Complex Numbers $z = re^{j\theta}$ where $r = \sqrt{x^2 + y^2}$ and $\theta = \tan^{-1}(\frac{y}{x})$ **Avg Power** $p_{avg} = V_m I_m \cos(\theta_v - \theta_i)$

Phasors

Time-domain: $v(t) = V_{max}\cos(\omega t + \theta_v)$, Phasor: $V = V_m\angle\theta_v$ where $V_m = \frac{V_{max}}{\sqrt{2}}$

Time-domain: $i(t) = I_{max}\cos(\omega t + \theta_i)$, Phasor: $I = I_m\angle\theta_i$ where $I_m = \frac{I_{max}}{\sqrt{2}}$

Power

$$p(t) = V_m I_m[\cos(2\omega t + \theta_v + \theta_i) + \cos(\theta_v - \theta_i)]$$

Purely resistive load

$$p(t) = v(t)i_R(t) = V_{max}I^R_{max}\cos^2(\omega t + \theta_v) = VI_R(1 + \cos(2[\omega t + \theta_v]))$$

Purely inductive load

$$p(t) = v(t)i_R(t) = V_{max}I^L_{max}\cos(\omega t + \theta_v)\cos(\omega t + \theta_v - 90^\circ) = VI_L(\sin(2[\omega t + \theta_v]))$$

Power Factor

PF is lagging if $\theta_i < \theta_v$ (e.g. inductive loads)

PF is leading if $\theta_i > \theta_v$ (e.g. capacitive loads)

General RLC load

Let $I_m\cos(\theta_v - \theta_i) = I_R$ and $I_m\cos(\theta_v - \theta_i) = I_X$ $\quad p(t) = V_m I_R(1 + \cos[2(\omega t + \theta_v)]) + V_m I_X \sin[2(\omega t + \theta_v)]$

Complex Power

$$S = VI^* = V_m I_m\angle(\theta_v - \theta_i) = V_m I_m\cos(\theta_v - \theta_i) + jV_m I_m \sin(\theta_v - \theta_i)$$

Real power (W) $\qquad$ Reactive power (VAR)

$$P = V_m I_m \cos(\theta_v - \theta_i) \qquad Q = V_m I_m \sin(\theta_v - \theta_i)$$

3 Phase Circuits

line-to-line voltages between the phases

$$E_{ab} = E_{an} - E_{bn}, \quad \text{Let } E_{an} = |E|\angle 0^\circ$$
$$E_{ab} = |E|\angle 0^\circ - |E|\angle{-120^\circ}$$
$$E_{ab} = |E| - |E|\frac{\left(-1 - j\sqrt{3}\right)}{2}$$
$$E_{ab} = |E|\sqrt{3}\angle 30^\circ$$

$$E_{bc} = E_{bn} - E_{cn}$$
$$E_{bc} = |E|\sqrt{3}\angle{-90^\circ}$$
$$E_{bc} = E_{bn} - E_{cn}$$
$$E_{bc} = |E|\sqrt{3}\angle 150^\circ$$

$$E_{ab} = \sqrt{3}E_{an}\angle 30^\circ$$
$$E_{bc} = \sqrt{3}E_{bn}\angle 30^\circ$$
$$E_{ca} = \sqrt{3}E_{cn}\angle 30^\circ$$

Convert Y - Delta

$Z_Y = Z_\Delta/3$

Line currents for the Y - Y

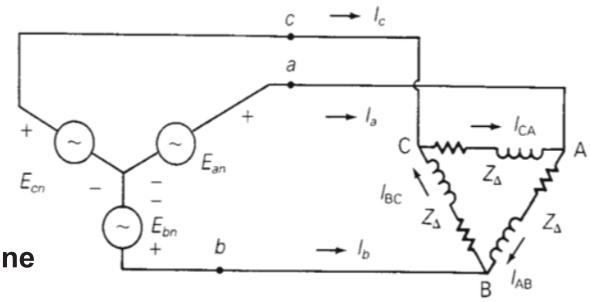
$$I_a = E_{an}/Z_y = E_{an}/(|Z_y|\angle\theta_z) = E_{an}/|Z_y|\angle{-\theta_z}$$
$$I_b = E_{bn}/Z_y = E_{bn}/(|Z_y|\angle\theta_z) = E_{bn}/|Z_y|\angle(-120^\circ - \theta_z)$$
$$I_c = E_{cn}/Z_y = E_{bn}/(|Z_y|\angle\theta_z) = E_{bn}/|Z_y|\angle(120^\circ - \theta_z)$$

Line current for Y- Delta

$$I_a = I_{AB} - I_{CA} = \sqrt{3}I_{AB}\angle{-30^\circ}$$
$$I_b = I_{BC} - I_{AB} = \sqrt{3}I_{BC}\angle{-30^\circ}$$
$$I_c = I_{CA} - I_{BC} = \sqrt{3}I_{CA}\angle{-30^\circ}$$

Load current for Y-Delta

$$I_{AB} = E_{ab}/Z_\Delta = \sqrt{3}|E|/|Z_\Delta|\angle(30^\circ - \theta_z)$$
$$I_{BC} = E_{bc}/Z_\Delta = \sqrt{3}|E|/|Z_\Delta|\angle(30^\circ - \theta_z - 120^\circ) = \sqrt{3}|E|/|Z_\Delta|\angle(-90^\circ - \theta_z)$$
$$I_{CA} = E_{ca}/Z_\Delta = \sqrt{3}|E|/|Z_\Delta|\angle(30^\circ - \theta_z + 120^\circ) = \sqrt{3}|E|/|Z_\Delta|\angle(150^\circ - \theta_z)$$



Instantaneous Power: Balanced

$$\begin{aligned} p_{3\phi}(t) &= p_a(t) + p_b(t) + p_c(t) \\ &= V_{LN}I_L\cos(\delta - \beta) + V_{LN}I_L\cos(2\omega t + \delta + \beta) \\ &+ V_{LN}I_L\cos(\delta - \beta) + V_{LN}I_L\cos(2\omega t + \delta + \beta - 240^\circ) \\ &+ V_{LN}I_L\cos(\delta - \beta) + V_{LN}I_L\cos(2\omega t + \delta + \beta + 240^\circ) \\ &= 3V_{LN}I_L\cos(\delta - \beta) \end{aligned}$$

In terms of Line to line

Recall: $V_{LN} = V_{LL}/\sqrt{3}$

$$p_{3\phi}(t) = \sqrt{3}V_{LL}I_L\cos(\delta - \beta)$$

Complex power 3 phase

$$S_{3\phi} = 3V_{LN}I_L\cos(\delta - \beta) + j3V_{LN}I_L\sin(\delta - \beta) \quad P_{3\phi} = \sqrt{3}V_{LL}I_L\cos(\delta - \beta)\ \text{W} \quad Q_{3\phi} = \sqrt{3}V_{LL}I_L\sin(\delta - \beta)\ \text{var}$$

$$|S_{3\phi}| = 3V_{LN}I_L = \sqrt{3}V_{LL}I_L\ \text{VA}$$