Assignment 1: Random Variables, Sampling and Estimation

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1 Properties of RV

1. Suppose a variable Y is an exact linear function of X:

$$Y = \lambda + \mu X$$

where λ and μ are constants, and suppose that Z is a third variable. Show that $\rho_{XZ} = \rho_{YZ}$.

2 Consistency and Unbiasedness

1. In the model

$$y_t = \alpha x_t + u_t \tag{2.1}$$

x_t is an explanatory variable which can be regarded as fixed in repeated samples. u_t is an unobserved disturbance for which it is assumed that

$$E(u_t) = 0, E(u_t u_s) = \begin{cases} 0, & \text{if } s = t. \\ \sigma^2, & \text{otherwise.} \end{cases} \tag{2.2}$$

An estimator of α is $\frac{1}{T}\sum_{t=1}^{T}\{\frac{y_t}{x_t}\}$.
Under the assumptions above show that the estimator is unbiased and consistent. Comment briefly on the efficiency of the estimator.

2. Show that, when you have n observations, the condition that the generalized estimator $(\lambda_1 X_1 + ... + \lambda_n X_n)$ should be an unbiased estimator of μ_x is $\lambda_1 + \lambda_2 + ...\lambda_n = 1$

3 Hypothesis Testing

1. Before beginning a certain course, 36 students are given an aptitude test. The scores and the course results(pass/fail) are given below:

student	test score	course result	student	test score	course result	student	test score	course result
1	30	fail	13	26	fail	25	9	fail
2	29	pass	14	43	pass	26	36	pass
3	33	fail	15	43	fail	27	61	pass
4	62	pass	16	68	pass	28	79	fail
5	59	fail	17	63	pass	29	57	fail
6	63	pass	18	42	fail	30	46	pass
7	80	pass	19	51	fail	31	70	fail
8	32	fail	20	45	fail	32	31	pass
9	60	pass	21	22	fail	33	68	pass
10	76	pass	22	30	pass	34	62	pass
11	13	fail	23	40	fail	35	56	pass
12	41	pass	24	26	fail	36	36	pass

Do you think that the aptitude test is useful for selecting students for admission to the course, and if so, how would you determine the pass mark? (Discuss the trade-off between Type I and Type II error associated with the choice of pass mark.)

2. You wish to test $H_o : \mu = 0$. You believe that μ cannot be negative and so the alternative hypothesis is $H_1 : \mu > 0$. Accordingly, you decide to perform a one-sided test. However, you are wrong. μ is actually equal to μ_1, and μ_1 is negative.What are the implications for your test results?