unit-2

Linear programming problem (L.P.P):

Definition:-

The General L.P.P called for optimizing (minimizing (or) maximizing) a Linear function of variables called the "objective function", subjected to the linear equation and (or) in equality's called the "constraint & Restriction"

General Formation of LPP:-

In order to find the 'n' decision variables of the objective function maximizing (or) minimizing of

$$z = c_1x_1 + c_2x_2 + \cdots\cdots + c_nx_n$$

$$\ni a_{11}x_1 + a_{12}x_2 + \cdots\cdots + a_{1n}x_n [\leq (or) \geq (or) \geq] b_1$$

$$a_{21}x_1 + a_{22}x_2 + \cdots\cdots + a_{2n}x_n [\leq (or) \geq (or) \geq] b_2$$

$$a_{m1}x_1 + a_{m2}x_2 + \cdots\cdots + a_{mn}x_n [\leq (or) \geq (or) \geq] b_2$$

h are m linear equation (or) constraints and all $x_1, x_2 \ldots x_n$ are greater than zero ($x_i \geq 0, \forall x = 1 \ldots n$)

NOTE 1:- $\max z = cx$

$$\ni \geq Ax (\leq) b$$
$$(z)$$

NOTE 2 :- $\min z = cx$

$$\ni Ax (\geq) b$$
$$(z)$$

SLACK variables :-

In order to reduce the constraints $\leq$ type to make equal we needed to add a variable, same x_i for sum(i) is called a slack variable.

Eg :- $2x+y = 9$

$\leq$ type we need to add 'a'

$2x+y+a = 9$

$\downarrow$

slack variable

where 'a' is called slack variable.

Surplues variables :-

In order to reduce the constraints $\geq$ type to make equal we needed to subtract a variable some x_i for sum i, is called surplues variable.

Eg :- $2x+4y \geq 3$

$x \geq 1$; $y=1$

$2+4-a = 3$

where 'a' is called surplues variable

Artificial variable :-

In order to reduce the constraints $\geq$ type to make equal we needed to subtract a variable some x_i for sum i. it does not have basis matric we needed to added some (bi) then the variables of bi called arítifical variable

Eg :- $x_1+2x_2-x_3+a_1=3$.

$2x_1+5x_2-x_4+a_2=9$.

$$\begin{bmatrix} 1 & 2 & -1 & 0 & \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \\ 2 & 5 & 0 & -1 & \\ x_1 & x_2 & x_3 & x_4 & a_1 \quad a_2 \end{bmatrix}$$

where x_3, x_4 - surplus variable

a_1, a_2 - Artifical variable:

stand form of linear programing problem:-

the given L.P.P is maximization type of the objective function.

$$\max z = c_1x_1+c_2x_2+ \cdots +c_nx_n$$

$$\exists = a_{11}x_1+a_{12}x_2+ \cdots +a_{1n}x_n \leq b_1$$

$$a_{m1}x_1+a_{m2}x_2+ \cdots +a_{mn}x_n \leq b_n$$

we need to added that slack variable of the above 'm' constant then

$$a_{11}x_1+a_{12}x_2+ \cdots +a_{1n}x_n+x_{n+1}=b_1$$

$$a_{21}x_1+a_{22}x_2+ \cdots +a_{2n}x_n+x_{n+2}=b_2$$

$$a_{m1}x_1+a_{m2}x_2+ \cdots +a_{m2}x_n+x_{m+n}=b_n$$

Matrix form of L.P.P:-

$$C = [c_1, c_2 \cdots\cdots c_n \; 1, 0 \cdots\cdots 0]_{1\times(m+n)}$$

$$A_{m\times n} = \begin{bmatrix} a_{11} & a_{12} & \cdots & a_{1n} & 1 & 0 & \cdots & 0 \\ a_{21} & a_{22} & \cdots & a_{2n} & 0 & 1 & \cdots & 0 \\ \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots \\ a_{m1} & a_{m2} & \cdots & a_{mn} & 0 & \cdots & \cdots & 1 \end{bmatrix}$$

$$x = \begin{bmatrix} x_1 \\ x_2 \\ x_n \\ x_{n+1} \\ x_{m+n} \end{bmatrix}$$

$$\max z = \min z = cx$$

$$AX = B, \quad X > 0$$

Graphical method :-

Extremum point :-

A point (x, y) is an extremum point if it is maximum (or) minimum exist.

① Solve L.P.P for graphically

$$\text{Min } z = 3x + 4y$$

→

$$3x + 4y \geq 12.$$

$$12x + 3y \geq 12$$

$$x \geq 0, \quad y \geq 0$$

sol. Given that

$\min z = 3x+4y \rightarrow M \rightarrow ①$

$\ni$

$3x+4y \geq 12$

$12x+2y \geq 12$

we need to write standard form

$\text{Max } z = 3x+4y$

$\ni$

$3x+4y=12 \rightarrow (a)$

$12x+2y=12 \rightarrow (b)$

from eq^n (a)

put $x=0, y=3$

The point A (0,3)

put $y=0, x=4$

The point B (4,0)

from eq (b)

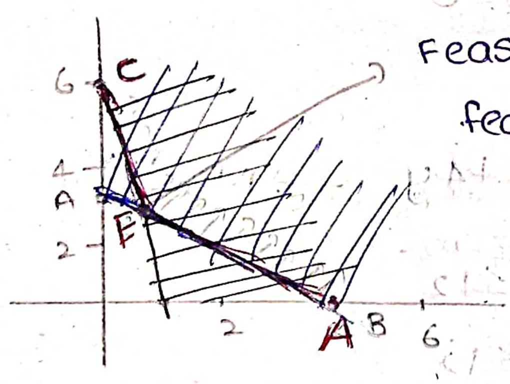
put $x=0, y=6$

The point c (0,6)

put $y=0, x=1$

The point of D (1,0)

Draw the Graph of above point (extreme point)



CEA

Feasable region & unbounded

feasable sol^n: c, e, B

Feasable means common region.

from eq^n (a) & (b)

$3x+4y=12$

$12x+2y=12$

The point $E\left(\frac{4}{7}, \frac{18}{7}\right)$

∴ The feasable region CEB & unbounded region

∴ The feasable regi soln C, E, B

from eq ①, $M = 3x+4y$

at $C(0,6)$, $M = 3(0) + 4(6) = 24$

at $E\left(\frac{4}{7}, \frac{18}{7}\right)$, $M = 3\left(\frac{4}{7}\right) + 4\left(\frac{18}{7}\right) = 12$

at $B(4,0)$, $M = 3(4) + 4(0) = 12$

The optimal soln Min $z = 12$

at E & B.

∴ Here Alternative optimal soln.

② Solve L.P.P

$\text{Max } z = 3x+4y$

∋

$3x+4y \leq 12$

$12x+2y \leq 12$

$x \geq 0, y \geq 0$

sol: Given that,

$\text{Max } z = 3x+4y \rightarrow M \rightarrow ①$

∋

$3x+4y \leq 12$

$12x+2y \leq 12$

$x \geq 0, y \geq 0$

We need to write the standard form

min $z = 3x + 4y$

$\Rightarrow$

$3x + 4y = 12 \rightarrow (a)$

$12x + 2y = 12 \rightarrow (b)$

from eq (a)

put $x = 0$, $y = 3$

The points A (0,3)

put $y = 0$, $x = 4$

The points B (4,0)

from eq (b)

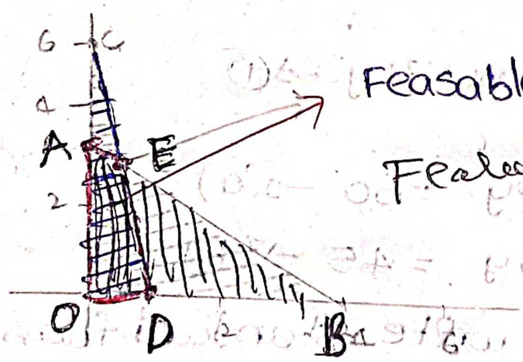
put $x = 0$, $y = 6$

The points C (0,6)

put $y = 0$, $x = 1$

The points D (1,0)

Draw the Graph of above points (extremepoint



Feasable region and bounded

Feasable Soln A, E, D, O

from eq^n (a) & (b)

$3x + 4y = 12$

$12x + 2y = 12$

The points e $\left(\frac{4}{7}, \frac{18}{7}\right)$

$\therefore$ The feasable region OAED is bounded

$\therefore$ The feasable solution are OAED

From eq^n (1) : $M = 3x + 4y$

at O (0,0) ; $M = 3(0) + 4(0) = 0$

at A (0,3) ; $M = 3(0) + 4(3) = 12$

at e $\left(\frac{4}{7}, \frac{18}{7}\right)$; $M = 12$

at D (1,0) ; $M = 3(1) + 4(0) = 3$

∴ The optimal value of Max $z = 12$ at A(0,3) & G$\left(\frac{4}{7}, \frac{18}{7}\right)$

∴ Here it has alternative and optimal solution.

(3) Solve the LPP for graphically

max $z = 20x + 30y \rightarrow M \rightarrow (1)$

∋

$10x + 30y \leq 60$

$9x + 15y \leq 45$

$x_1 \geq 0, x_2 \geq 0$

Given that

max $z = 20x + 30y \rightarrow (1)$

∋

$10x + 30y = 60 \rightarrow (a)$

$9x + 15y = 45 \rightarrow (b)$

we need to write standard form

from eq (a)

Put $x = 0$, $y = 2$

The points A (0,2)

put $y = 0$, $x = 6$

The points B(6,0)

from eq^n (b)

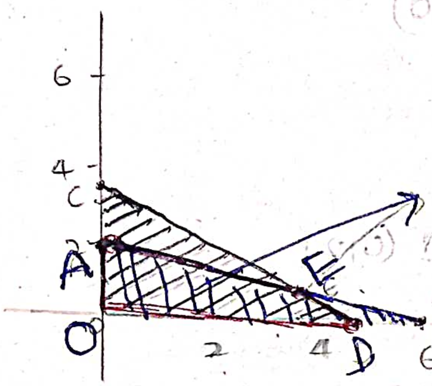
put $x=0$, $y=3$

The points $C(0,3)$

put $y=0$, $x=5$

The points $D(5,0)$

we draw the graph of above external points



feasible region & bounded region (OAED)

feasible solution

(O, A, E, D)

from eq^n (a) & (b)

$10x+30y=60$

$9x+15y=45$

The points $E(3.75, 0.75)$

at $O(0,0)$ $M=20(0)+30(0)=0$

at $A(0,2)$ $M=20(0)+30(2)=60$

at $D(5,0)$ $M=20(5)+30(0)=100$

at $E(3.75, 0.75)$ $M=20(3.75)+30(0.75)=97.5$

at $D(5,0)$ we have unique optimal solution

$\text{Max } z=100$

(5) solve the L.P.P for graphically

$\max z=5x+6y$

$\ni$

$8x+4y\geq 64$

$25x+25y\leq 50$

Sol

Given $\max z=5x+6y=M \rightarrow ①$

$\ni$

$8x+4y=64 \rightarrow a$

$25x+25y=50 \rightarrow b$

⑤ solve LPP by Graphically

$$Max\ z = 6x+3y$$

$$\ni\ 2x+3y \le 6$$

$$9x+4y \ge 36$$

$$x \ge 0, y \ge 0$$

Given that,

$$Max\ z = 6x+3y$$

$\ni$

$$2x+3y \le 6$$

$$9x+4y \ge 36$$

$$x \ge 0, y \ge 0$$

we need to write standard form

$$Max\ z = 6x+3y = M \to ①$$

$$2x+3y = 6 \to (a)$$

$$9x+4y = 36 \to (b)$$

from eq (a)

put $x=0$, $y=2$

The points $A(x,y) = (0,2)$

put $y=0$, $x=3$

the points $B(x,y) = (3,0)$

from eq (b)

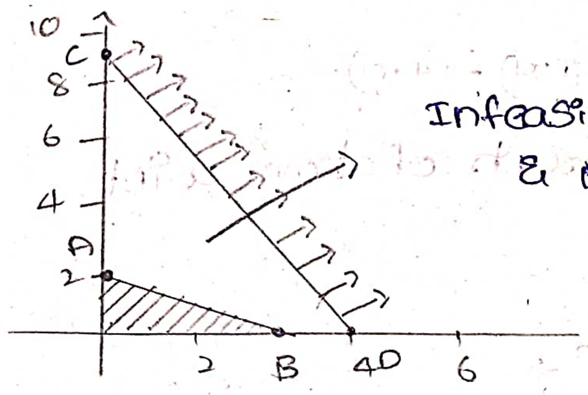
put $x=0$, $y=9$

The points $C(x,y) = (0,9)$

put $y=0$ $x=4$

The points $D(x,y) = (4,0)$

we need to draw the graph for above points



Infeasiable region & No solution exist

⑧ solve LPP by Graphically

$$\text{Max } z = 6x + 3y$$

$\ni$

$$2x + 3y \geq 6$$

$$9x + 4y \leq 36$$

$$x \geq 0, y \geq 0$$

Given,

$$\text{Max } z = 6x + 3y$$

$\ni$

$$2x + 3y \geq 6$$

$$9x + 4y \leq 36$$

we need to write standard form

$$\text{Max } z = 6x + 3y \rightarrow ①$$

$\ni$

$$2x + 3y = 6 \rightarrow a$$

$$9x + 4y = 36 \rightarrow b$$

from eq (a)

put $x = 0, y = 2$

The points A (0,2)

put $y = 0, x = 3$

The points $B(x, y) = 3, 0$

from eq (b)

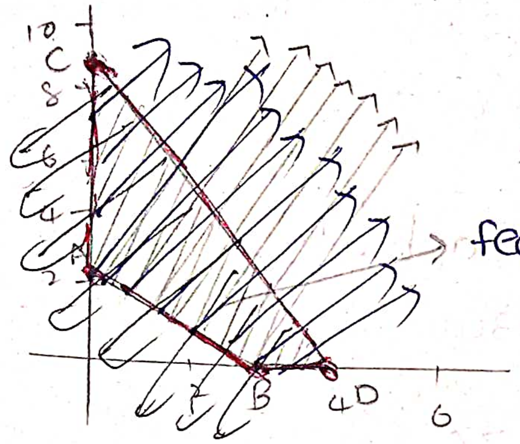
put $x = 0, y = 9$

The points $C(x, y) = (0, 9)$

put $y=0$, $x=4$

The points $D(x,y) = (4,0)$

we draw the graph of above point.



→ feasible region ABCD & bounded

feasible sol^n A, B, C, D

we find optimum sol^n of eq ① $M = 6x+3y$

at A $(0,2)$ = $M = 6(0)+3(2) = 6$

at B $(3,0)$ = $M = 6(3)+3(0) = 18$

at C $(0,9)$, $M = 6(0)+3(9) = 27$

at D $(4,0)$, $M = 6(4)+3(0) = 24$

∴ The optimum soln Max $z = 27$ at $C(0,9)$

∴ It has unique sol^n.

⑨ Solve L.P.P by Graphically

$\text{Min } z = 13x+12y$

∋

$4x+8y \geq 32$

$6x+5y \geq 30$

$x \geq 0,\ y \geq 0$

Given

$\text{Min } z = 13x+12y$

∋

$4x+8y \geq 32$

$6x+5y \geq 30$

$x \geq 0,\ y \geq 0$

The standard form of LPP.

$Min\ z = 13x + 12y$

$\ni$

$4x + 8y = 32 \rightarrow (a)$

$6x + 5y = 30 \rightarrow (b)$

from eq (a)

put $x = 0$; $y = 4$

$\therefore$ The points $A(x,y) = (0,4)$

put $y = 0$, $x = 8$

$\therefore$ The points $B(x,y) = (8,0)$

from eq (b)

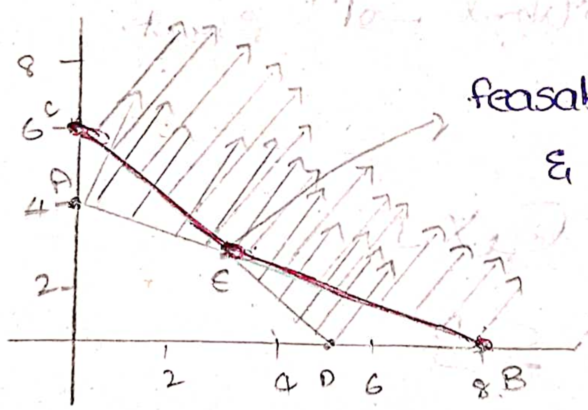
put $x = 0$, $y = 6$

The points $C(x,y) = (0,6)$

put $y = 0$, $x = 5$

$\therefore$ The points $D(x,y) = (5,0)$

we draw the graph of above points.



feasable region CEB & unbounded.

feasable sol^n C, E, B

The intersect point E : in eq (a) & (b)

$4x + 8y = 32$

$6x + 5y = 30$

at $E(x,y) = \left(\frac{20}{7}, \frac{18}{7}\right)$

The optimum soln of eq ① $M = 13x + 12y$

at $C(0,6)$, $M = 13(0) + 12(6) = 12$

at $E\left(\frac{20}{7}, \frac{18}{7}\right)$, $M = 13\left(\frac{20}{7}\right) + 12\left(\frac{18}{7}\right) = 68$

at $B(8,0)$, $M = 13(8) + 12(0) = 104$

$\therefore$ The optional soln of eq (1)

$\min z = 68$ at $E\left(\frac{20}{7}, \frac{18}{7}\right)$

$\therefore$ It has unique soln.

Note:- 1) $\operatorname{Max} z = C_i X_i$

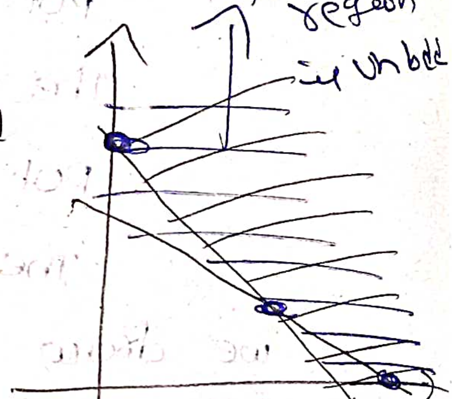
$\ni AX \geq B$

$X \geq 0$

→ feasible region is unbdd

→ feasible soln does not exist

→ No optimal soln exist.



2) $\operatorname{Min} z = C_i X_i$

$\ni AX \leq B$

$X \geq 0$

→ feasible region is bdd

→ feasible soln exist

→ optimal soln exist [Min z = 0, Max = ?]