Assignment 2: Simple Linear Regression

October 10, 2018

1 DERIVATION OF REGRESSION COEFFICIENTS

1. Consider a simple linear regression model

$$Y_i = \beta_0 + \beta_1 X_i + u_i \tag{1.1}$$

under the classical linear regression model assumptions, where X_i is fixed under repeated sampling. The usual OLS estimators $\hat{\beta}_0$ and $\hat{\beta}_1$ are unbiased for their respective population parameters. Let β_1 be the estimator of β_1 obained by assuming the intercept is zero.

- a) What is the meaning of X_i being fixed and what is an alternative for this ?
- b) Does unbiasedness of usual OLS estimators $\hat{\beta}_0$ and $\hat{\beta}_1$ change if population size is increased ?
- c) Show that the restricted least squares estimator of β_1 is given by

$$\tilde{\beta}_1 = \frac{\sum_{i=1}^{n} X_i Y_i}{\sum_{i=1}^{n} X_i^2} \tag{1.2}$$

- d) Find $E(\tilde{\beta}_1)$ in terms of the X_i, β_0 and β_1. Verify that $\tilde{\beta}_1$ is unbiased for β_1, when the population intercept is zero. Are there other cases where $\tilde{\beta}_1$ is unbiased?

2 GOODNESS OF FIT

1. Show that the R^2 in the regression of Y on X (with an intercept) is the squared value of the sample correlation between X and Y (i.e. $R^2 = r_{XY}^2$).