

Introduction to Econometrics, 5th edition

Chapter 2: Properties of the Regression Coefficients and Hypothesis Testing

	Review chapter	Regression model
Model	X unknown μ , σ^2	
Estimator	$ar{X}$	

This sequence describes the testing of a hypotheses relating to regression coefficients. It is concerned only with procedures, not with theory.

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Hypothesis testing forms a major part of the foundation of econometrics and it is essential to have a clear understanding of the theory.

	Review chapter	Regression model
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Estimator	$ar{X}$	

The theory, discussed in sections R.9 to R.11 of the Review chapter, is non-trivial and requires careful study. This sequence is purely mechanical and is not in any way a substitute.

	Review chapter	Regression model
Model	X unknown μ , σ^2	
Estimator	$ar{X}$	

If you do not understand, for example, the trade-off between the size (significance level) and the power of a test, you should study the material in those sections before looking at this sequence.

	Review chapter	Regression model
Model	X unknown μ , σ^2	
Estimator	$ar{X}$	

In our standard example in the Review chapter, we had a random variable X with unknown population mean μ and variance σ^2 . Given a sample of data, we used the sample mean as an estimator of μ .

	Review chapter	Regression model
Model	X unknown μ , $\sigma^{\!\scriptscriptstyle 2}$	$Y = \beta_1 + \beta_2 X + u$
Estimator	$ar{X}$	$\hat{\beta}_{2} = \frac{(X_{i} - \bar{X})(Y_{i} - \bar{Y})}{(X_{i} - \bar{X})^{2}}$

In the context of the regression model, we have unknown parameters β_1 and β_2 and we have derived estimators $\hat{\beta}_2$ and for them. In what follows, we shall focus on β_2 and its estimator $\hat{\beta}_2$.

	Review chapter	Regression model
Model	X unknown μ , σ^2	$Y = \beta_1 + \beta_2 X + u$
Estimator	$ar{X}$	$\hat{\beta}_{2} = \frac{(X_{i} - \overline{X})(Y_{i} - \overline{Y})}{(X_{i} - \overline{X})^{2}}$
Null hypothesis Alternative hypothesis	$H_0: \mu = \mu_0$ $H_1: \mu \triangleleft \mu_0$	$H_0: \beta_2 = \beta_2^0$ $H_1: \beta_2 \ \ $

In the case of the random variable X, our standard null hypothesis was that μ was equal to some specific value μ_0 . In the case of the regression model, our null hypothesis is that β_2 is equal to some specific value β_2^0 .

	Review chapter	Regression model
Model	X unknown μ , $\sigma^{\!\scriptscriptstyle 2}$	$Y = \beta_1 + \beta_2 X + u$
Estimator	$ar{X}$	$\hat{\beta}_{2} = \frac{(X_{i} - \overline{X})(Y_{i} - \overline{Y})}{(X_{i} - \overline{X})^{2}}$
Null hypothesis Alternative hypothesis	H_0 : $\mu = \mu_0$ H_1 : $\mu \triangleleft \mu_0$	$H_0: m{eta}_2 = m{eta}_2^0 \ H_1: m{eta}_2 \ m{\Phi} m{eta}_2^0$
Test statistic	$t = \frac{\overline{X} - \mu_0}{\text{s.e.}(\overline{X})}$	$t = \frac{\hat{\beta}_2 - \beta_2^0}{\text{s.e.}(\hat{\beta}_2)}$

For both the population mean μ of the random variable X and the regression coefficient β_2 , the test statistic is a t statistic.

	Review chapter	Regression model
Model	X unknown μ , σ^2	$Y = \beta_1 + \beta_2 X + u$
Estimator	$ar{X}$	$\hat{\beta}_{2} = \frac{(X_{i} - \bar{X})(Y_{i} - \bar{Y})}{(X_{i} - \bar{X})^{2}}$
Null hypothesis	H_0 : $\mu = \mu_0$	$\boldsymbol{H}_0 \colon \boldsymbol{\beta}_2 = \boldsymbol{\beta}_2^0$
Alternative hypothesis	$H_1: \mu \ \mathbf{\hat{\psi}} \mu_0$	$H_1: \boldsymbol{\beta}_2 \ \boldsymbol{\delta} \boldsymbol{\beta}_2^0$
Test statistic	$t = \frac{\bar{X} - \mu_0}{\text{s.e.}(\bar{X})}$	$t = \frac{\hat{\beta}_2 - \beta_2^0}{\text{s.e.}(\hat{\beta}_2)}$

In both cases, it is defined as the difference between the estimated coefficient and its hypothesized value, divided by the standard error of the coefficient.

	Review chapter	Regression model
Model	X unknown μ , σ^2	$Y = \beta_1 + \beta_2 X + u$
Estimator	$ar{X}$	$\hat{\beta}_{2} = \frac{(X_{i} - \overline{X})(Y_{i} - \overline{Y})}{(X_{i} - \overline{X})^{2}}$
Null hypothesis	H_0 : $\mu = \mu_0$	$\boldsymbol{H}_0 \colon \boldsymbol{\beta}_2 = \boldsymbol{\beta}_2^{0}$
Alternative hypothesis	$H_1: \mu \ lackbox{}{} \mu_0$	$H_1: \boldsymbol{\beta}_2 \ \boldsymbol{\delta} \boldsymbol{\beta}_2^0$
Test statistic	$t = \frac{\bar{X} - \mu_0}{\text{s.e.}(\bar{X})}$	$t = \frac{\hat{\beta}_2 - \beta_2^0}{\text{s.e.}(\hat{\beta}_2)}$
Reject H ₀ if	$ t > t_{\rm crit}$	$ t > t_{\text{crit}}$

We reject the null hypothesis if the absolute value is greater than the critical value of t, given the chosen significance level.

	Review chapter	Regression model
Model	X unknown μ , $\sigma^{\!\scriptscriptstyle 2}$	$Y = \beta_1 + \beta_2 X + u$
Estimator	$ar{X}$	$\hat{\beta}_{2} = \frac{(X_{i} - \bar{X})(Y_{i} - \bar{Y})}{(X_{i} - \bar{X})^{2}}$
Null hypothesis	H_0 : $\mu = \mu_0$	$\boldsymbol{H}_0 \colon \boldsymbol{\beta}_2 = \boldsymbol{\beta}_2^{0}$
Alternative hypothesis	$H_1: \mu \ \mathbf{\hat{\psi}} \mu_0$	$H_1: \boldsymbol{\beta}_2 \ \boldsymbol{\Diamond} \boldsymbol{\beta}_2^0$
Test statistic	$t = \frac{\bar{X} - \mu_0}{\text{s.e.}(\bar{X})}$	$t = \frac{\hat{\beta}_2 - \beta_2^0}{\text{s.e.}(\hat{\beta}_2)}$
Reject H ₀ if	$ t > t_{\rm crit}$	$ t > t_{\rm crit}$
Degrees of freedom	<i>n</i> – 1	n-k=n-2

There is one important difference. When locating the critical value of t, one must take account of the number of degrees of freedom. In the case of the random variable X, this is n-1, where n is the number of observations in the sample.

	Review chapter	Regression model
Model	X unknown μ , σ^2	$Y = \beta_1 + \beta_2 X + u$
Estimator	$ar{X}$	$\hat{\beta}_{2} = \frac{(X_{i} - \bar{X})(Y_{i} - \bar{Y})}{(X_{i} - \bar{X})^{2}}$
Null hypothesis	H_0 : $\mu = \mu_0$	$\boldsymbol{H}_0:\boldsymbol{\beta}_2=\boldsymbol{\beta}_2^0$
Alternative hypothesis	$H_1: \mu \ \mathbf{\hat{\psi}} \mu_0$	$H_1: oldsymbol{eta}_2 \ oldsymbol{\phi} oldsymbol{eta}_2^0$
Test statistic	$t = \frac{\overline{X} - \mu_0}{\text{s.e.}(\overline{X})}$	$t = \frac{\hat{\beta}_2 - \beta_2^0}{\text{s.e.}(\hat{\beta}_2)}$
Reject H ₀ if	$ t > t_{\rm crit}$	$ t > t_{\rm crit}$
Degrees of freedom	n-1	n-k=n-2

In the case of the regression model, the number of degrees of freedom is n - k, where n is the number of observations in the sample and k is the number of parameters (β coefficients). For the simple regression model above, it is n - 2.

Example: $p = \beta_1 + \beta_2 w + u$

Null hypothesis: H_0 : $\beta_2 = 1.0$

Alternative hypothesis: $H_1: \beta_2 \neq 1.0$

As an illustration, we will consider a model relating price inflation to wage inflation. p is the percentage annual rate of growth of prices and w is the percentage annual rate of growth of wages.

Example: $p = \beta_1 + \beta_2 w + u$

Null hypothesis: H_0 : $\beta_2 = 1.0$

Alternative hypothesis: $H_1: \beta_2 \neq 1.0$

We will test the hypothesis that the rate of price inflation is equal to the rate of wage inflation. The null hypothesis is therefore H_0 : $\beta_2 = 1.0$. (We should also test $\beta_1 = 0$.)

Example: $p = \beta_1 + \beta_2 w + u$

Null hypothesis: H_0 : $\beta_2 = 1.0$

Alternative hypothesis: $H_1: \beta_2 \neq 1.0$

$$\hat{p} = 1.21 + 0.82w$$
 $(0.05)(0.10)$

Suppose that the regression result is as shown (standard errors in parentheses). Our actual estimate of the slope coefficient is only 0.82. We will check whether we should reject the null hypothesis.

Example: $p = \beta_1 + \beta_2 w + u$

Null hypothesis: H_0 : $\beta_2 = 1.0$

Alternative hypothesis: $H_1: \beta_2 \neq 1.0$

$$\hat{p} = 1.21 + 0.82w$$
 $(0.05)(0.10)$

$$t = \frac{\hat{\beta}_2 - \beta_2^0}{\text{s.e.}(\hat{\beta}_2)} = \frac{0.82 - 1.00}{0.10} = -1.80.$$

We compute the t statistic by subtracting the hypothetical true value from the sample estimate and dividing by the standard error. It comes to -1.80.

Example:

$$p = \beta_1 + \beta_2 w + u$$

Null hypothesis:

$$H_0$$
: $\beta_2 = 1.0$

Alternative hypothesis:

$$H_1: \beta_2 \neq 1.0$$

$$\hat{p} = 1.21 + 0.82w$$
 $(0.05)(0.10)$

$$t = \frac{\hat{\beta}_2 - \beta_2^0}{\text{s.e.}(\hat{\beta}_2)} = \frac{0.82 - 1.00}{0.10} = -1.80.$$

$$n = 20$$

degrees of freedom =18

$$t_{\text{crit}, 5\%} = 2.101$$

There are 20 observations in the sample. We have estimated 2 parameters, so there are 18 degrees of freedom.

Example: $p = \beta_1 + \beta_2 w + u$

Null hypothesis: H_0 : $\beta_2 = 1.0$

Alternative hypothesis: $H_1: \beta_2 \neq 1.0$

$$\hat{p} = 1.21 + 0.82w$$
 $(0.05)(0.10)$

$$t = \frac{\hat{\beta}_2 - \beta_2^0}{\text{s.e.}(\hat{\beta}_2)} = \frac{0.82 - 1.00}{0.10} = -1.80.$$

n = 20 degrees of freedom = 18 $t_{crit, 5\%} = 2.101$

The critical value of t with 18 degrees of freedom is 2.101 at the 5% level. The absolute value of the t statistic is less than this, so we do not reject the null hypothesis.

Model

$$Y = \beta_1 + \beta_2 X + u$$

In practice it is unusual to have a feeling for the actual value of the coefficients. Very often the objective of the analysis is to demonstrate that Y is influenced by X, without having any specific prior notion of the actual coefficients of the relationship.

Model $Y = \beta_1 + \beta_2 X + u$

Null hypothesis: H_0 : $\beta_2 = 0$

Alternative hypothesis: $H_1: \beta_2 \neq 0$

In this case it is usual to define β_2 = 0 as the null hypothesis. In words, the null hypothesis is that X does not influence Y. We then try to demonstrate that the null hypothesis is false.

Model $Y = \beta_1 + \beta_2 X + u$

Null hypothesis: H_0 : $\beta_2 = 0$

Alternative hypothesis: $H_1: \beta_2 \neq 0$

$$t = \frac{\hat{\beta}_2 - \beta_2^0}{\text{s.e.}(\hat{\beta}_2)} = \frac{\hat{\beta}_2}{\text{s.e.}(\hat{\beta}_2)}$$

For the null hypothesis β_2 = 0, the t statistic reduces to the estimate of the coefficient divided by its standard error.

Model $Y = \beta_1 + \beta_2 X + u$

Null hypothesis: H_0 : $\beta_2 = 0$

Alternative hypothesis: $H_1: \beta_2 \neq 0$

$$t = \frac{\hat{\beta}_2 - \beta_2^0}{\text{s.e.}(\hat{\beta}_2)} = \frac{\hat{\beta}_2}{\text{s.e.}(\hat{\beta}_2)}$$

This ratio is commonly called the *t* statistic for the coefficient and it is automatically printed out as part of the regression results. To perform the test for a given significance level, we compare the *t* statistic directly with the critical value of *t* for that significance level.

. reg EARNING	s s					
Source	ss 		MS		Number of obs F(1, 498)	
Model Residual	64314.9215	1 6014. 498 129.1			Prob > F R-squared	= 0.0000 = 0.0855
Total	70328.9662	499 140.9	39812		Adj R-squared Root MSE	= 0.0837
EARNINGS	Coef.	Std. Err.	t	P> t	[95% Conf.	Interval]
S _cons	1.265712 .7646844	.1854782 2.803765	6.82 0.27	0.000 0.785		1.630128 6.273351

Here is the output from the earnings function fitted in a previous slideshow, with the t statistics highlighted.

. reg EARNI	NGS S					
Source	 ss +		MS		Number of obs = 500 F(1, 498) = 46.57	
Model Residual	6014.04474 64314.9215	1 6014. 498 129.1	04474 .46429		Prob > F = 0.0000 R-squared = 0.0855) 5
	+ 70328.9662				Adj R-squared = 0.0837 Root MSE = 11.364	
EARNINGS	•				[95% Conf. Interval]	-
S _cons	1.265712		6.82 0.27	0.000 0.785	.9012959 1.630128	

You can see that the *t* statistic for the coefficient of *S* is enormous. We would reject the null hypothesis that schooling does not affect earnings at the 1% significance level (critical value about 2.59).

. reg EARNING	s s					
Source	ss 		MS		Number of obs F(1, 498)	
Model Residual	64314.9215	1 6014. 498 129.1			Prob > F R-squared	= 0.0000 = 0.0855
Total	70328.9662	499 140.9	39812		Adj R-squared Root MSE	= 0.0837
EARNINGS	Coef.	Std. Err.	t	P> t	[95% Conf.	Interval]
S _cons	1.265712 .7646844	.1854782 2.803765	6.82 0.27	0.000 0.785		1.630128 6.273351

In this case we could go further and reject the null hypothesis that schooling does not affect earnings at the 0.1% significance level.

. reg EARNIN	GS S				
Source	 SS 		MS		Number of obs = 500 F(1, 498) = 46.57
Model Residual	6014.04474 64314.9215	1 6014. 498 129.1	.04474 L46429		Prob > F = 0.0000 R-squared = 0.0855
·	70328.9662				Adj R-squared = 0.0837 Root MSE = 11.364
-	Coef.				[95% Conf. Interval]
S _cons	1.265712 .7646844		6.82 0.27	0.000 0.785	.9012959 1.630128

The advantage of reporting rejection at the 0.1% level, instead of the 1% level, is that the risk of mistakenly rejecting the null hypothesis of no effect is now only 0.1% instead of 1%. The result is therefore even more convincing.

. reg EARNING	SS S					
Source	SS		MS		Number of obs = F(1, 498) =	
Model Residual	6014.04474 64314.9215	1 6014. 498 129.1	04474		Prob > F = R-squared =	0.0000 0.0855 0.0837
Total	70328.9662	499 140.9	39812		Adj R-squared = Root MSE =	11.364
EARNINGS	Coef.			P> t	[95% Conf. In	_
S _cons	1.265712 .7646844	.1854782 2.803765	6.82 0.27	0.000 0.785	.9012959 1	630128

We have seen that the intercept does not have any plausible meaning, so it does not make sense to perform a t test on it.

. reg EARNING	SS S					
Source	SS	df	MS		Number of obs = F(1, 498) =	
Model Residual	6014.04474 64314.9215	1 6014 498 129.	.04474		Prob > F = R-squared =	0.0000 0.0855
Total	70328.9662		939812		Adj R-squared = Root MSE =	0.0837 11.364
EARNINGS	Coef.		t	P> t	[95% Conf. In	terval]
S _cons	1.265712 .7646844	.1854782 2.803765	6.82	0.000 0.785	.9012959 1	.630128 .273351

The next column in the output gives what are known as the p values for each coefficient. This is the probability of obtaining the corresponding t statistic as a matter of chance, if the null hypothesis H_0 : $\beta = 0$ is true.

. reg EARNIN	GS S				
Source	ss		MS		Number of obs = 500 F(1, 498) = 46.57
Model Residual	6014.04474 64314.9215	1 6014 498 129.:	.04474 146429		Prob > F = 0.0000 R-squared = 0.0855
•	70328.9662				Adj R-squared = 0.0837 Root MSE = 11.364
EARNINGS					[95% Conf. Interval]
S _cons	1.265712		6.82 0.27	0.000 0.785	.9012959 1.630128

If you reject the null hypothesis H_0 : β = 0, this is the probability that you are making a mistake and making a Type I error. It therefore gives the significance level at which the null hypothesis would just be rejected.

. reg EARNING	SS S					
Source	SS	df	MS		Number of obs = F(1, 498) =	
Model Residual	6014.04474 64314.9215	1 6014 498 129.	.04474		Prob > F = R-squared =	0.0000 0.0855 0.0837
Total	70328.9662		939812		· J	11.364
EARNINGS	Coef.		t	P> t	-	terval]
S _cons	1.265712 .7646844	.1854782 2.803765	6.82 0.27	0.000 0.785	.9012959 1	. 630128

If p = 0.05, the null hypothesis could just be rejected at the 5% level. If it were 0.01, it could just be rejected at the 1% level. If it were 0.001, it could just be rejected at the 0.1% level. This is assuming that you are using two-sided tests.

. reg EARNING	SS S					
Source	SS	df	MS		Number of obs = F(1, 498) =	
Model Residual	6014.04474 64314.9215	1 6014 498 129.	.04474		Prob > F = R-squared =	0.0000 0.0855
Total	70328.9662		939812		Adj R-squared = Root MSE =	0.0837 11.364
EARNINGS	Coef.		t	P> t	[95% Conf. In	terval]
S _cons	1.265712 .7646844	.1854782 2.803765	6.82	0.000 0.785	.9012959 1	.630128 .273351

In the present case p = 0 to three decimal places for the coefficient of S. This means that we can reject the null hypothesis H_0 : $\beta_2 = 0$ at the 0.1% level, without having to refer to the table of critical values of t. (Testing the intercept does not make sense in this regression.)

. reg EARNING	SS S					
Source	SS	df	MS		Number of obs F(1, 498)	
Model Residual	6014.04474 64314.9215	1 6014 498 129.3	.04474		Prob > F R-squared	= 0.0000 = 0.0855
Total	70328.9662	499 140.9	939812		Adj R-squared Root MSE	= 0.0837
EARNINGS	Coef.		t	P> t	-	Interval]
S _cons	1.265712 .7646844	.1854782 2.803765	6.82 0.27	0.000 0.785	.9012959	1.630128 6.273351

The use of *p* values is a more informative approach to reporting the results of tests It is widely used in the medical literature.

. reg EARNING	SS S					
Source	SS	df	MS		Number of obs F(1, 498)	
Model Residual	6014.04474 64314.9215	1 6014 498 129.3	.04474		Prob > F R-squared	= 0.0000 = 0.0855
Total	70328.9662	499 140.9	939812		Adj R-squared Root MSE	= 0.0837
EARNINGS	Coef.		t	P> t	-	Interval]
S _cons	1.265712 .7646844	.1854782 2.803765	6.82 0.27	0.000 0.785	.9012959	1.630128 6.273351

However, in economics, standard practice is to report results referring to 5% and 1% significance levels, and sometimes to the 0.1% level (when one can reject at that level).

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