# Properties of the Regression Coefficients and Hypothesis Testing

## Assignment 3

October 18, 2018

## 1 ASSUMPTIONS OF REGRESSION MODELS

1. Explain why inclusion of intercept term in a regression model makes it reasonable to assume zero expectation for disturbance term and also how it could be a potential problem.

2. Deduce value of covariance between disturbance terms of any two observations of a sample using regression assumptions.

3. State the theorem in *full*, which is the basis for normal distribution assumption of the disturbance term.

## 2 RANDOM COMPONENTS OF REGRESSION COEFFICIENTS

1. Demonstrate that

$$b_1 = \beta_1 + \sum c_i u_i, \tag{2.1}$$

where $c_i = \frac{1}{n} - a_i \bar{X}$ and $a_i = \frac{(X_i - \bar{X})}{\sum_{i=1}^{n} (X_i - \bar{X})^2}$

## 3 Monte Carlo Experiment

Suppose true model for a population is assumed to be

$$Y_i = 3 + 3.4X_i \tag{3.1}$$

and disturbance term follows t-distribution. Explain steps involved in conducting Monte Carlo experiment to deduce distribution of OLS estimators of parameters for this model.

## 4 Unbiasedness of Regressio Coefficients

An investigator correctly believes that the relationship between two variables and Y is given by

$$Y_i = \beta_1 + \beta_2 X_i + u_i \tag{4.1}$$

Given a sample of observations on Y, X and a third variable Z (which is not a determinant of Y), the investigator estimates $\beta_2$ as

$$\frac{\sum_{i=1}^{n}(Z_i - \bar{Z})(Y_i - \bar{Y})}{\sum_{i=1}^{n}(Z_i - \bar{Z})(X_i - \bar{X})} \tag{4.2}$$

Demonstrate that this estimator is unbiased.

## 5 Precision of Regression Coefficients

Explain intutively with diagrams, why actual size of MSD(X) and $\sigma_u^2$ is less important than the relative sizes, when talking about variance of the OLS estimator.