part is the `Main` scene. The other part are the *autoloads*.

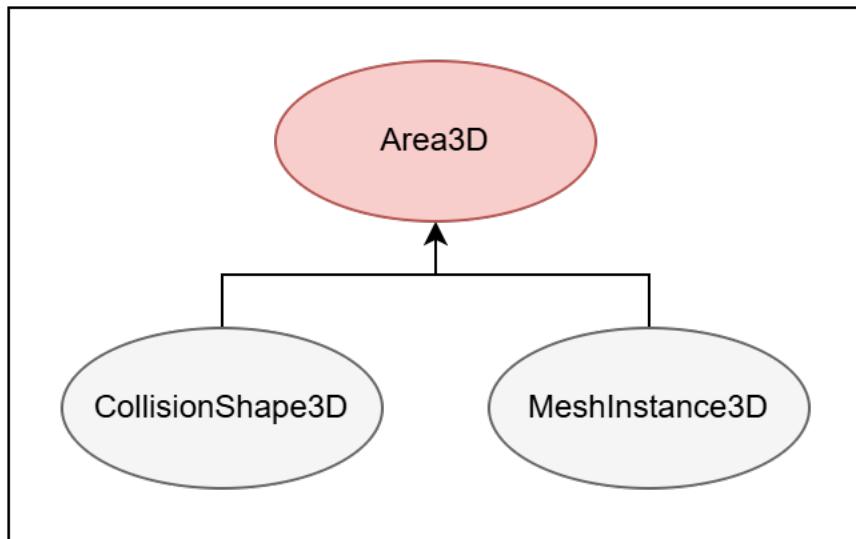


Figure 5.2: A simple 3D object scene. The red node is the root node of the scene.

5.3 The Main scene

The **Main** scene represents the whole UI of the application. It is composed of all of the UI screens. One of its children is the background of the application. Figure 5.3 shows the structure of the **Main** scene.

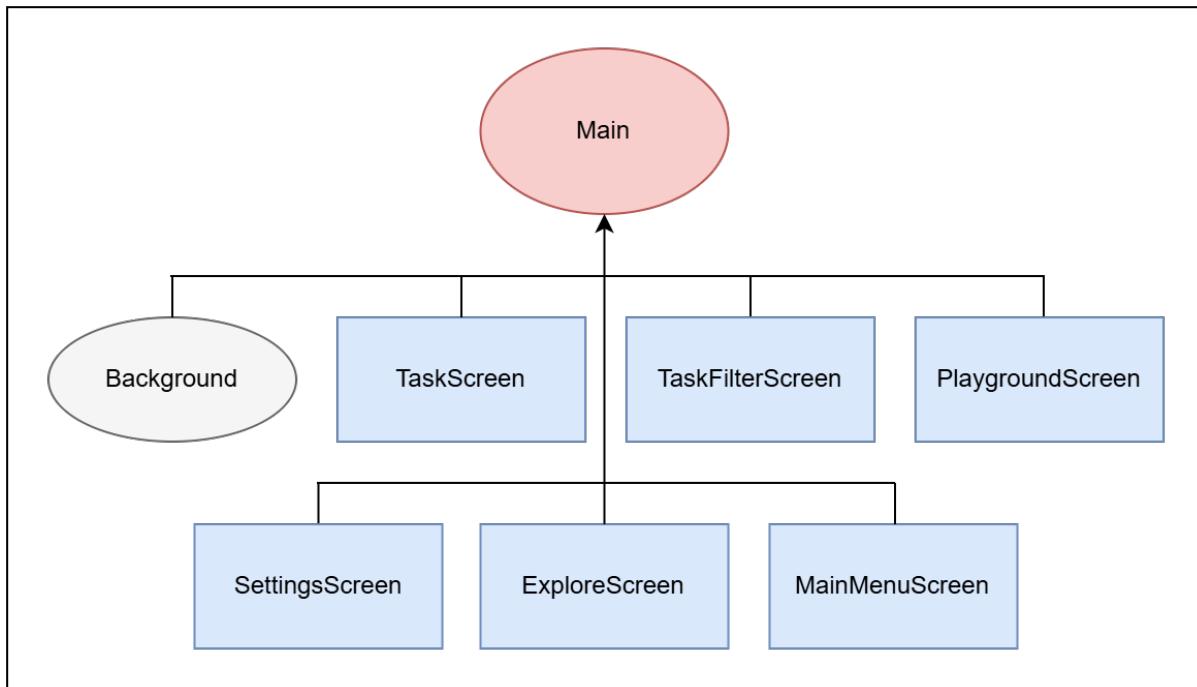


Figure 5.3: The Main scene.

The ellipses represent nodes, while rectangles represent scenes. The **Main** node has a script attached. The script controls transitions between screens. Each screen scene has a script attached. A screen's script is responsible for the internal logic of the screen.

5.4 Autoloads

Autoloads are Godot's implementation of the singleton design pattern. An autoload is a node which is loaded when the application starts. The members of an autoload are accessible from every node in the scene tree. The autoloads used in GeoApp are:

- **Global** - contains utility functions related to rounding of floating-point values and clearing of UI grids.
- **Poly** - contains the predefined vertices of certain polyhedrons and functions related to polyhedrons.
- **Settings** - contains all of the application's settings and functions for storing and loading them.
- **Tasks** - is responsible for loading all of the tasks.

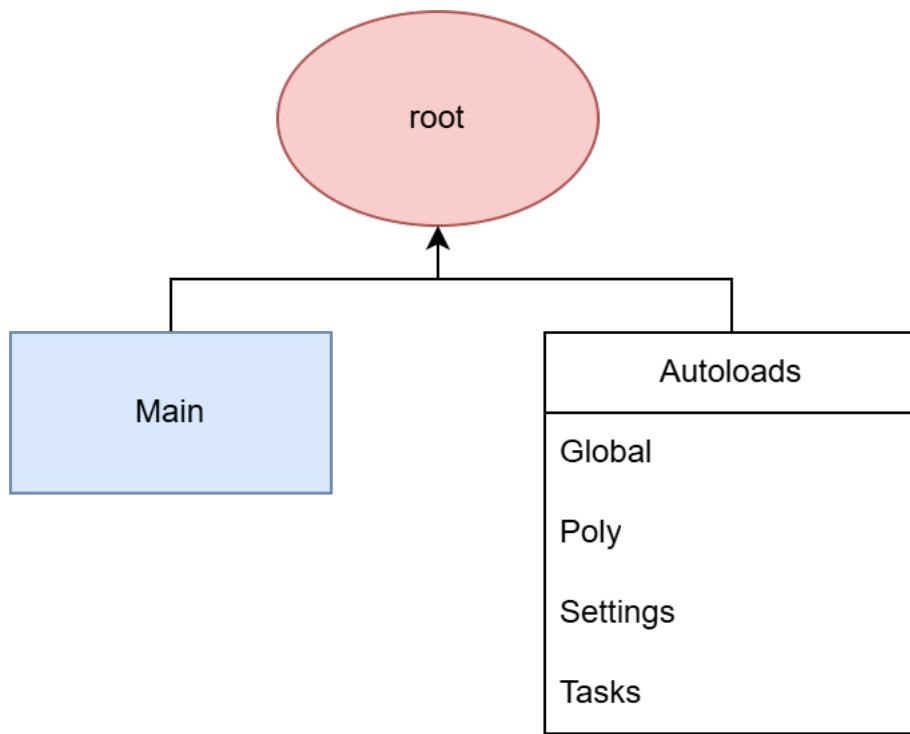


Figure 5.4: The complete application structure starting from the `root` node.

5.5 Signals

Signals are Godot's implementation of the Observer design pattern. Each node defines a set of signals. These signals can be connected to a node's method. This node may be any node in the scene tree, including the node which exposes the signal. The method is called when the signal is emitted. A signal is emitted when the action associated with it happens.

For example, a `Button` node's `pressed` signal is emitted when the button is pressed. The signal may be connected to a `_on_button_pressed` method which will be called whenever the button is pressed. The script containing the `_on_button_pressed` method may be attached to any node, including the button itself.

Signals allow for loose coupling between nodes. A node does not have to know where it is in the scene tree when communication with it is handled using signals.

5.6 Representation of solids

Each solid in the application is represented by a scene. The structure of the scene similiar to the one in figure 5.2. The difference is that the type of the root node is a class which inherits from the custom `Figure` class. For example, the root node of the cube scene if of the `Cube` type.

5.6.1 Figure

The `Figure` class is the base class of each solid. It:

- Stores the mesh and the collision shape,
- Handles behavior that is triggered when the solid is hovered,
- Contains the `area` and `volume` methods,
- Emits signals when the properties of the solid change.

5.6.2 Polyhedron

The `Polyhedron` class extends the `Figure` class. It represents a convex polyhedron. The class handles the additional vertex and edge objects. The class handles the additional objects that are spawned alongside the main body of the polyhedron. Those objects are the spheres on the vertices and cylinders on the edges. The class implements `Figure`'s `area` and `volume` methods to calculate these properties for any polyhedron.

Area

The total area of a polyhedron can be calculated as the sum of the areas of all faces. A face of a valid polyhedron¹ is a convex polygon. A convex polygon can be decomposed into a set of triangles. If the polygon is defined a list of vertices lying on a plane, the face can be decomposed by fixing one vertex and creating triangles from pairs of adjacent vertices.

¹This refers to valid polyhedron as defined in section 4.3.6

The area of each triangle is calculated using the magnitude of the cross product of two edge vectors. For a triangle with vertices v_1 , v_2 , and v_3 , its area A is calculated as:

$$A = \frac{1}{2} \| (v_2 - v_1) \times (v_3 - v_1) \| . \quad (5.1)$$

The total face area is the sum of the areas of all triangles.

Algorithm 1 Face area computation

```

area ← 0
for  $i \leftarrow 1$  to  $n - 2$  do
     $v_1 \leftarrow vertices[0]$ 
     $v_2 \leftarrow vertices[i]$ 
     $v_3 \leftarrow vertices[i + 1]$ 
    area ← area +  $\frac{1}{2} \| (v_2 - v_1) \times (v_3 - v_1) \|$ 
end for
return area

```

Volume

The volume of a polyhedron can be calculated by dividing it into pyramids. For each face of the polyhedron, the algorithm computes the volume of the pyramid associated with the face. The base of the pyramid is the corresponding face. Let q be any of the face's vertices, n the surface normal of the face and A the surface area of the face. The dot product of q and n is equal to the pyramid's height. The volume V_P of the pyramid is given by the formula:

$$V_P = \frac{1}{3}(q \cdot n) \cdot A \quad (5.2)$$

The total volume of the polyhedron is the sum of all of the pyramid volumes. This approach assumes that the polyhedron is closed and that all face normals point outward.

Algorithm 2 Polyhedron volume computation

```

V ← 0
for each face  $f$  in faces do
    Select three vertices  $v_1$ ,  $v_2$ ,  $v_3$  from  $f$ 
     $n \leftarrow \text{normalize}((v_3 - v_1) \times (v_2 - v_1))$ 
     $A \leftarrow \text{area}(f)$ 
     $q \leftarrow v_1$ 
     $V \leftarrow V + \frac{1}{3}(q \cdot n) \cdot A$ 
end for
return V

```

5.7 Tasks

Each task is a scene whose root node is of a type derived from the `Task` class. The `Task` class defines methods for the task difficulty, description, parameters and steps. Each derived task class then overrides those methods to create a unique task. The only children of the task scene's root node are the the solids that make up the task.

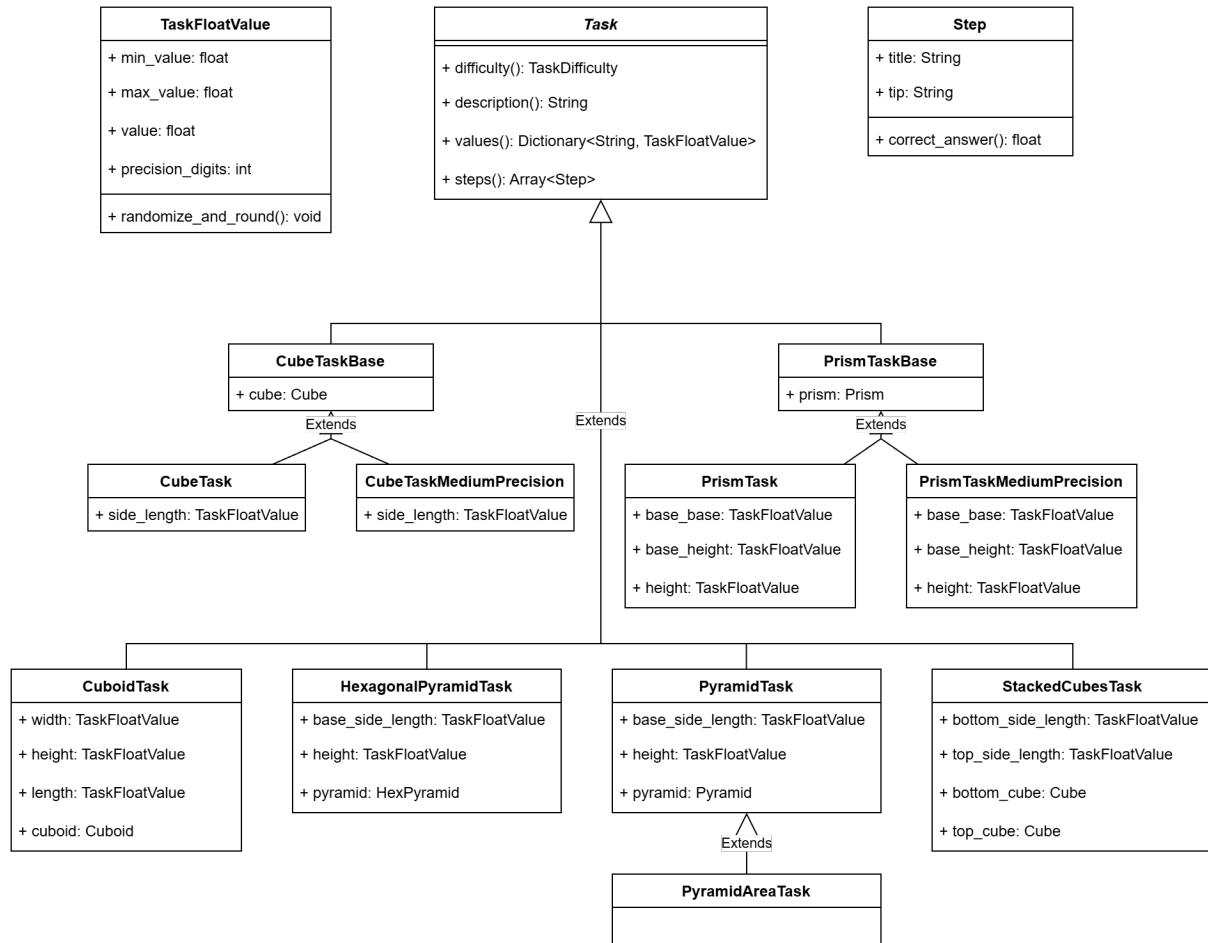


Figure 5.5: Task class diagram. Less important class members were omitted for conciseness.

5.8 User interface

User interface in Godot is created with the use of `Control` nodes. Control nodes used in GeoApp may be split into three categories: containers, interactive elements and labels. Containers contain other UI elements and are responsible for their layout. They are nodes that derive from the `Container` class. There are many types of containers. Each type of container lays out its children in a different way. For instance, the `HBoxContainer` lays out its children horizontally. The `HBoxContainer` container lays out its children vertically. Other types of containers include the `GridContainer` and `ScrollView`.

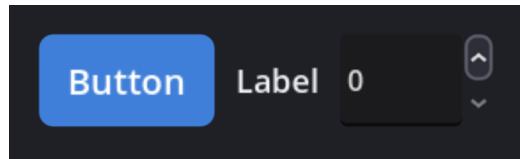


Figure 5.6: A Button, Label and SpinBox in a HBoxContainer.

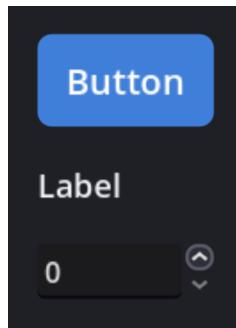


Figure 5.7: The same nodes in a VBoxContainer.

Interactive elements allow the user to interact with the application. They define signals which are emitted when the user interacts with them. The two main interactive elements in GeoApp are the **Button** and **SpinBox** nodes. The **Button** node exposes the **pressed** signal which is emitted when the button is pressed. The **SpinBox** node exposes the **value_changed** signal which is emitted when the value inside of the spin box changes.

Labels are non-interactive elements which display text. The text can be changed dynamically. Labels are represented by two nodes: **Label** and **RichTextLabel**. **Label** is the simpler of the two. It allows for displaying text without additional formatting. **RichTextLabel** allow the text to be formatted with the BBCode markup language [1]. BBCode allows sections of the text to be bolded, italicized or underlined. It also allows other format specifiers.

5.9 Polyhedron files

Polyhedron files store polyhedrons used in the playground module. They are introduced in section 4.3.6. This section explains their internal structure.

Polyhedron files store their content in the JSON (JavaScript Object Notation) format [5]. The file stores a JSON array of vertex objects. A vertex object is a JSON object with name-value pairs defined in table 5.1. A vertex object can contain other name-value pairs. Name-value pairs other than the expected ones will be ignored.

Table 5.1: Name-value pairs of a valid vertex JSON object.

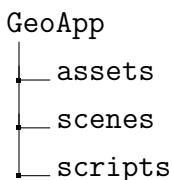
| Name | Value | Note |
|-------------|--------|--------------------------------|
| <i>name</i> | string | The name of the vertex |
| <i>x</i> | number | The x coordinate of the vertex |
| <i>y</i> | number | The y coordinate of the vertex |
| <i>z</i> | number | The z coordinate of the vertex |

Listing 5.1: The content of a polyhedron file containing two vertices.

```
1 [  
2   {  
3     "name": "Vertex 1",  
4     "x": -1.0,  
5     "y": 0.0,  
6     "z": 0.0  
7   },  
8   {  
9     "name": "Vertex 2",  
10    "x": 1.0,  
11    "y": 0.0,  
12    "z": 0.0  
13  }  
14 ]
```

5.10 Project file structure

Godot does not enforce a strict file structure. Any file can be placed anywhere in the project directory or its subdirectories. Despite that, files were organized into subdirectories to group them logically and reduce the size of the main directory. The project directory is divided into three subdirectories: **assets**, **scenes** and **scripts**. Visually, the project file structure looks like:



- **assets** contains non-code resources used by the application, such as icons, and the application's theme.
- **scenes** contains Godot scene files, which define the structure and composition of

the user interface and 3D objects. Each scene represents a self-contained part of the application.

- **scripts** contains script files implementing the application logic, including input hanlding and geometric calculations.

This structure improves clarity and simplifies project navigation.

Chapter 6

Verification and validation

- testing paradigm (eg V model)
- test cases, testing scope (full / partial)
- detected and fixed bugs
- results of experiments (optional)

Table 6.1: A caption of a table is **above** it.

| ζ | method | | | | | | | |
|---------|---------|---------|----------------|--------------|--------------|----------------------|----------------|--|
| | alg. 1 | alg. 2 | alg. 3 | | | alg. 4, $\gamma = 2$ | | |
| | | | $\alpha = 1.5$ | $\alpha = 2$ | $\alpha = 3$ | $\beta = 0.1$ | $\beta = -0.1$ | |
| 0 | 8.3250 | 1.45305 | 7.5791 | 14.8517 | 20.0028 | 1.16396 | 1.1365 | |
| 5 | 0.6111 | 2.27126 | 6.9952 | 13.8560 | 18.6064 | 1.18659 | 1.1630 | |
| 10 | 11.6126 | 2.69218 | 6.2520 | 12.5202 | 16.8278 | 1.23180 | 1.2045 | |
| 15 | 0.5665 | 2.95046 | 5.7753 | 11.4588 | 15.4837 | 1.25131 | 1.2614 | |
| 20 | 15.8728 | 3.07225 | 5.3071 | 10.3935 | 13.8738 | 1.25307 | 1.2217 | |
| 25 | 0.9791 | 3.19034 | 5.4575 | 9.9533 | 13.0721 | 1.27104 | 1.2640 | |
| 30 | 2.0228 | 3.27474 | 5.7461 | 9.7164 | 12.2637 | 1.33404 | 1.3209 | |
| 35 | 13.4210 | 3.36086 | 6.6735 | 10.0442 | 12.0270 | 1.35385 | 1.3059 | |
| 40 | 13.2226 | 3.36420 | 7.7248 | 10.4495 | 12.0379 | 1.34919 | 1.2768 | |
| 45 | 12.8445 | 3.47436 | 8.5539 | 10.8552 | 12.2773 | 1.42303 | 1.4362 | |
| 50 | 12.9245 | 3.58228 | 9.2702 | 11.2183 | 12.3990 | 1.40922 | 1.3724 | |

Chapter 7

Conclusions

7.1 Obtained result

The goal of the project was to create an educational application which made learning stereometry easier. The finished result achieves this goal with the use of three modules, namely task exploration, task solving and the playground.

7.2 Further development

The application meets the specified functional and non-functional requirements. Despite that, there is room for improving the existing features and adding new ones.

The keyboard controls for the 3D view currently cannot be modified. This could be improved by allowing the user to change the controls in the settings.

Another improvement for the playground is related to the polyhedron's vertices. Currently, the only way to move a vertex is through the vertex list. Another way of moving a vertex could be accomplished through the 3D view. Upon clicking a vertex, a control would appear. The control would allow the user to easily move the vertex on each axis.

The set of tasks is small and could thus be expanded. New tasks could feature new solids, including non-polyhedrons.

7.3 Encountered difficulties

Even though the used tools are well suited for the project, some issues arose during the development. The most common issue that appeared multiple times was screen state management. Some elements, like the polyhedron or task in the 3D view, were not being reset correctly when entering or exiting the screen they were in. This caused some properties to be set to null and the application to crash. Godot's debugger helped track the source of these issues. A potential solution would be to instantiate the screen from scratch when it

is entered. When done this way, care should be taken regarding the performance of such approach. Another issue was the class structure. The classes were refactored several times to accomodate for changing requirements. One example are classes representing solids like cube and prism. Originally, they were derived from the general `Figure` class. Later, a new `Polyhedron` class was added and the `Cube` and `Prism` classes were rewritten to inherit from it.

Bibliography

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- [7] *System requirements.* URL: https://docs.godotengine.org/en/stable/about/system_requirements.html (visited on 18/01/2026).

Appendices

Index of abbreviations and symbols

DNA deoxyribonucleic acid

MVC model–view–controller

N cardinality of data set

μ membership function of a fuzzy set

\mathbb{E} set of edges of a graph

\mathcal{L} Laplace transformation

Listings

(Put long listings here.)

```
1 if (_nClusters < 1)
2   throw std::string ("unknown number of clusters");
3 if (_nIterations < 1 and _epsilon < 0)
4   throw std::string ("You should set a maximal number of
      iteration or minimal difference --- epsilon .");
5 if (_nIterations > 0 and _epsilon > 0)
6   throw std::string ("Both number of iterations and minimal
      epsilon set --- you should set either number of iterations
      or minimal epsilon .");
```

List of additional files in electronic submission (if applicable)

Additional files uploaded to the system include:

- source code of the application,
- test data,
- a video file showing how software or hardware developed for thesis is used,
- etc.

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