Dougherty

Introduction to Econometrics, 5th edition

Chapter 2: Properties of the Regression Coefficients and Hypothesis Testing

CONFIDENCE INTERVALS FOR REGRESSION COEFFICIENTS

Model $Y = \beta_1 + \beta_2 X + u$

Null hypothesis: $H_0 : \beta_2 = \beta_2^0$

Alternative hypothesis: $H_1 : \beta_2 \neq \beta_2^0$

Confidence intervals were treated at length in the Review chapter and their application to regression analysis presents no problems. We will not repeat the graphical explanation. We will just provide the mathematical derivation in the context of a regression.

CONFIDENCE INTERVALS FOR REGRESSION COEFFICIENTS

Model $Y = \beta_1 + \beta_2 X + u$

Null hypothesis: $H_0 : \beta_2 = \beta_2^0$

Alternative hypothesis: $H_1 : \beta_2 \neq \beta_2^0$

Reject H_0 if $\dfrac{\hat{\beta}_2 - \beta_2^0}{\text{s.e.}\left(\hat{\beta}_2\right)} > t_{\text{crit}}$ **or** $\dfrac{\hat{\beta}_2 - \beta_2^0}{\text{s.e.}\left(\hat{\beta}_2\right)} < -t_{\text{crit}}$

From the initial discussion in this section, we saw that, given the theoretical model $Y = \beta_1 + \beta_2 X + u$ and a fitted model, the regression coefficient $\hat{\beta}_2$ and the hypothetical value of β_2 are incompatible if either of the inequalities shown is valid.

CONFIDENCE INTERVALS FOR REGRESSION COEFFICIENTS

Model $$Y = \beta_1 + \beta_2 X + u$$

Null hypothesis: $$H_0 : \beta_2 = \beta_2^0$$

Alternative hypothesis: $$H_1 : \beta_2 \neq \beta_2^0$$

Reject H_0 if $$\frac{\hat{\beta}_2 - \beta_2^0}{\text{s.e.}\left(\hat{\beta}_2\right)} > t_{\text{crit}} \quad \textbf{or} \quad \frac{\hat{\beta}_2 - \beta_2^0}{\text{s.e.}\left(\hat{\beta}_2\right)} < -t_{\text{crit}}$$

Reject H_0 if $$\hat{\beta}_2 - \beta_2^0 > \text{s.e.}\left(\hat{\beta}_2\right) \times t_{\text{crit}} \quad \textbf{or} \quad \hat{\beta}_2 - \beta_2^0 < -\text{s.e.}\left(\hat{\beta}_2\right) \times t_{\text{crit}}$$

Multiplying through by the standard error of $\hat{\beta}_2$, the conditions for rejecting H_0 can be written as shown.

CONFIDENCE INTERVALS FOR REGRESSION COEFFICIENTS

Model $Y = \beta_1 + \beta_2 X + u$

Null hypothesis: $H_0 : \beta_2 = \beta_2^0$

Alternative hypothesis: $H_1 : \beta_2 \neq \beta_2^0$

Reject H_0 if $\dfrac{\hat{\beta}_2 - \beta_2^0}{\text{s.e.}\left(\hat{\beta}_2\right)} > t_{\text{crit}}$ **or** $\dfrac{\hat{\beta}_2 - \beta_2^0}{\text{s.e.}\left(\hat{\beta}_2\right)} < -t_{\text{crit}}$

Reject H_0 if $\hat{\beta}_2 - \beta_2^0 > \text{s.e.}\left(\hat{\beta}_2\right) \times t_{\text{crit}}$ **or** $\hat{\beta}_2 - \beta_2^0 < -\text{s.e.}\left(\hat{\beta}_2\right) \times t_{\text{crit}}$

Reject H_0 if $\hat{\beta}_2 - \text{s.e.}\left(\hat{\beta}_2\right) \times t_{\text{crit}} > \beta_2^0$ **or** $\hat{\beta}_2 + \text{s.e.}\left(\hat{\beta}_2\right) \times t_{\text{crit}} < \beta_2^0$

The inequalities may then be re-arranged as shown.

CONFIDENCE INTERVALS FOR REGRESSION COEFFICIENTS

Model	$Y = \beta_1 + \beta_2 X + u$
Null hypothesis:	$H_0 : \beta_2 = \beta_2^0$
Alternative hypothesis:	$H_1 : \beta_2 \neq \beta_2^0$

Reject H_0 if $\dfrac{\hat{\beta}_2 - \beta_2^0}{\text{s.e.}\left(\hat{\beta}_2\right)} > t_{\text{crit}}$ **or** $\dfrac{\hat{\beta}_2 - \beta_2^0}{\text{s.e.}\left(\hat{\beta}_2\right)} < -t_{\text{crit}}$

Reject H_0 if $\hat{\beta}_2 - \beta_2^0 > \text{s.e.}\left(\hat{\beta}_2\right) \times t_{\text{crit}}$ **or** $\hat{\beta}_2 - \beta_2^0 < -\text{s.e.}\left(\hat{\beta}_2\right) \times t_{\text{crit}}$

Reject H_0 if $\hat{\beta}_2 - \text{s.e.}\left(\hat{\beta}_2\right) \times t_{\text{crit}} > \beta_2^0$ **or** $\hat{\beta}_2 + \text{s.e.}\left(\hat{\beta}_2\right) \times t_{\text{crit}} < \beta_2^0$

Do not reject H_0 if $\hat{\beta}_2 - \text{s.e.}\left(\hat{\beta}_2\right) \times t_{\text{crit}} \leq \beta_2 \leq \hat{\beta}_2 + \text{s.e.}\left(\hat{\beta}_2\right) \times t_{\text{crit}}$

We can then obtain the confidence interval for β_2, being the set of all values that would not be rejected, given the sample estimate $\hat{\beta}_2$. To make it operational, we need to select a significance level and determine the corresponding critical value of *t*.

CONFIDENCE INTERVALS FOR REGRESSION COEFFICIENTS

```
. reg EARNINGS S

----------------------------------------------------------------------------
      Source |       SS       df       MS              Number of obs =     500
-------------+------------------------------           F(  1,   498) =   46.57
       Model |  6014.04474     1  6014.04474           Prob > F      =  0.0000
    Residual |  64314.9215   498  129.146429           R-squared     =  0.0855
-------------+------------------------------           Adj R-squared =  0.0837
       Total |  70328.9662   499  140.939812           Root MSE      =  11.364
----------------------------------------------------------------------------
    EARNINGS |      Coef.   Std. Err.      t    P>|t|     [95% Conf. Interval]
-------------+--------------------------------------------------------------
           S |   1.265712   .1854782     6.82   0.000     .9012959    1.630128
       _cons |   .7646844   2.803765     0.27   0.785    -4.743982    6.273351
----------------------------------------------------------------------------
```

$$\hat{\beta}_2 - \text{s.e.}\left(\hat{\beta}_2\right) \times t_{crit} \le \beta_2 \le \hat{\beta}_2 + \text{s.e.}\left(\hat{\beta}_2\right) \times t_{crit}$$

For an example of the construction of a confidence interval, we will return to the wage equation fitted earlier. We will construct a 95% confidence interval for β_2.

```
. reg EARNINGS S

----------------------------------------------------------------------------
      Source |       SS       df       MS              Number of obs =     500
-------------+------------------------------           F(  1,   498) =   46.57
       Model |  6014.04474     1  6014.04474           Prob > F      =  0.0000
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           S |   1.265712   .1854782     6.82   0.000     .9012959    1.630128
       _cons |   .7646844   2.803765     0.27   0.785    -4.743982    6.273351
----------------------------------------------------------------------------
```

$$\hat{\beta}_2 - \text{s.e.}\left(\hat{\beta}_2\right) \times t_{\text{crit}} \le \beta_2 \le \hat{\beta}_2 + \text{s.e.}\left(\hat{\beta}_2\right) \times t_{\text{crit}}$$

$$1.266 - 0.185 \text{ x } 1.965 \le \beta_2 \le 1.266 + 0.185 \text{ x } 1.965$$

The point estimate β_2 is 1.266 and its standard error is 0.185.

CONFIDENCE INTERVALS FOR REGRESSION COEFFICIENTS

```
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----------------------------------------------------------------------------
    EARNINGS |      Coef.   Std. Err.      t    P>|t|     [95% Conf. Interval]
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           S |   1.265712   .1854782     6.82   0.000     .9012959    1.630128
       _cons |   .7646844   2.803765     0.27   0.785    -4.743982    6.273351
----------------------------------------------------------------------------
```

$$\hat{\beta}_2 - \text{s.e.}\left(\hat{\beta}_2\right) \times t_{\text{crit}} \le \beta_2 \le \hat{\beta}_2 + \text{s.e.}\left(\hat{\beta}_2\right) \times t_{\text{crit}}$$

$$1.266 - 0.185 \times 1.965 \le \beta_2 \le 1.266 + 0.185 \times 1.965$$

The critical value of *t* at the 5% significance level with 498 degrees of freedom is 1.965.

CONFIDENCE INTERVALS FOR REGRESSION COEFFICIENTS

```
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------------------------------------------------------------------------------
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       Model |  6014.04474     1  6014.04474           Prob > F      =  0.0000
    Residual |  64314.9215   498  129.146429           R-squared     =  0.0855
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------------------------------------------------------------------------------
    EARNINGS |      Coef.   Std. Err.       t     P>|t|     [95% Conf. Interval]
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           S |   1.265712   .1854782     6.82    0.000     .9012959    1.630128
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------------------------------------------------------------------------------
```

$$\hat{\beta}_2 - \text{s.e.}\left(\hat{\beta}_2\right) \times t_{\text{crit}} \leq \beta_2 \leq \hat{\beta}_2 + \text{s.e.}\left(\hat{\beta}_2\right) \times t_{\text{crit}}$$

$$1.266 - 0.185 \times 1.965 \leq \beta_2 \leq 1.266 + 0.185 \times 1.965$$

$$0.902 \leq \beta_2 \leq 1.630$$

Hence we establish that the confidence interval is from 0.902 to 1.630. Stata actually computes the 95% confidence interval as part of its default output, 0.901 to 1.630. The discrepancy in the lower limit is due to rounding error in the calculations we have made.

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EC212 Introduction to Econometrics
http://www2.lse.ac.uk/study/summerSchools/summerSchool/Home.aspx
or the University of London International Programmes distance learning course
EC2020 Elements of Econometrics
www.londoninternational.ac.uk/lse.