POKHARA UNIVERSITY

Level: Bachelor Semester: Spring Year : 2019
Programme: BE Full Marks: 100
Course: Engineering Mathematics III Pass Marks: 45
Time : 3hrs.

Candidates are required to give their answers in their own words as far as practicable.
The figures in the margin indicate full marks.
Attempt all the questions.

1. a) Solve by using Gauss elimination method 7
   $4x - 8y + 3z = 16$, $-x + 2y - 5z = -21$, $3x - 6y + z = 7$

   b) Using cayley Hamiltan Theorem find the inverse of 8

$$A = \begin{bmatrix} 7 & -1 & 3 \\ 6 & 1 & 4 \\ 2 & 4 & 8 \end{bmatrix}$$

2. a) State and prove the leibnitz's theorem for alternating series and hence test the convergence of the series $\sum_{n=1}^{\infty}(-1)^{n+1}\frac{n}{n^3+1}$ 7

   b) Find the centre, radius and interval of convergence : 8

$$\sum_{1}^{\infty}\frac{(-1)^n(2x-1)^n}{n6^n}$$

OR

Define Maclaurin series of function f(x) and find the expansion of **tanx** upto three terms and hence obtain the expansion of **sec²x**.

3. a) A function f(x) defined by $f(x) = x^2$ for $0 \le x \le L$ Find the Fourier cosine series. 7

   b) Find Fourier sine as well as cosine series of the function $f(x) = x$ for $0 < x < 1$ 8

4. a) Find the directional derivative of the function $f = x^2 - y^2 + 2z^2$ at the point $P(1, 2, 3)$ in the directional of line PQ where Q is the point $(5, 0, 4)$ 7

   b) If θ is the acute angle between the surfaces $xy^2z = 3x + z^2$ and $3x^2 - y^2 + 2z = 1$ at the point (1,-2,1). Show that $Cos\,\theta = \frac{3}{7\sqrt{6}}$ 8

5. a) Evaluate the line integral $\oint_C \vec{F}.d\vec{r}$, where $\vec{F} = (1, xy, yz)$ and S is the surface $x^2 + y^2 \le z$, $y \ge 0$, $z \le 4$ 7

   b) State Gauss divergence theorem and using it, evaluate $\iint_s \vec{F}\cdot\hat{n}ds$, where $\vec{F} = 2x^2y\vec{i} - y^2\vec{j} + 4xz^2\vec{k}$ and S is the region bounded by the cylinder $y^2 + z^2 = 3$ and $0 \le x \le 2$, $y \ge 0$ and $z \ge 0$. 8

6. a) Solve the following Linear programming problem using the simplex method. 7

   Maximize $z = 30x_1 + 20x_2$ subject to:
   $-x_1 + x_2 \le 5$
   $2x_1 + x_2 \le 10$
   $x_1, x_2 \ge 0$

   b) Solve the linear programming problem by Simplex method constructing its duality: Minimize $z = 20x_1 + 30x_2$ Subject to $x_1 + 4x_2 \ge 8$, $x_1 + x_2 \ge 5$, $2x_1 + x_2 \ge 7$, $x_1 \ge 0$, $x_2 \ge 0$ 8

7. Attempt all the questions. 4×2.5

   a) Check the following transformation is linear or not ?
   $T: R^2 \to R^2$ be defined by $T(u, v) = (u, v + 3)$

   b) Find the unit tangent vector to the curve $\vec{r} = (t, t^2, t^3)$ at t = 1

   c) Test the convergence of the series: $1 + \frac{1}{\sqrt{2}} + \frac{1}{\sqrt{3}} + \frac{1}{\sqrt{4}} + \dots\dots$

   d) Evaluate the integral

$$\int_{(-1,2)}^{(3,1)}[(y^2 + 2xy)dx + (x^2 + 2xy)dy]$$