# Topological Stasis Meets Spectral Gap: Connecting Charlton's PACK Principle to Toroidal Coherence Protection

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**Abstract**

We identify a structural correspondence between two independent results: Charlton's PACK (Packing-Lock) principle for the Navier–Stokes equations in $\mathbb{R}^3$ and the toroidal spectral gap bound for quantum coherence protection on $T^2$. Both derive dynamical constraints—stasis in one case, exponential dephasing suppression in the other—not from analytical control of the evolution equations, but from global topological properties of the domain. We make the correspondence precise via a shared "topological inadmissibility" structure: domain geometry forbids the dynamical patterns that would produce pathological behavior. This suggests a broader principle: regularity, coherence, and stability are geometric phenomena when the domain topology is sufficiently constrained.

## 1 Introduction

Two recent results, developed independently in different domains, share a striking structural identity.

**Charlton (2026)** addresses the Clay Navier–Stokes Millennium Problem by showing that in the infinite simply-connected plenum $\mathbb{R}^3$, the incompressibility constraint $\nabla \cdot u = 0$ is not merely a local differential identity but a *global geometric constraint* that forces the acceleration field to vanish identically: $a \equiv 0$ [1, 2]. The mechanism is PACK (Packing-Lock): any strictly positive directional acceleration on a set of nonzero measure forces a forward-half-space momentum demand that diverges to infinity, contradicting finite forcing.

**Cormier (2026)** shows that the spectral gap $\lambda_1 = 2 - 2\cos(2\pi/N)$ of the discrete Laplacian on the Tonnetz torus $T^2$ bounds the decay rate of incoherent modes, providing exponential dephasing suppression for quantum states and transformer latent dynamics alike [3, 4]. The mechanism: high-frequency (noisy) components on the compact toroidal manifold decay as $e^{-\lambda_1 t}$ while the zero mode (coherent signal) is topologically protected.

Both results derive dynamical constraints from domain topology rather than from analytical estimates on the evolution equations themselves. This note makes the correspondence precise.

## 2 The Shared Structure: Topological Inadmissibility

**Definition 1** (Topological Inadmissibility). *Let $X$ be a domain, $\mathcal{E}$ an evolution equation on $X$, and $\mathcal{P}$ a pathological behavior (e.g. blowup, decoherence, drift). We say $\mathcal{P}$ is* topologically inadmissible *on $X$ if the global topology of $X$ prohibits the dynamical patterns required for $\mathcal{P}$, independent of initial conditions and the specific form of $\mathcal{E}$.*

Both results instantiate this structure:

| | **PACK (Charlton)** | **Spectral Gap (Cormier)** |
|---|---|---|
| Domain $X$ | $\mathbb{R}^3$ (infinite plenum) | $T^2$ (compact torus) |
| Constraint | $\nabla \cdot u = 0$ (incompressibility) | $\Delta_{T^2}$ has gap $\lambda_1 > 0$ |
| Pathology $\mathcal{P}$ | Finite-time blowup | Decoherence / drift |
| Mechanism | Infinite forward demand | Exponential mode decay |
| Result | $a \equiv 0$ (stasis) | $\|\delta(t)\| \leq e^{-\lambda_1 t}$ |
| Character | Topological, not analytical | Geometric, not perturbative |

The key parallel: in both cases, the topology of the domain itself does the work. No energy estimates, no Sobolev embeddings, no Grönwall inequalities. The geometry *forbids* the pathological dynamics.

## 3 PACK as an Infinite-Domain Spectral Gap

We can sharpen the analogy. On the compact torus $T^2$, the Laplacian has discrete spectrum $\{0, \lambda_1, \lambda_2, \ldots\}$ with $\lambda_1 > 0$. The spectral gap separates the coherent zero mode from all noisy modes, and the Poincaré inequality gives:

$$\|f - \bar{f}\|^2 \leq \frac{1}{\lambda_1} \|\nabla f\|^2 \tag{1}$$

This bounds fluctuations in terms of gradients. On $T^2$, the gap is finite and positive, giving exponential decay of perturbations.

On $\mathbb{R}^3$, the Laplacian has *continuous* spectrum starting at zero—no spectral gap in the classical sense. But PACK provides a different kind of gap:

not spectral but *topological*. The incompressibility constraint $\nabla \cdot u = 0$ in an infinite simply-connected domain forces the projection onto divergence-free fields to be so rigid that the "effective gap" is infinite:

**Observation 1.** *PACK can be interpreted as $\lambda_{\text{eff}} = \infty$ for the incompressibility constraint on $\mathbb{R}^3$. While $T^2$ gives finite exponential suppression ($e^{-\lambda_1 t}$), the infinite plenum gives total suppression ($a \equiv 0$).*

This places the two results on a continuum:

| Domain | Spectral gap | Suppression | Character |
|---|---|---|---|
| $\mathbb{R}^n$ (unconstrained) | $\lambda_1 = 0$ | None | Drift/blowup possible |
| $T^2$ (compact torus) | $\lambda_1 > 0$ (finite) | Exponential | Coherence protection |
| $\mathbb{R}^3$ (incompressible plenum) | $\lambda_{\text{eff}} = \infty$ | Total | Topological stasis |

The hierarchy $0 < \lambda_1 < \infty$ corresponds to the hierarchy of protection: no protection, exponential protection, absolute protection.

# 4 Geometric Thermodynamics Interpretation

McGinty (2026) frames thermodynamic systems as Riemannian manifolds where curvature encodes interaction strength and phase transitions appear as curvature singularities [5]. This provides the unifying language:

- **Curvature = constraint strength.** On $T^2$, the constant positive curvature of the spectral gap manifold provides uniform dephasing suppression. On $\mathbb{R}^3$ with incompressibility, the effective curvature of the constraint manifold is infinite—so rigid that no acceleration is admissible.

- **Phase transitions = curvature divergences.** In the optomechanical systems of [4], the cooperativity parameter $C$ acts as a curvature parameter. When $C \to \infty$, the bare-state description undergoes a geometric phase transition to the dressed-state regime. Charlton's PACK is the fluid-dynamical analog: incompressibility is so strong that the velocity-field manifold admits no non-trivial flows.

- **Stability = positive-definite metric.** The toric code threshold ($p < p_c \approx 0.09$) requires the Tonnetz metric to remain positive-definite. PACK requires the momentum-balance metric on $\mathbb{R}^3$ to remain finite. In both cases, metric degeneration corresponds to loss of protection.

# 5 Implications

## 5.1 For Quantum Coherence

The PACK principle suggests that stronger topological constraints yield stronger coherence protection. Our current model uses $T^2$ (finite spectral gap, exponential suppression). Embedding the torus in a higher-dimensional incompressible structure could provide additional protection channels—a "PACK shield" around the quantum state.

Concretely: nanotori bundle geometries (ring-of-rings, Hopf links) already improve cooperative Q by factors of 4–6× via collective phase matching. These bundles increase the effective topological rigidity of the system, moving along the continuum toward stronger suppression.

## 5.2 For Navier–Stokes

The spectral gap perspective suggests a quantitative refinement of PACK: rather than total stasis ($a \equiv 0$), one might characterize the *rate* at which near-incompressible perturbations are suppressed as a function of the domain's topological complexity (genus, connectivity, boundary conditions). This would connect to the well-known result that Navier–Stokes regularity is easier to establish on $T^3$ (periodic domain) than on $\mathbb{R}^3$—precisely because $T^3$ has a finite spectral gap.

## 5.3 For Machine Learning

The toroidal logit bias [3] already demonstrates that imposing $T^2$ geometry on transformer latent dynamics reduces hallucination by +2.8pp on TruthfulQA across four models. The PACK analogy suggests that even stronger geometric constraints (e.g. volume-preserving flows in latent space, divergence-free attention) could yield correspondingly stronger coherence guarantees.

# 6 Conclusion

Charlton's PACK and Paraxiom's spectral gap bound are manifestations of a single principle: **domain topology constrains dynamics**. Regularity in fluid mechanics, coherence in quantum systems, and stability in neural networks are not achieved by analytical control of the evolution equations but by geometric properties of the space in which evolution occurs.

The two results sit on a continuum parameterized by the effective spectral gap: $\lambda_1 = 0$ (no protection, pathology possible), $0 < \lambda_1 < \infty$ (exponential protection, toroidal coherence), $\lambda_{\text{eff}} = \infty$ (total protection, topological stasis). The unifying thesis is that *geometry is not merely descriptive—it* *is prescriptive.* The shape of the domain determines what dynamics are admissible.

## References


[1] J. P. Charlton Jr. Acceleration as Topology: The Inertial Lock of the Infinite Simply-Connected Plenum. Zenodo, 2026. DOI: 10.5281/zenodo.18630611.

[2] J. P. Charlton Jr. Enabling Cross-Silo Innovation for the CPNS: A Public Specification Resolving Ambiguity. Zenodo, 2026. DOI: 10.5281/zenodo.18548206.

[3] S. Cormier. Topological Constraints for Coherent Language Models: Why Geometry Prevents Hallucination. Zenodo, 2026. DOI: 10.5281/zenodo.18624950.

[4] S. Cormier. Toroidal Coherence Architecture: Defensive Technical Disclosure. Zenodo, 2026. DOI: 10.5281/zenodo.18595753.

[5] C. McGinty. Geometric Thermodynamics: Curvature, Entropy, and the Geometry of Physical Reality. Plenum of Energy Prophecies (LinkedIn Newsletter), February 2026.

[6] C. Fefferman. Existence and Smoothness of the Navier–Stokes Equation. Clay Mathematics Institute Millennium Problem Description, 2000.