Multiple Criteria Decision Making (MCDM) using TOPSIS

By

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Case Study I

M3 model is best

Model	Corr	R^2	RMSE	Accuracy
M1	0.79	0.62	1.57	60.89
M2	0.66	0.44	2.89	63.07
M3	**0.82**	**0.67**	**1.25**	**80.39**
M4	0.56	0.31	2.68	70.19
M5	0.75	0.56	1.3	62.87

Case Study II

Which model is best?

Model	Corr	Rseq	RMSE	Accuracy
M1	0.79	0.62	**1.25**	60.89
M2	0.66	0.44	2.89	63.07
M3	0.56	0.31	1.57	62.87
M4	**0.82**	**0.67**	2.68	70.19
M5	0.75	0.56	1.3	**80.39**

Poor Approach

We cannot add or average all the parameters.

Example:

Selection of a cricket player.

- Batsman
- Bowler
- All rounder (Hardik Pandya)

This is known as multiple-criteria decision making problem.

Solution is TOPSIS

(Technique for Order of Preference by Similarity to Ideal Solution)

Case Study II

Output

Model	Corr	Rseq	RMSE	Accuracy
M1	0.79	0.62	1.25	60.89
M2	0.66	0.44	2.89	63.07
M3	0.56	0.31	1.57	62.87
M4	0.82	0.67	2.68	70.19
M5	0.75	0.56	1.3	80.39

Topsis Score	Rank
0.55	5
0.87	1
0.6	4
0.79	2
0.66	3

Implementation in R

```
#install.packages("topsis")


library(topsis)
mydata=read.csv('data.csv')
d ← as.matrix(mydata[,-1])     // Drop 1st column
w ← c(1, 1, 1, 1)              // Weights
i ← c("+", "+", "-", "+")      // Impacts
topsis(d, w, i)
```

Applications

Selection of Mobile Phone

Attribute Or Criteria ⇨	Price or Cost	Storage Space	Camera	Looks
Mobile 1	250 $	16 GB	12 MP	Excellent
Mobile 2	200 $	16 GB	8 MP	Average
Mobile 3	300 $	32 GB	16MP	Good
Mobile 4	275 $	32 GB	8MP	Good
Mobile 5	225 $	16 GB	16 MP	Below Average

Other Applications

- Selection of Car
- Selection of Home
- Selection of Life Partner
- ………. and many more

Project Idea 1

Develop an APP for selection of singers for Indian Idol.

Singer ID	Sur	Taal	Laye	Pitch	Pace	Topsis Score	Rank
S1	0.79	0.62	1.25	60.89	11	0.55	5
S2	0.66	0.44	2.89	63.07	20	0.87	1
S3	0.56	0.31	1.57	62.87	16	0.6	4
S4	0.82	0.67	2.68	70.19	16	0.79	2
S5	0.75	0.56	1.3	80.39	20	0.66	3

Algorithmic approach give same result always

Some Points

1. Advantage: Algorithmic approach give same result always rather than human judge.
2. Organize a Hackthon with leaderboard.
3. Helpful in data collection
4. Validate the approach with multiple human Don't depend 100% on algorithm selection.
5. Reduction in labor intensive job; select top 10% or 20% or 30%.

Project Idea 2

- Selection of models for an Ad
- Selection of Photogenic face

ID	Eyes	Nose	Forehead	Lips	Chin	Topsis Score	Rank
S1	0.79	0.62	1.25	60.89	11	0.55	5
S2	0.66	0.44	2.89	63.07	20	0.87	1
S3	0.56	0.31	1.57	62.87	16	0.6	4
S4	0.82	0.67	2.68	70.19	16	0.79	2
S5	0.75	0.56	1.3	80.39	20	0.66	3

Self Study

Mathematics for Topsis

Learn the Mathematics for Topsis from Youtube.

https://www.youtube.com/watch?v=aRBdrCB1K4k

Mathematics of TOPSIS

Input: **Given Dataset of mobile phones**

Attribute Or Criteria ⇨	Price or Cost	Storage Space	Camera	Looks
Mobile 1	250 $	16 GB	12 MP	Excellent
Mobile 2	200 $	16 GB	8 MP	Average
Mobile 3	300 $	32 GB	16MP	Good
Mobile 4	275 $	32 GB	8MP	Good
Mobile 5	225 $	16 GB	16 MP	Below Average

Output: **Select the best Mobile**

Given*: **(1) Weights (2) Impacts**

* Assume yourself if not given

Step 1: Convert categorical to numeric

Convert Looks to Numerical

Using 5 point Scale	
Low	1
Below Average	2
Average	3
Good	4
Excellent	5

Attribute Or Criteria ⇨	Price or Cost	Storage Space	Camera	Looks
Mobile 1	250 $	16 GB	12 MP	Excellent
Mobile 2	200 $	16 GB	8 MP	Average
Mobile 3	300 $	32 GB	16MP	Good
Mobile 4	275 $	32 GB	8MP	Good
Mobile 5	225 $	16 GB	16 MP	Below Average

Attribute Or Criteria ⇨	Price or Cost	Storage Space	Camera	Looks
Mobile 1	250 $	16 GB	12 MP	5
Mobile 2	200 $	16 GB	8 MP	3
Mobile 3	300 $	32 GB	16MP	4
Mobile 4	275 $	32 GB	8MP	4
Mobile 5	225 $	16 GB	16 MP	2

Step 2.1: Vector Normalization

Calculate Root of Sum of Squares

$$\overline{X}_{ij} = \frac{X_{ij}}{\sqrt{\sum_{j=1}^{n} X_{ij}^2}}$$

Attribute Or Criteria	Price or Cost	Storage Space	Camera	Looks
Mobile 1	250 $	16 GB	12 MP	5
Mobile 2	200 $	16 GB	8 MP	3
Mobile 3	300 $	32 GB	16MP	4
Mobile 4	275 $	32 GB	8MP	4
Mobile 5	225 $	16 GB	16 MP	2
	564.579	53.0659	28	8.3666

For Price $250^2 + 200^2 + 300^2 + 275^2 + 225^2 = 318750$

$$\sqrt{318750} = 564.579$$

Step 2.2: Vector Normalization

Find Normalized Decision Metrix -

Divide every column value its Root of Sum of Squares

Attribute Or Criteria ⇨	Price or Cost	Storage Space	Camera	Looks
Mobile 1	$\frac{250}{564.579}$	$\frac{16}{53.0659}$	$\frac{12}{28}$	$\frac{5}{8.3666}$
Mobile 2	$\frac{200}{564.579}$	$\frac{16}{53.0659}$	$\frac{8}{28}$	$\frac{3}{8.3666}$
Mobile 3	$\frac{300}{564.579}$	$\frac{32}{53.0659}$	$\frac{16}{28}$	$\frac{4}{8.3666}$
Mobile 4	$\frac{275}{564.579}$	$\frac{32}{53.0659}$	$\frac{8}{28}$	$\frac{4}{8.3666}$
Mobile 5	$\frac{225}{564.579}$	$\frac{16}{53.0659}$	$\frac{16}{28}$	$\frac{2}{8.3666}$

Attribute Or Criteria ⇨	Price or Cost	Storage Space	Camera	Looks
Mobile 1	0.4428	0.3015	0.4286	0.5976
Mobile 2	0.3542	0.3015	0.2857	0.3586
Mobile 3	0.5314	0.6030	0.5714	0.4781
Mobile 4	0.4871	0.6030	0.2857	0.4781
Mobile 5	0.3985	0.3015	0.5714	0.2390

Value in every cell is known as Normalized performance value.

Step 3.1: Weight Assignment

Assign weight to every columns (0.25)

Weightage →	0.25	0.25	0.25	0.25
Attribute Or Criteria →	Price or Cost	Storage Space	Camera	Looks
Mobile 1	0.4428	0.3015	0.4286	0.5976
Mobile 2	0.3542	0.3015	0.2857	0.3586
Mobile 3	0.5314	0.6030	0.5714	0.4781
Mobile 4	0.4871	0.6030	0.2857	0.4781
Mobile 5	0.3985	0.3015	0.5714	0.2390

***Weights** can be (1,1,1,1) or (1,1,0.5,0.5) or (1,1,2,2)

Step 3.2: Weight Assignment

Calculate Weight × Normalized performance value

Weightage →	0.25	0.25	0.25	0.25
Attribute Or Criteria →	Price or Cost	Storage Space	Camera	Looks
Mobile 1	0.4428*0.25	0.3015*0.25	0.4286*0.25	0.5976*0.25
Mobile 2	0.3542*0.25	0.3015*0.25	0.2857*0.25	0.3586*0.25
Mobile 3	0.5314*0.25	0.6030*0.25	0.5714*0.25	0.4781*0.25
Mobile 4	0.4871*0.25	0.6030*0.25	0.2857*0.25	0.4781*0.25
Mobile 5	0.3985*0.25	0.3015*0.25	0.5714*0.25	0.2390*0.25

Weightage →	0.25	0.25	0.25	0.25
Attribute Or Criteria →	Price or Cost	Storage Space	Camera	Looks
Mobile 1	0.1107	0.0754	0.1071	0.1494
Mobile 2	0.0886	0.0754	0.0714	0.0896
Mobile 3	0.1328	0.1508	0.1429	0.1195
Mobile 4	0.1218	0.1508	0.0714	0.1195
Mobile 5	0.0996	0.0754	0.1429	0.0598

Known as weighted normalized decision matrix

Step 4: Find Ideal Best and Ideal Worst

- **Calculate <u>ideal best</u> value and <u>ideal worst</u> value**
- **Impacts: Price (-) Storage (+) Camera(+) Looks (+)**
  -ve means → min is best | +ve means → max is best

- **For price <u>min</u> value is best**
- **For storage, camera, Looks <u>max</u> value is best**

Weightage →	0.25	0.25	0.25	0.25
Attribute Or Criteria	Price or Cost	Storage Space	Camera	Looks
Mobile 1	0.1107	0.0754	0.1071	0.1494
Mobile 2	0.0886	0.0754	0.0714	0.0896
Mobile 3	0.1328	0.1508	0.1429	0.1195
Mobile 4	0.1218	0.1508	0.0714	0.1195
Mobile 5	0.0996	0.0754	0.1429	0.0598
V_j^+	0.0886	0.1508	0.1429	0.1494
V_j^-	0.1328	0.0754	0.0714	0.0598

V_j^+ = ideal best

V_j^- = ideal worst

Step 5: Calculate Euclidean distance

Calculate Euclidean distance from <u>ideal best</u> value and <u>ideal worst</u> value

$$S_i^+ = \left[\sum_{j=1}^{m} (V_{ij} - V_j^+)^2 \right]^{0.5}$$

$$S_i^- = \left[\sum_{j=1}^{m} (V_{ij} - V_j^-)^2 \right]^{0.5}$$

Step 5: Calculate Euclidean distance

Calculate Euclidean distance from ideal best value and ideal worst value

$$((0.1107 - 0\ 0886)^2 + (0.0754 - 0.1508)^2 + (0.1071 - 0.1429)^2 + (0.1494 - 0.1494)^2)^{0.5} = 0.0863$$

Attribute Or Criteria	Price or Cost	Storage Space	Camera	Looks	S_i^+	S_i^-
Mobile 1	0.1107	0.0754	0.1071	0.1494	0.0863	0.0990
Mobile 2	0.0886	0.0754	0.0714	0.0896	0.1198	0.0534
Mobile 3	0.1328	0.1508	0.1429	0.1195	0.0534	0.1198
Mobile 4	0.1218	0.1508	0.0714	0.1195	0.0842	0.0968
Mobile 5	0.0996	0.0754	0.1429	0.0598	0.1176	0.0788
V_j^+	0.0886	0.1508	0.1429	0.1494		
V_j^-	0.1328	0.0754	0.0714	0.0598		

$$S_i^+ = \left[\sum_{j=1}^{m} (v_{ij} - v_j^+)^2 \right]^{0.5} \qquad S_i^- = \left[\sum_{j=1}^{m} (v_{ij} - v_j^-)^2 \right]^{0.5}$$

Step 6: Calculate Performance Score

Calculate Performance Score

TOPSIS

performance score

Attribute Or Criteria	S_i^+	S_i^-	$S_i^+ + S_i^-$	P_i
Mobile 1	0.0863	0.0990	0.1853	0.534269
Mobile 2	0.1198	0.0534	0.1732	0.308314
Mobile 3	0.0534	0.1198	0.1732	0.691686
Mobile 4	0.0842	0.0968	0.181	0.534807
Mobile 5	0.1176	0.0788	0.1964	0.401222

$$P_i = \frac{S_i^-}{S_i^+ + S_i^-}$$

Step 7: TOPSIS Score and Rank

Final Result

Attribute Or Criteria	P_i	Rank
Mobile 1	0.534269	3
Mobile 2	0.308314	5
Mobile 3	0.691686	1
Mobile 4	0.534807	2
Mobile 5	0.401222	4

Assignment

Given Input Data

Impacts: Corr(+) Rseq (+) RMSE (-) Accuracy (+)

Model	Corr	Rseq	RMSE	Accuracy
M1	0.79	0.62	1.25	60.89
M2	0.66	0.44	2.89	63.07
M3	0.56	0.31	1.57	62.87
M4	0.82	0.67	2.68	70.19
M5	0.75	0.56	1.3	80.39

Find the TOPSIS score and model rank for:

1. Weights (1, 1, 1, 1)
2. Weights (1, 1, 2, 1)
3. Weights (1, 1, 0.5, 1)
4. Weights (2, 2, 1, 1)
5. Weights (2, 2, 0.5, 1)

Project Work 1

1. Learn the mathematics for TOPSIS from given below link.

   https://www.youtube.com/watch?v=aRBdrCB1K4k

2. Implement the technique in python and develop a cmd line solution (i.e. run through cmd line), <u>handle all the exceptions like file format, number of parameters,</u> etc :

   Usages: python topsis.py <InputDataFile> <Weights> <Impacts> <ResultFileName>

   Example: python topsis.py myData.csv “1,2,1,1” “+,+,-+” result.csv

3. Develop a python package.
4. Test and validate it on different datasets
5. Upload the package on pypi.org

Thanks

Learning by Doing

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