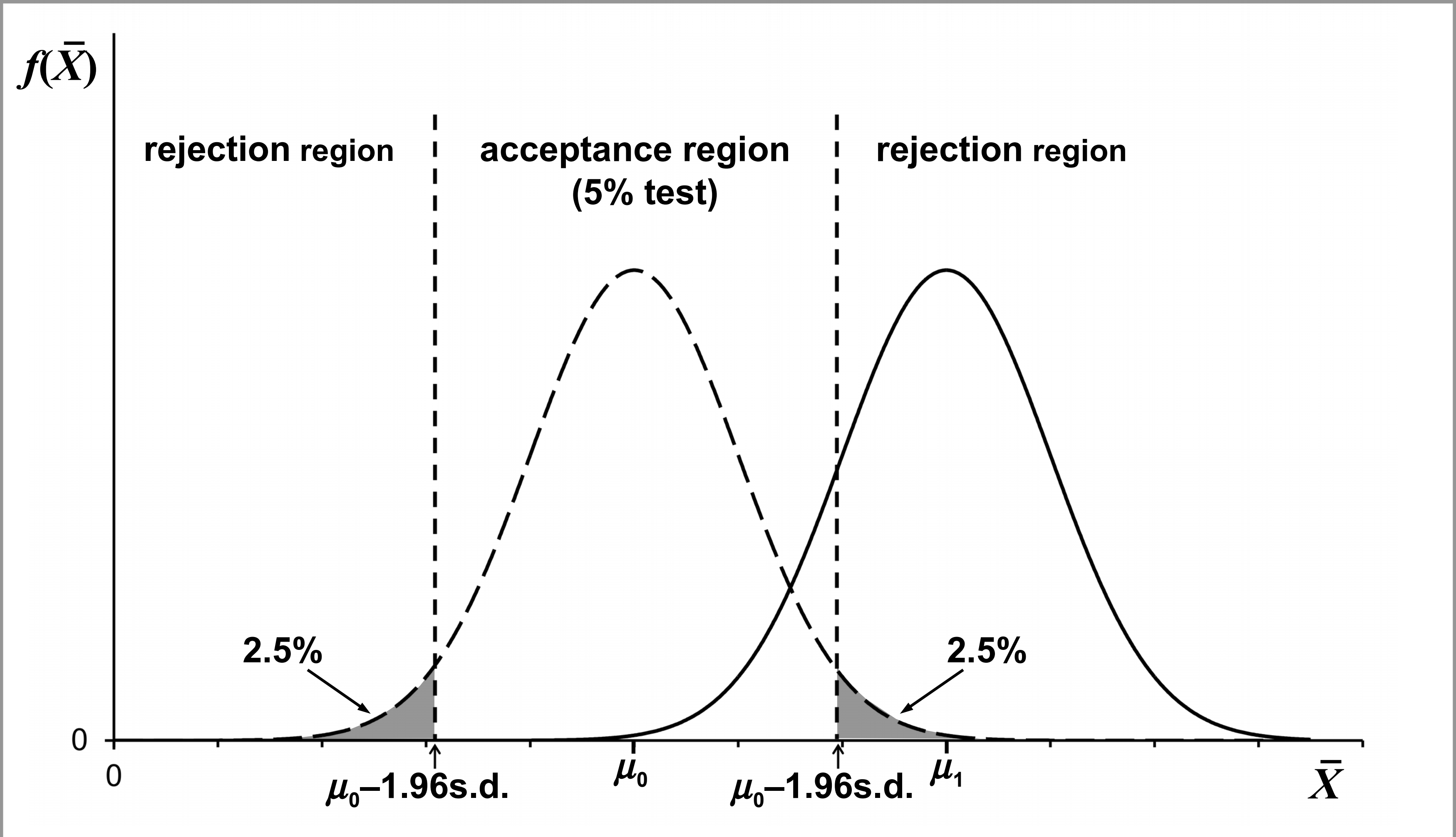
Dougherty

Introduction to Econometrics, 5th edition

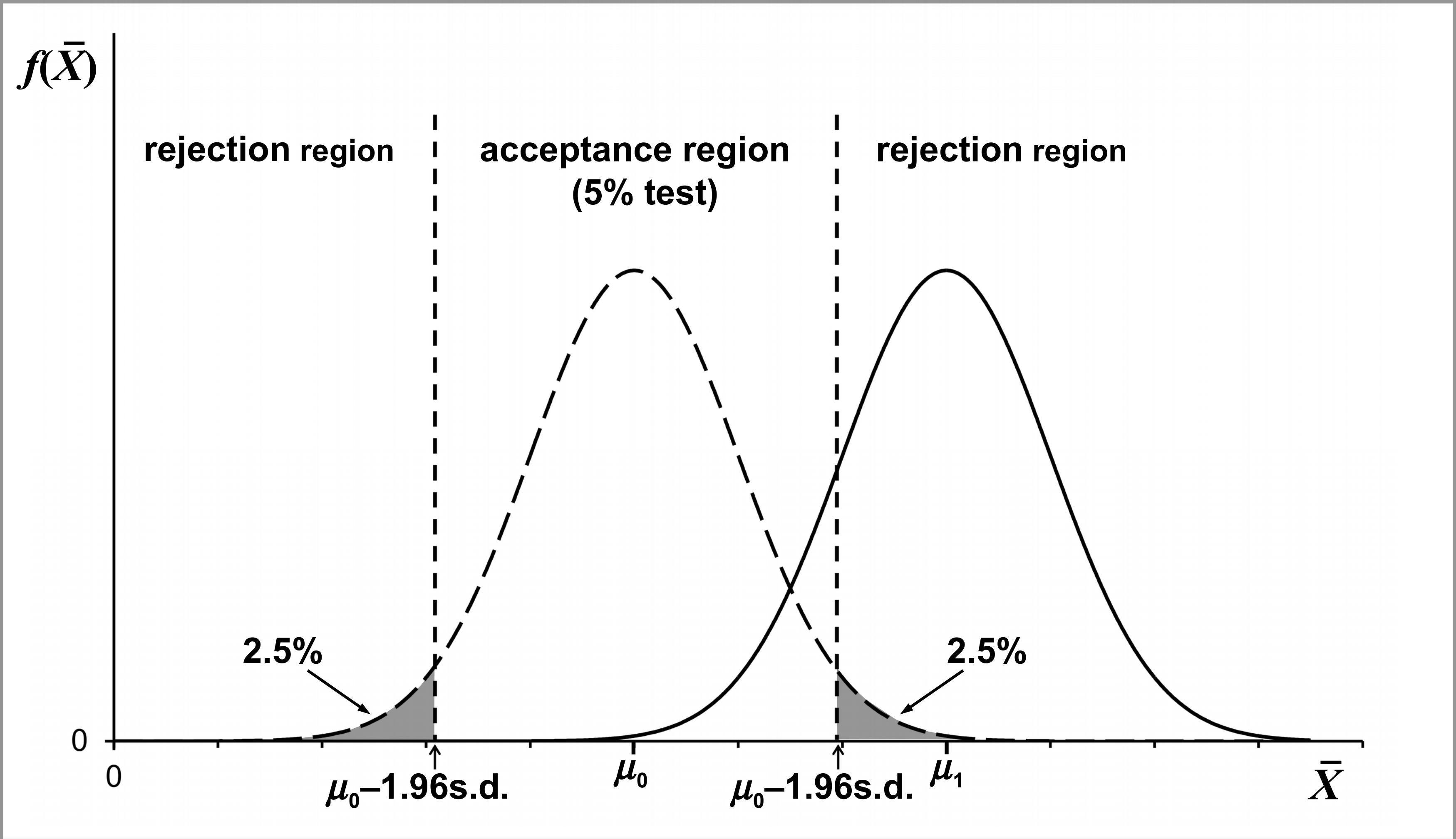
Review: Random Variables, Sampling, Estimation, and Inference

TYPE II ERROR AND THE POWER OF A TEST



A Type I error occurs when the null hypothesis is rejected when it is in fact true. A Type II error occurs when the null hypothesis is not rejected when it is in fact false.

TYPE II ERROR AND THE POWER OF A TEST



We will see that, in general, there is a trade-off between the risk of making a Type I error and the risk of making a Type II error.

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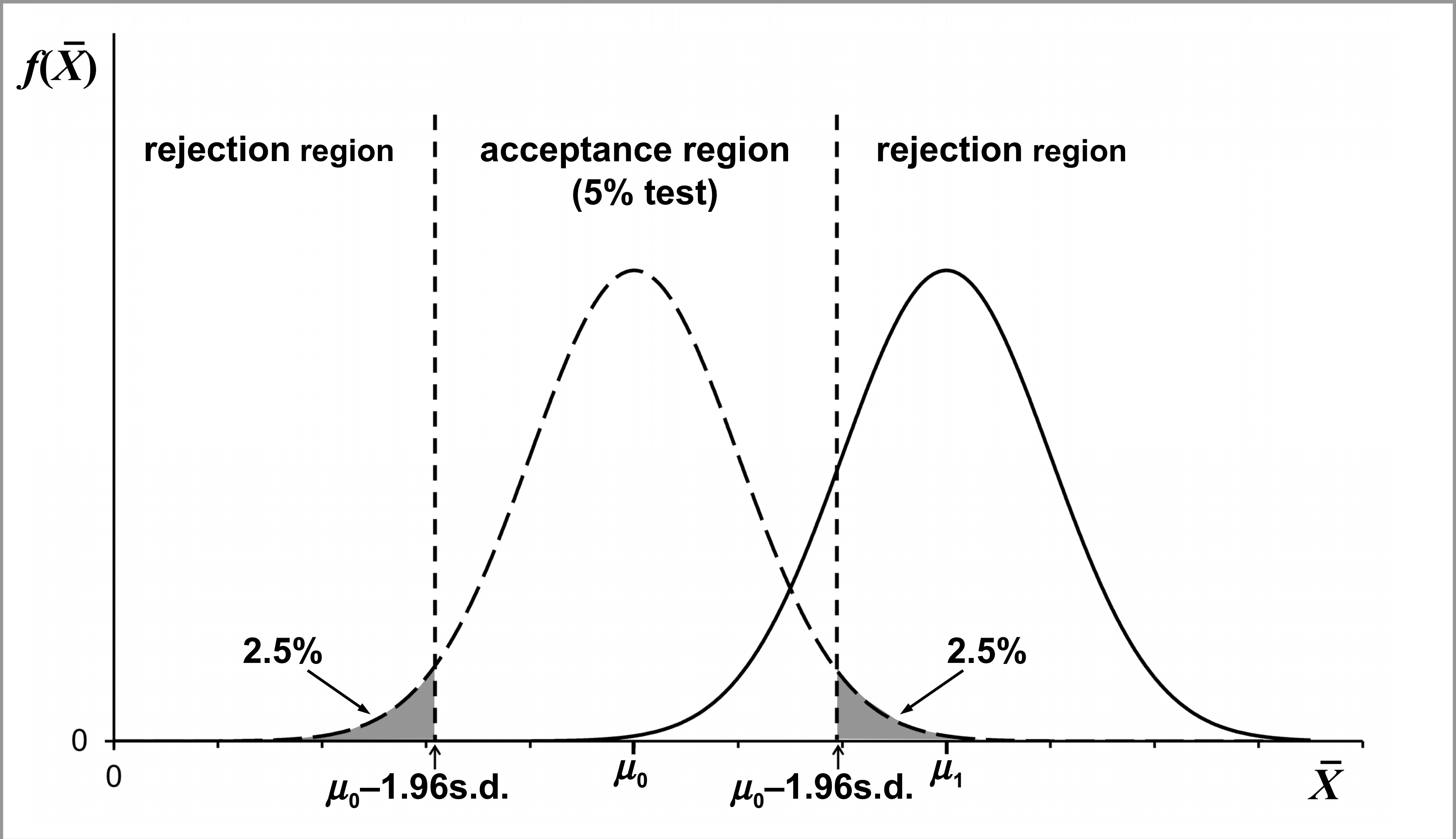

We will consider the case where the null hypothesis, H_0: $\mu = \mu_0$ is false and the actual value of μ is μ_1. This is shown in the figure.

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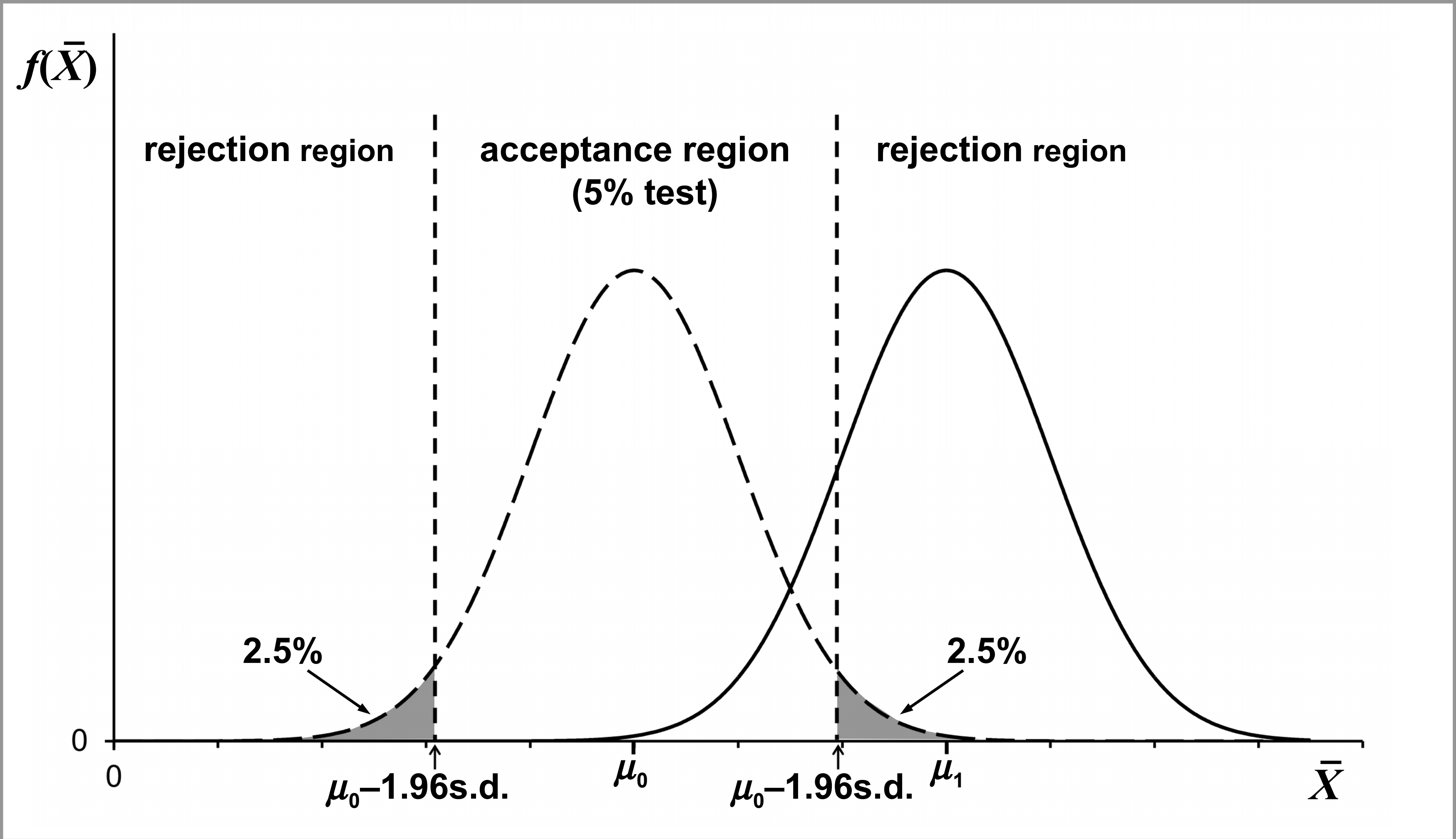

If the null hypothesis is tested, it will be rejected only if *X* lies in one of the rejection regions associated with it.

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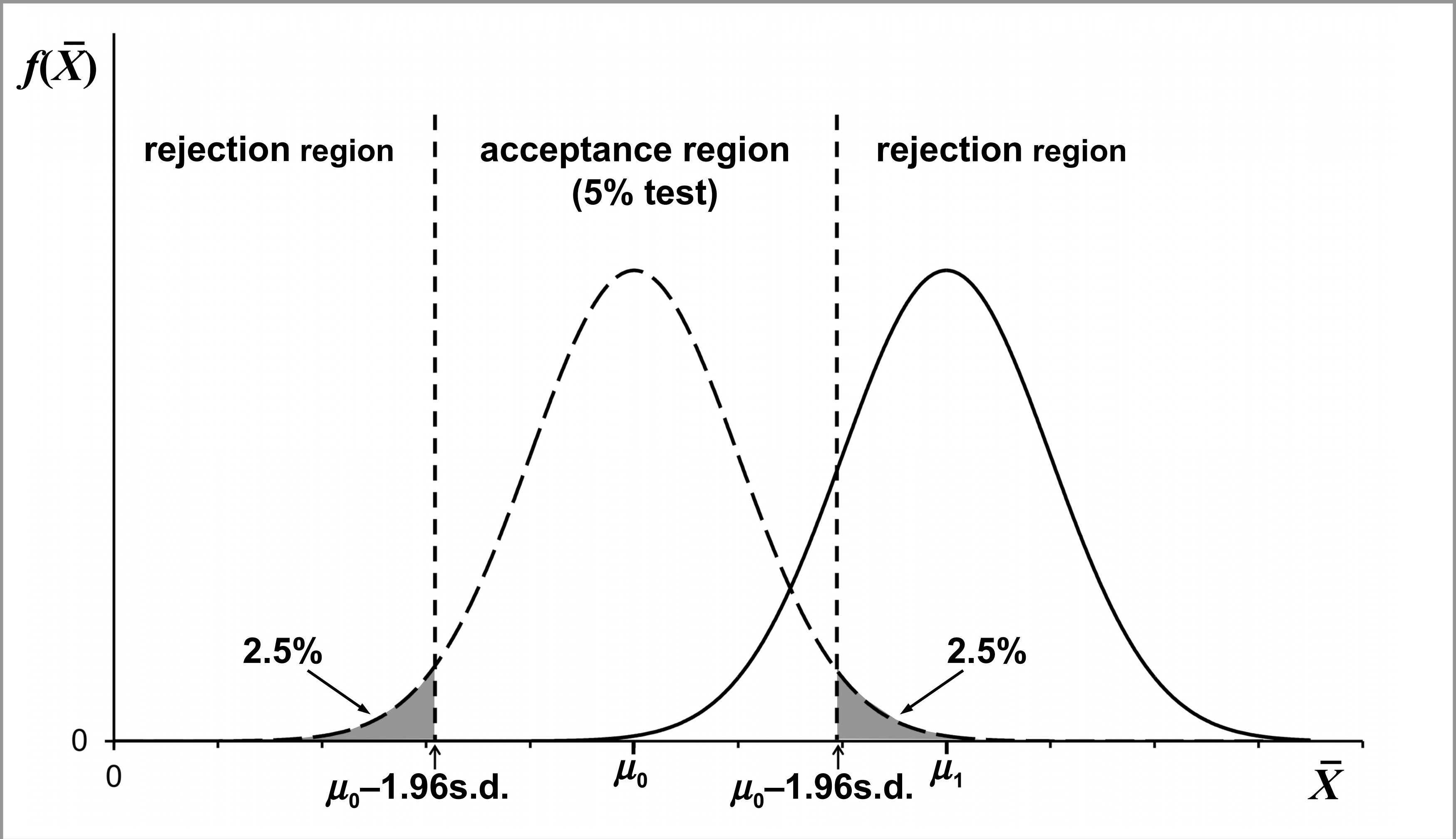
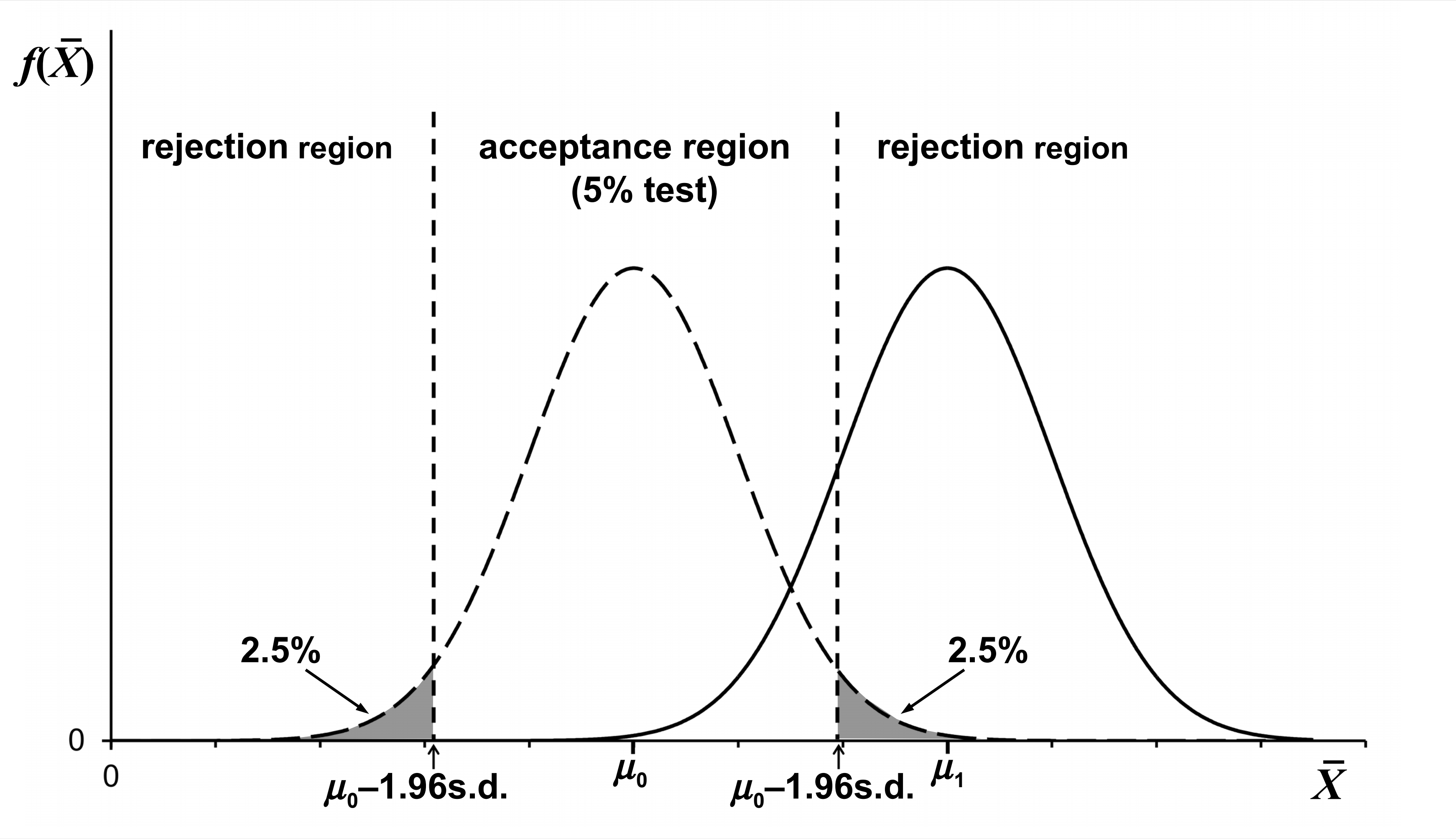


To determine the rejection regions, we draw the distribution of *X* conditional on H_0 being true. The distribution is marked with a dashed curve to emphasize that H_0 is not actually true.



The rejection regions for a (two-sided) 5 percent test, given this distribution, are marked on the diagram.

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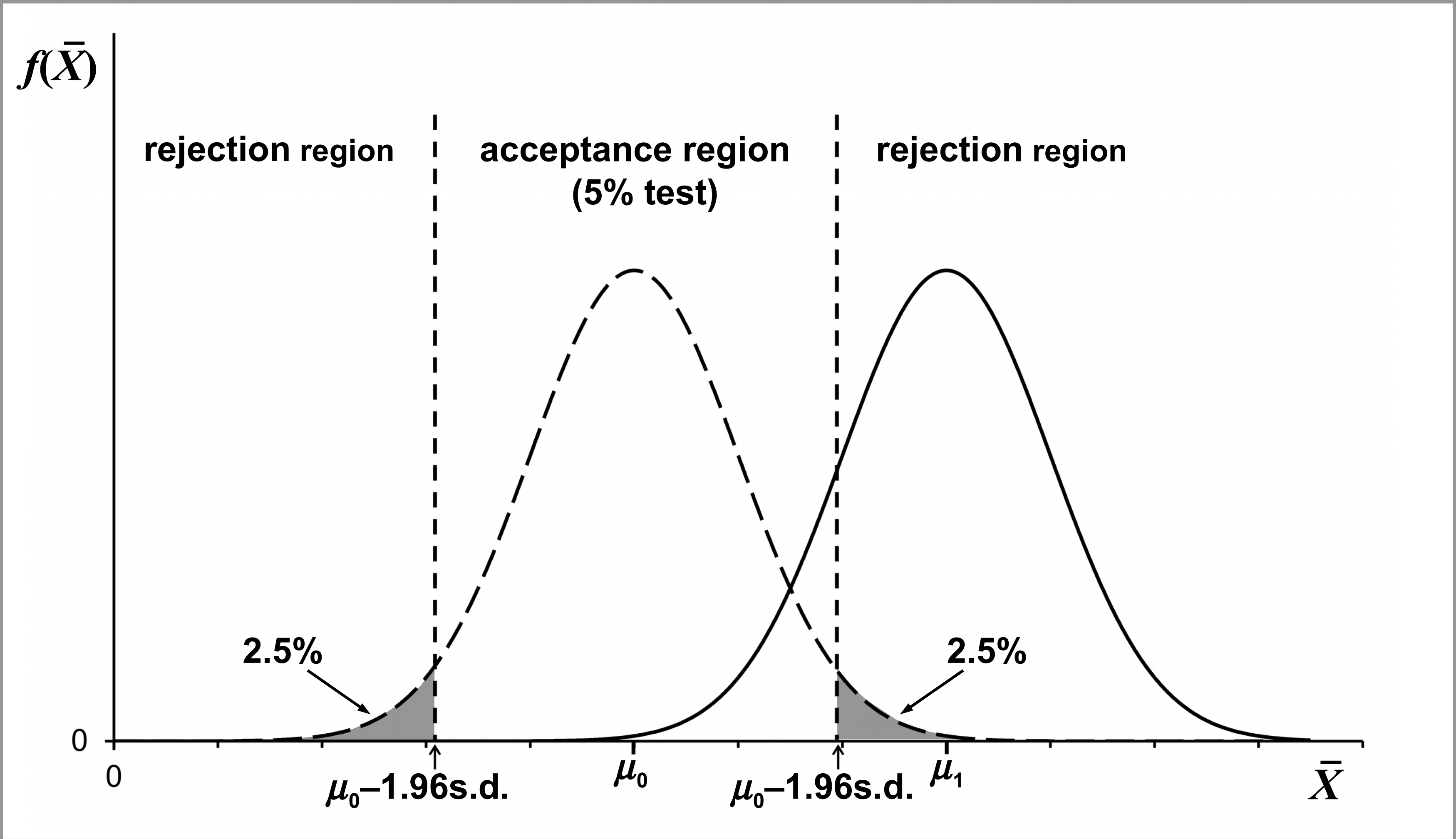

If X lies in the acceptance region, H_0 will not be rejected, and so a Type II error will occur. What is the probability of this happening? To determine this, we now turn to the actual distribution of $\bar{X}$, given that $\mu = \mu_1$. This is the solid curve on the right.

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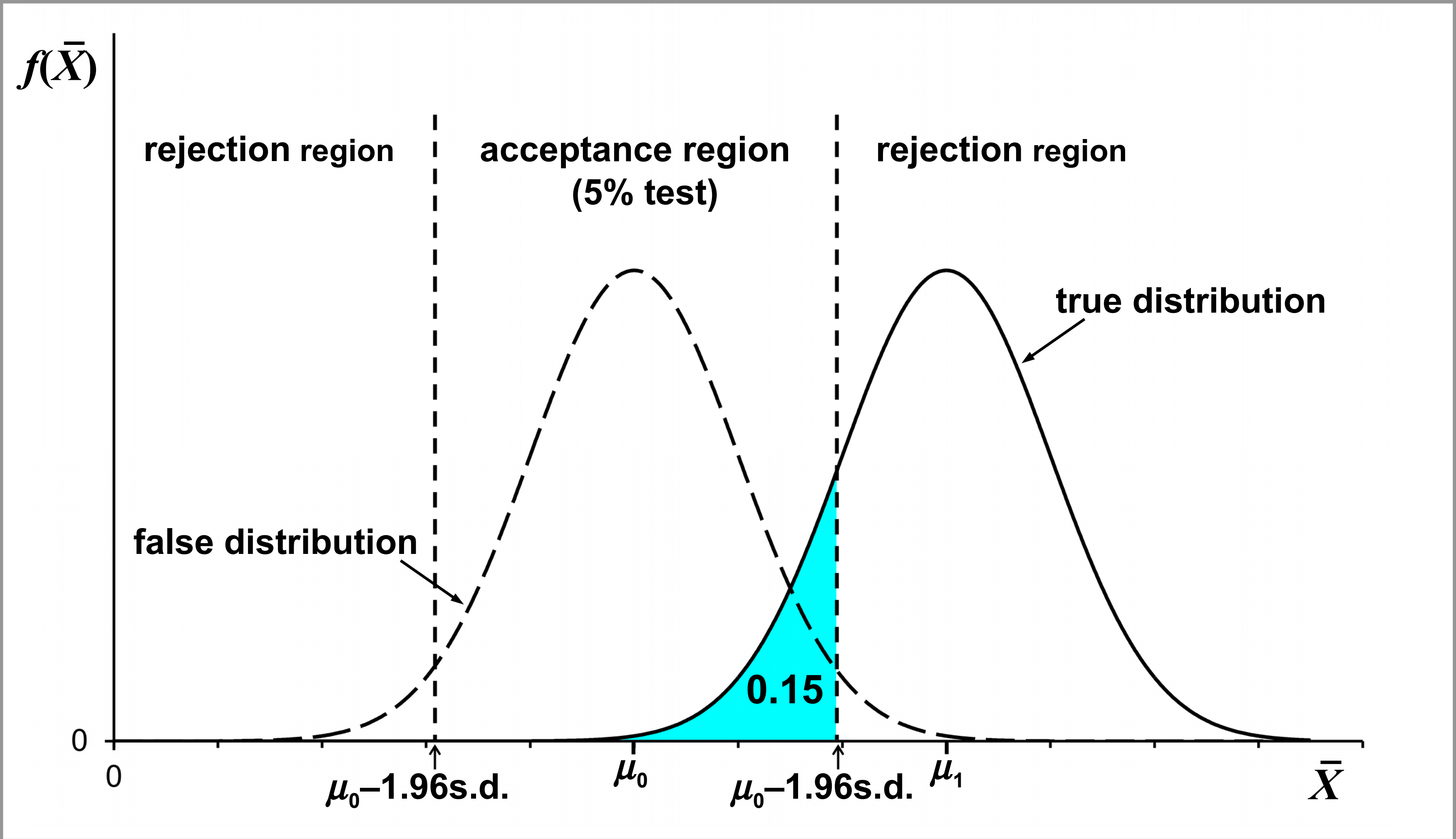
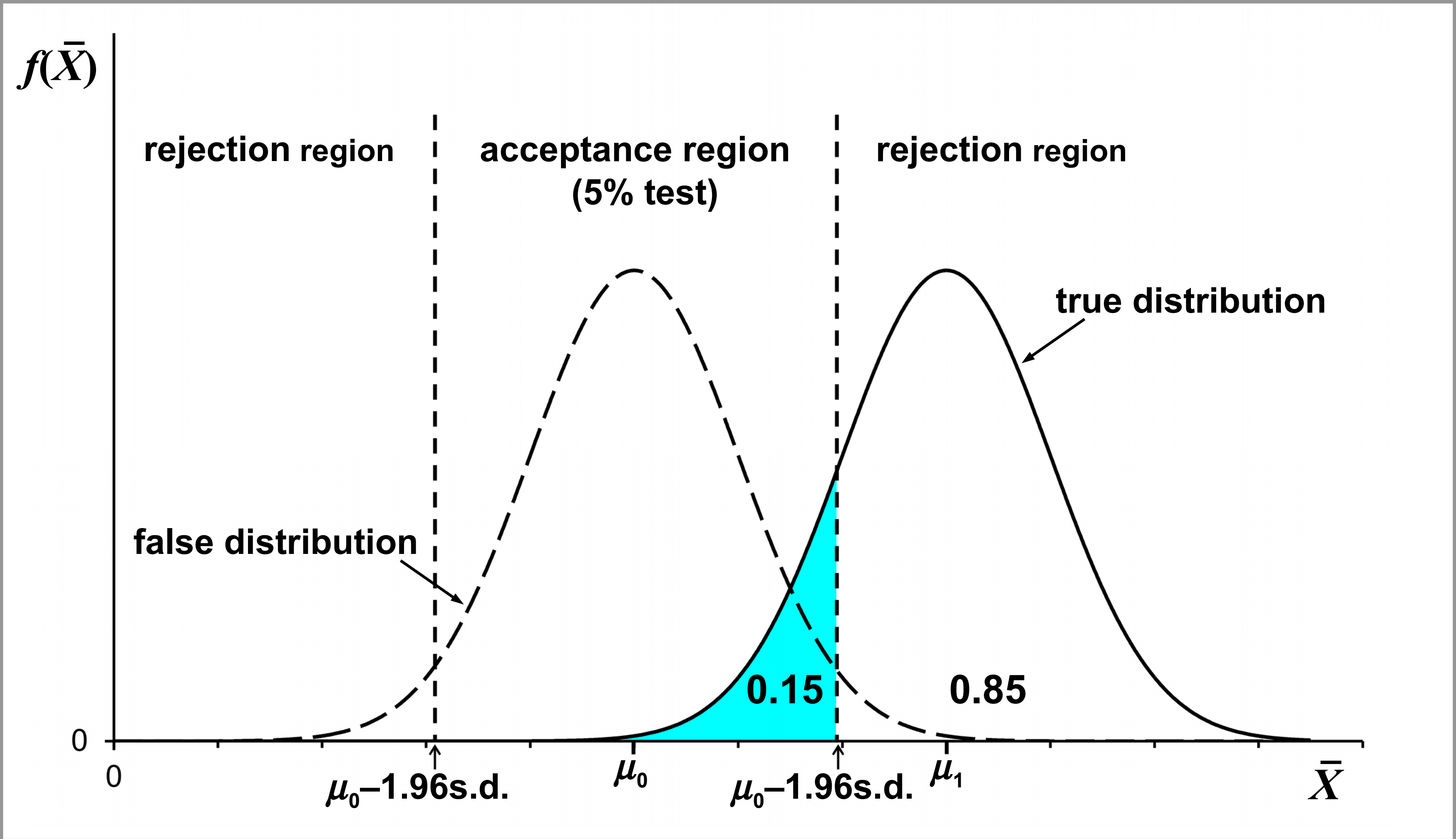


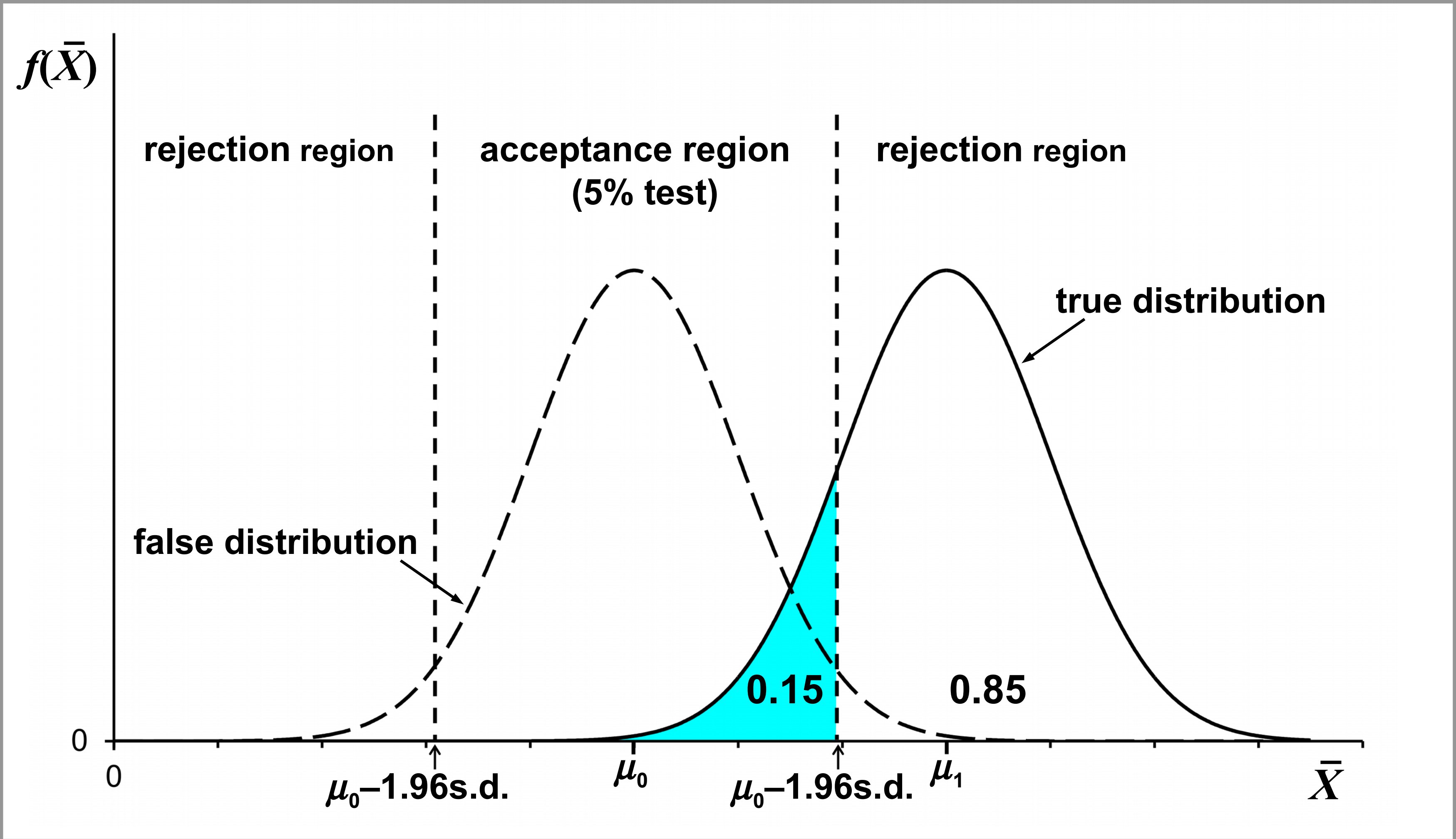
The probability of $\bar{X}$ lying in the acceptance region for H_0 is the area under this curve in the acceptance region. It is the blue area in the figure. In this particular case, the probability of X lying within the acceptance region for H_0, thus causing a Type II error, is 0.15.

TYPE II ERROR AND THE POWER OF A TEST



The probability of rejecting the null hypothesis, when it is false, is known as the power of a test. By definition, it is equal to 1 minus the probability of making a Type II error. It is therefore 0.85 in this example.

TYPE II ERROR AND THE POWER OF A TEST



The power depends on the distance between the value of μ under the false null hypothesis and its actual value. The closer that the actual value is to μ_0, the harder it is to demonstrate that H_0: $\mu = \mu_0$ is false.

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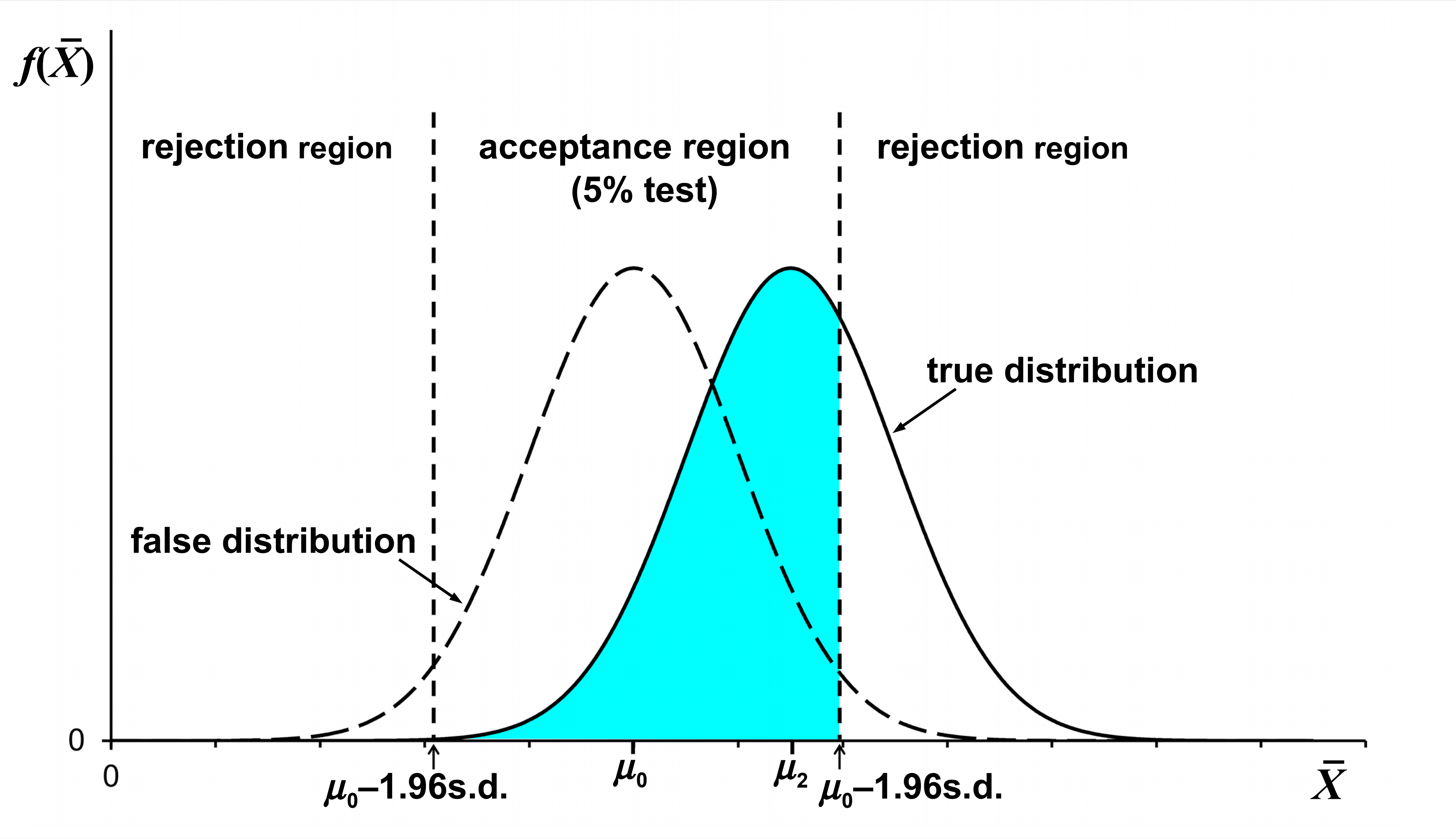
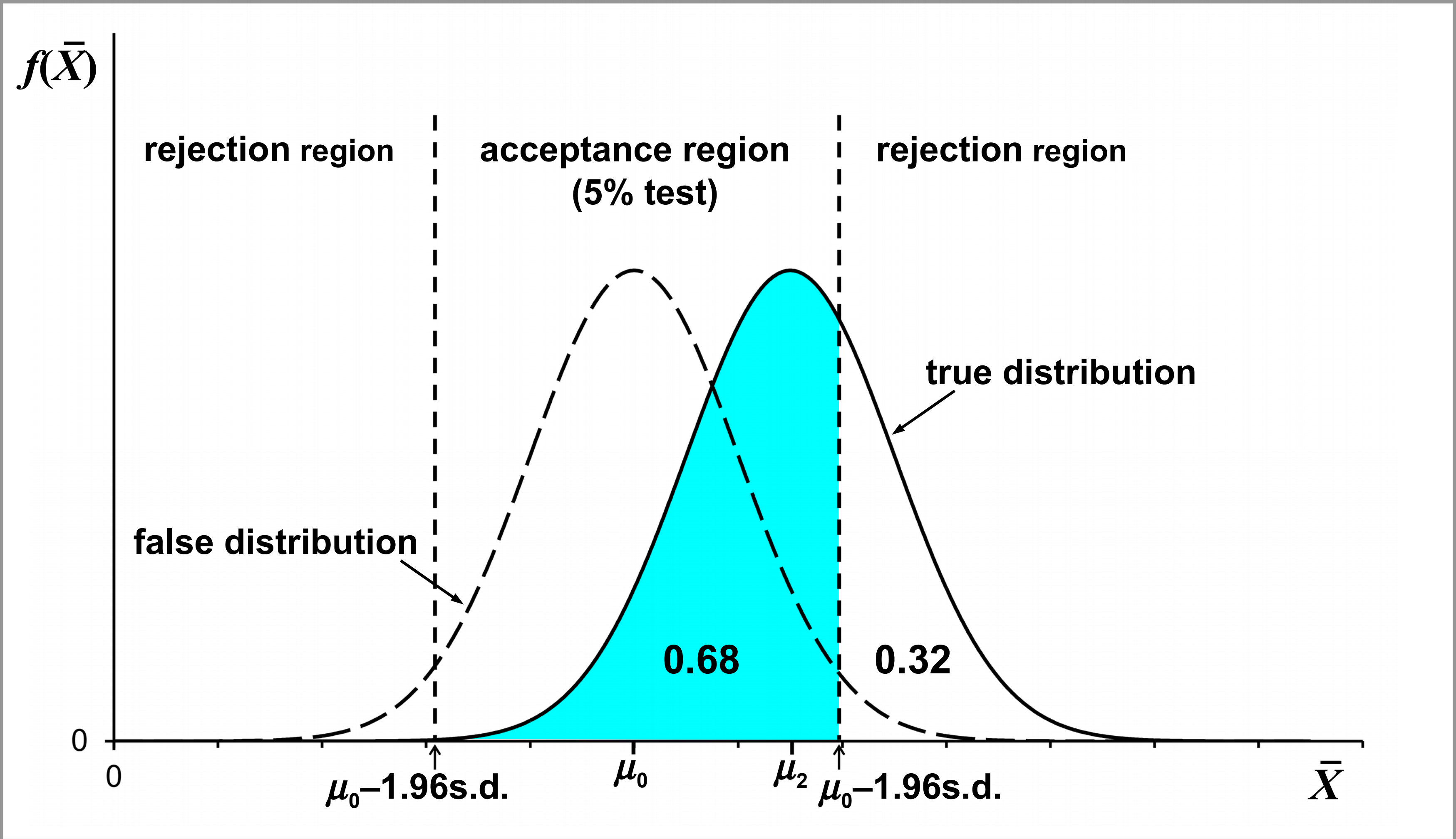


This is illustrated in the figure. μ_0 is the same as in the previous figure, and so the acceptance region and rejection regions for the test of H_0: $\mu = \mu_0$ are the same as in the previous figure.

TYPE II ERROR AND THE POWER OF A TEST



As in the previous figure, H_0 is fa<u>l</u>se, but now the true value is μ_2, and μ_2 is closer to μ_0. As a consequence, the probability of X lying in the acceptance region for H_0 is much greater, 0.68 instead of 0.15, and so the power of the test, 0.32, is much lower.

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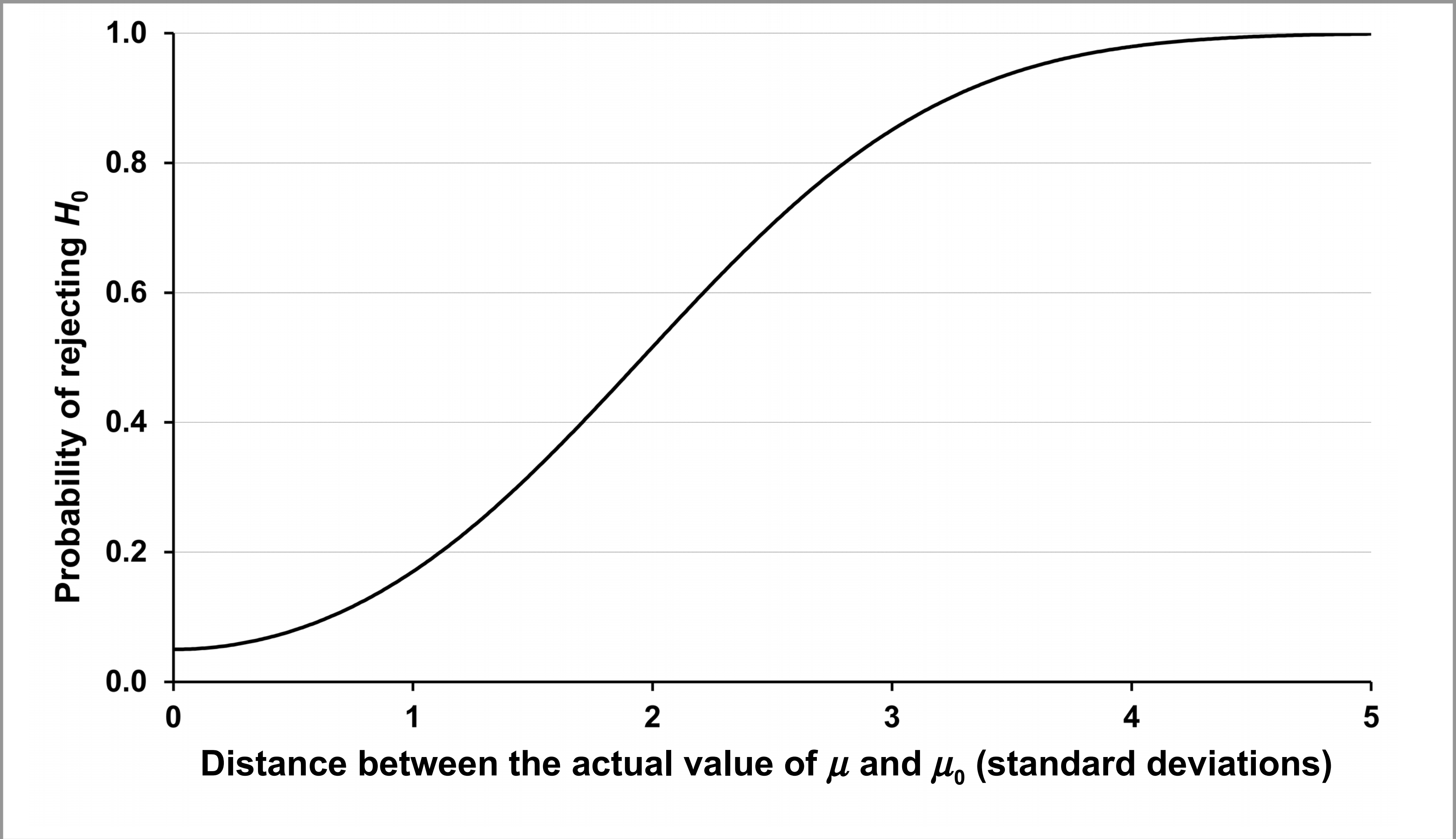

The figure plots the power of a 5 percent significance test as a function of the distance separating the actual value of μ and μ_0, measured in terms of the standard deviation of the distribution of X.

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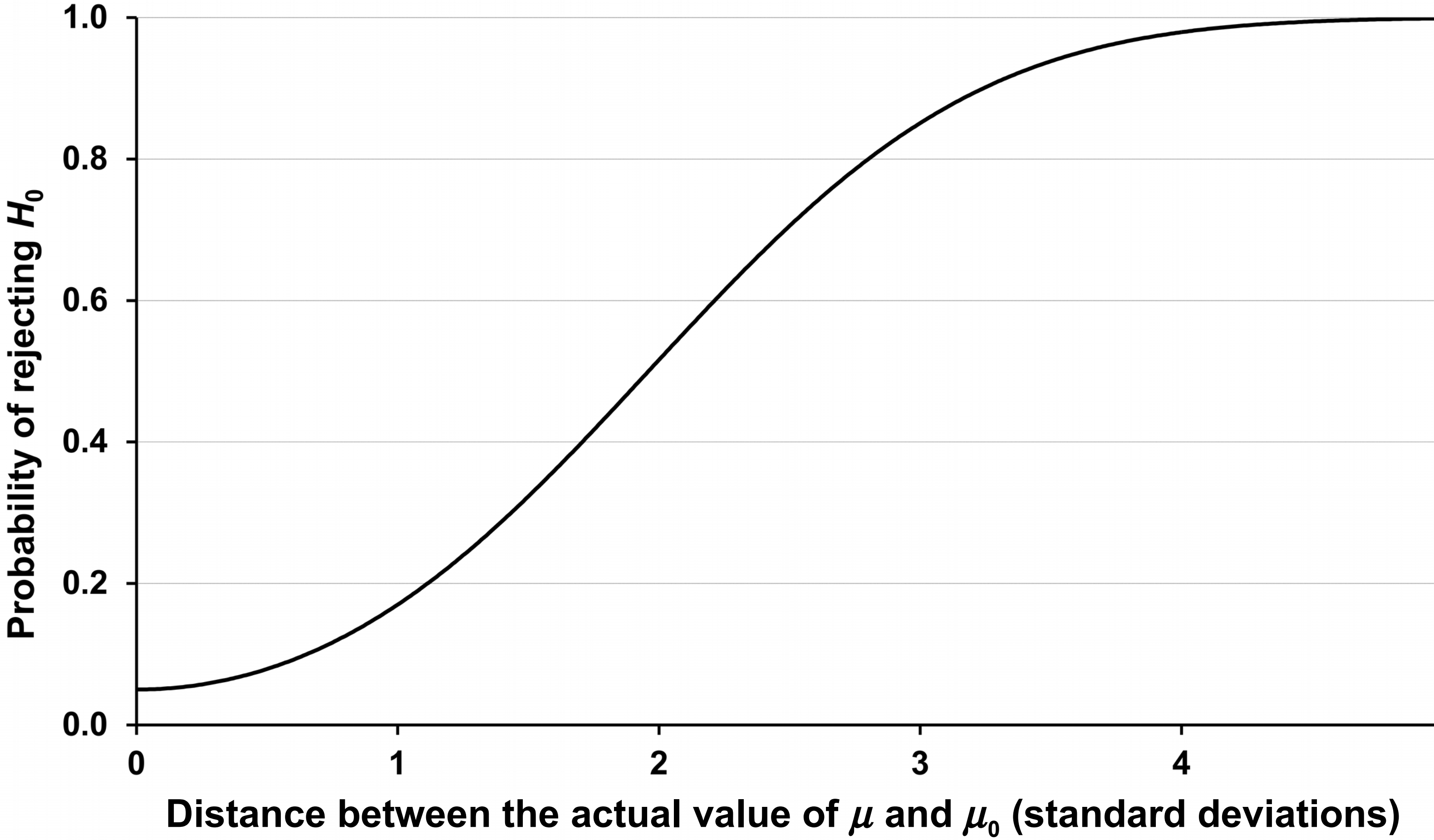

As is intuitively obvious, the greater is the discrepancy, the greater is the probability of H_0: $\mu = \mu_0$ being rejected.

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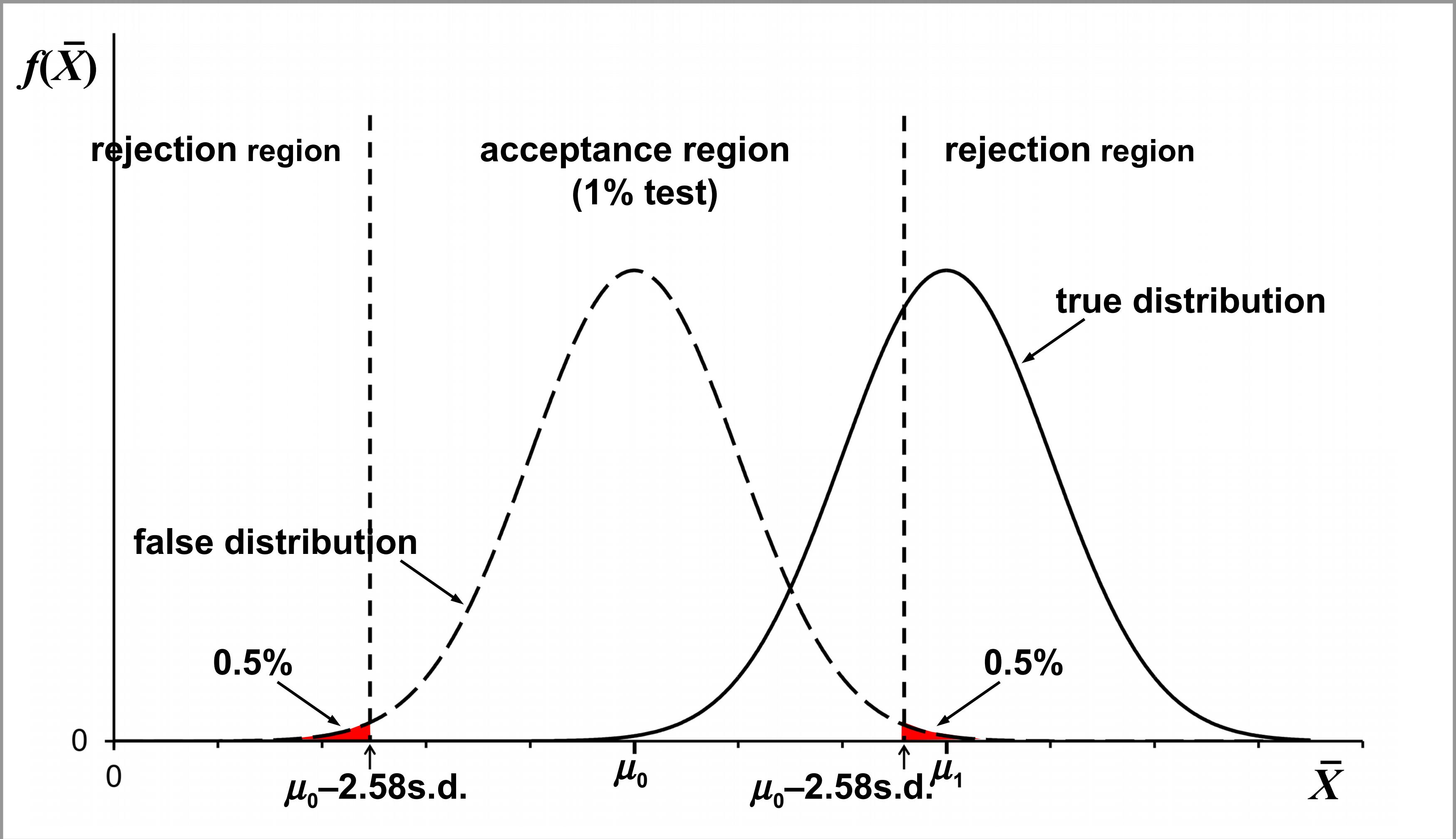

We now return to the original value of μ_1 and again consider the case where H_0: $\mu = \mu_0$ is false and H_1: $\mu = \mu_1$ is true. What difference does it make if we perform a 1 percent test, instead of a 5 percent test?

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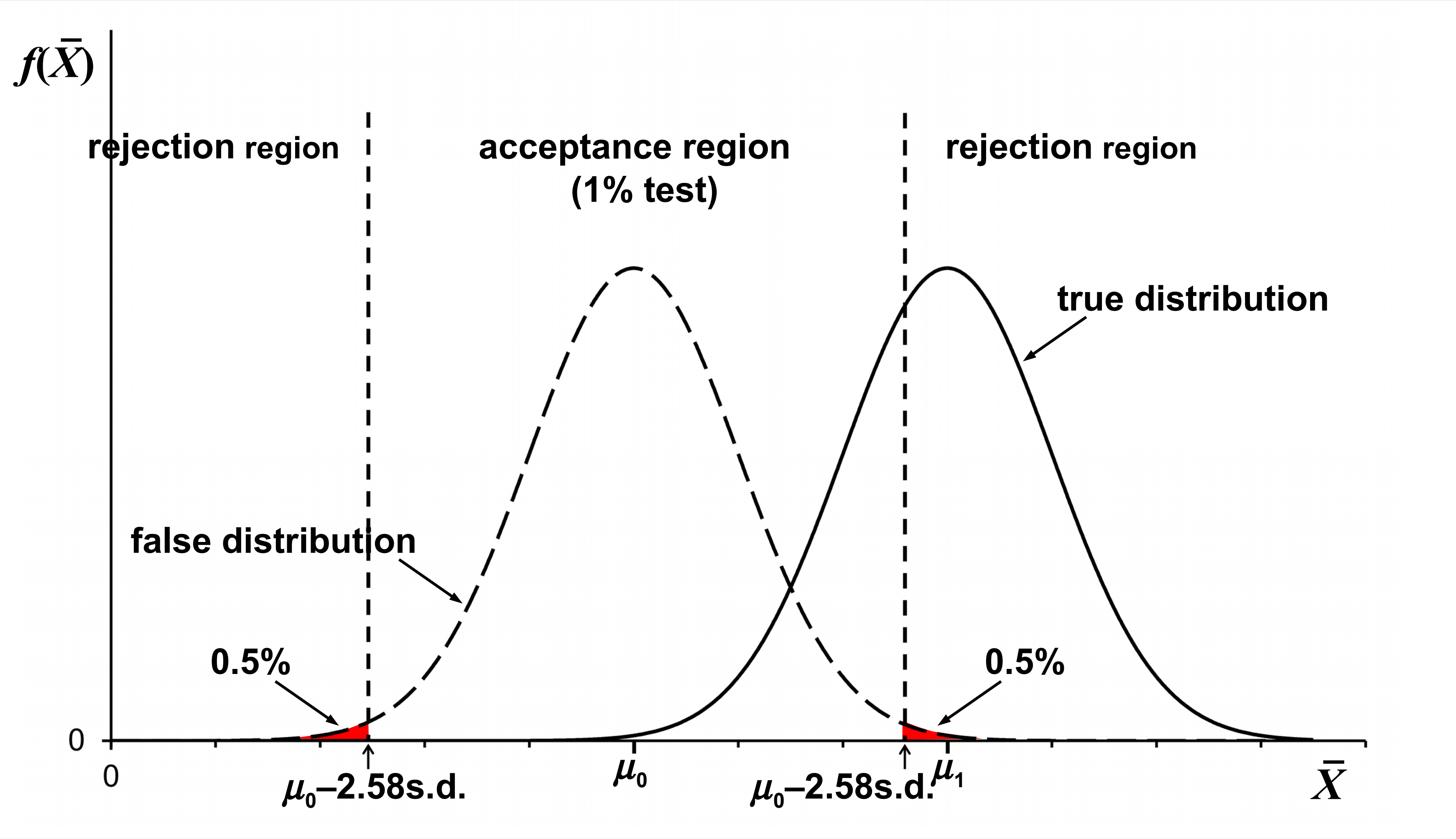

The figure shows the acceptance region for the 1 percent test.

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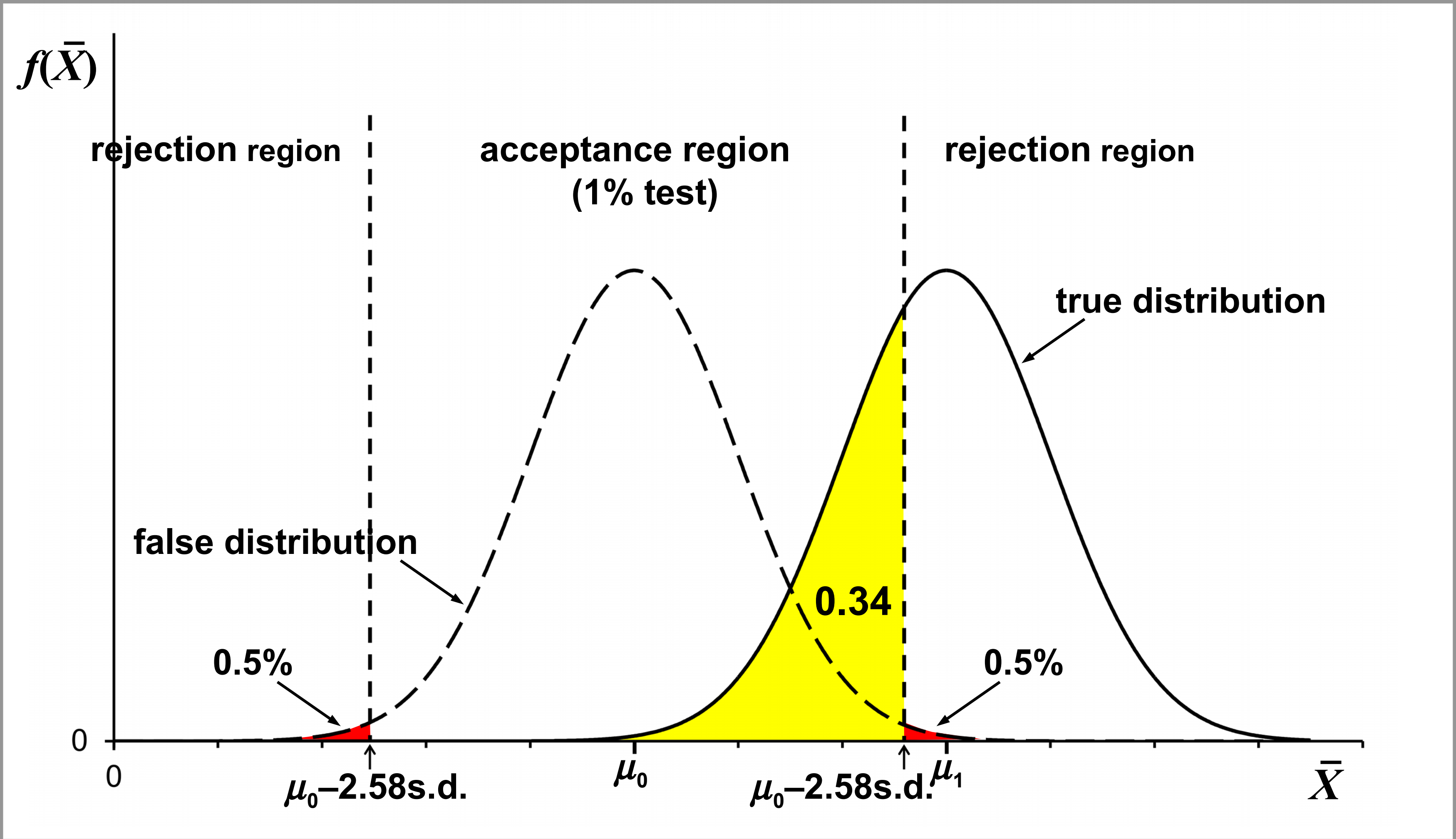

The probability of X lying in this region, given that it is actually distributed with mean μ_1, is shown as the yellow shaded area. It is 0.34. The probability of making a Type II error is therefore 0.34.

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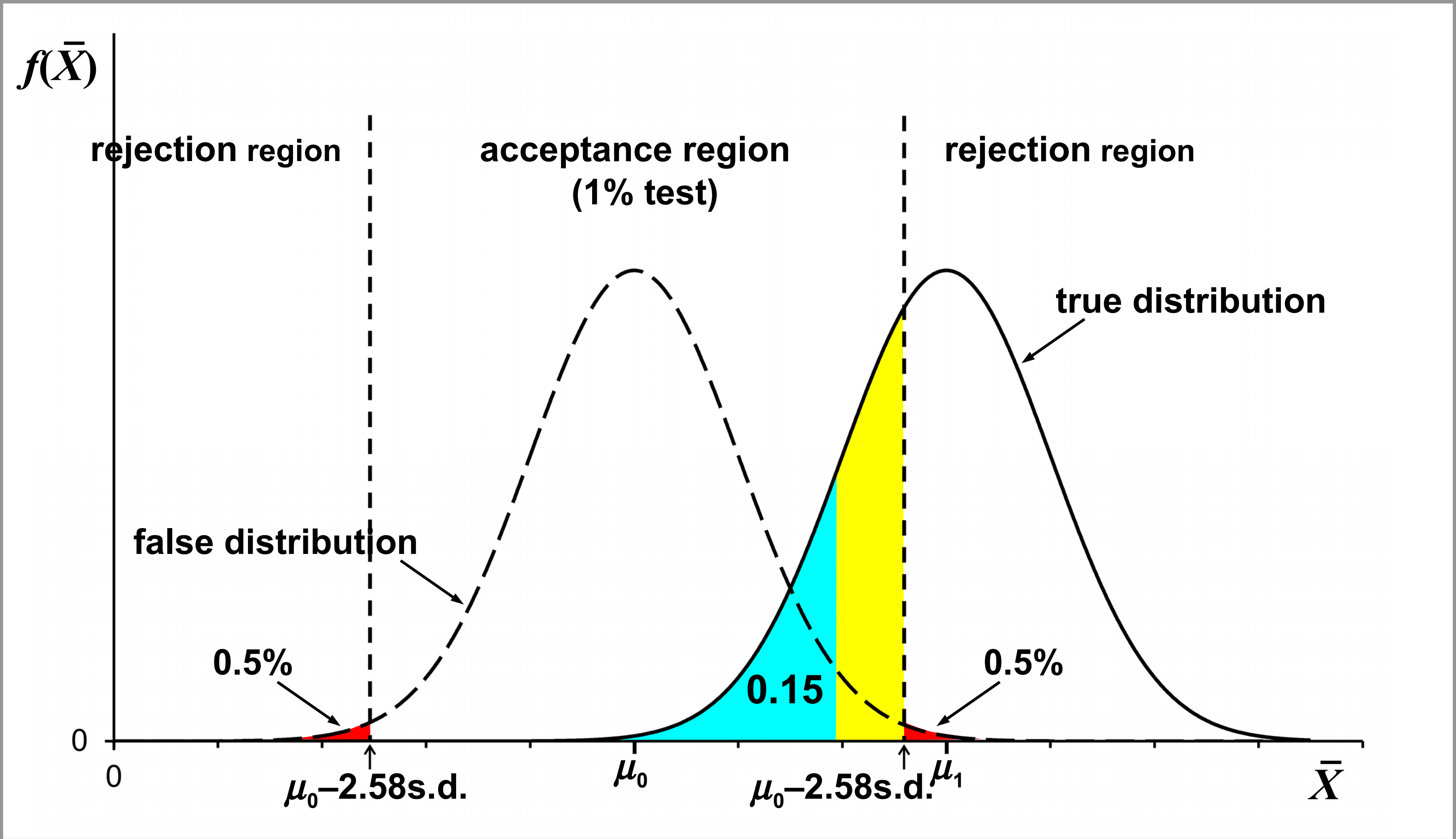

We have seen that the probability of making a Type II error with a 5 percent test, given by the blue shaded area, was 0.15. This illustrates the trade-off between the risks of Type I and Type II error.

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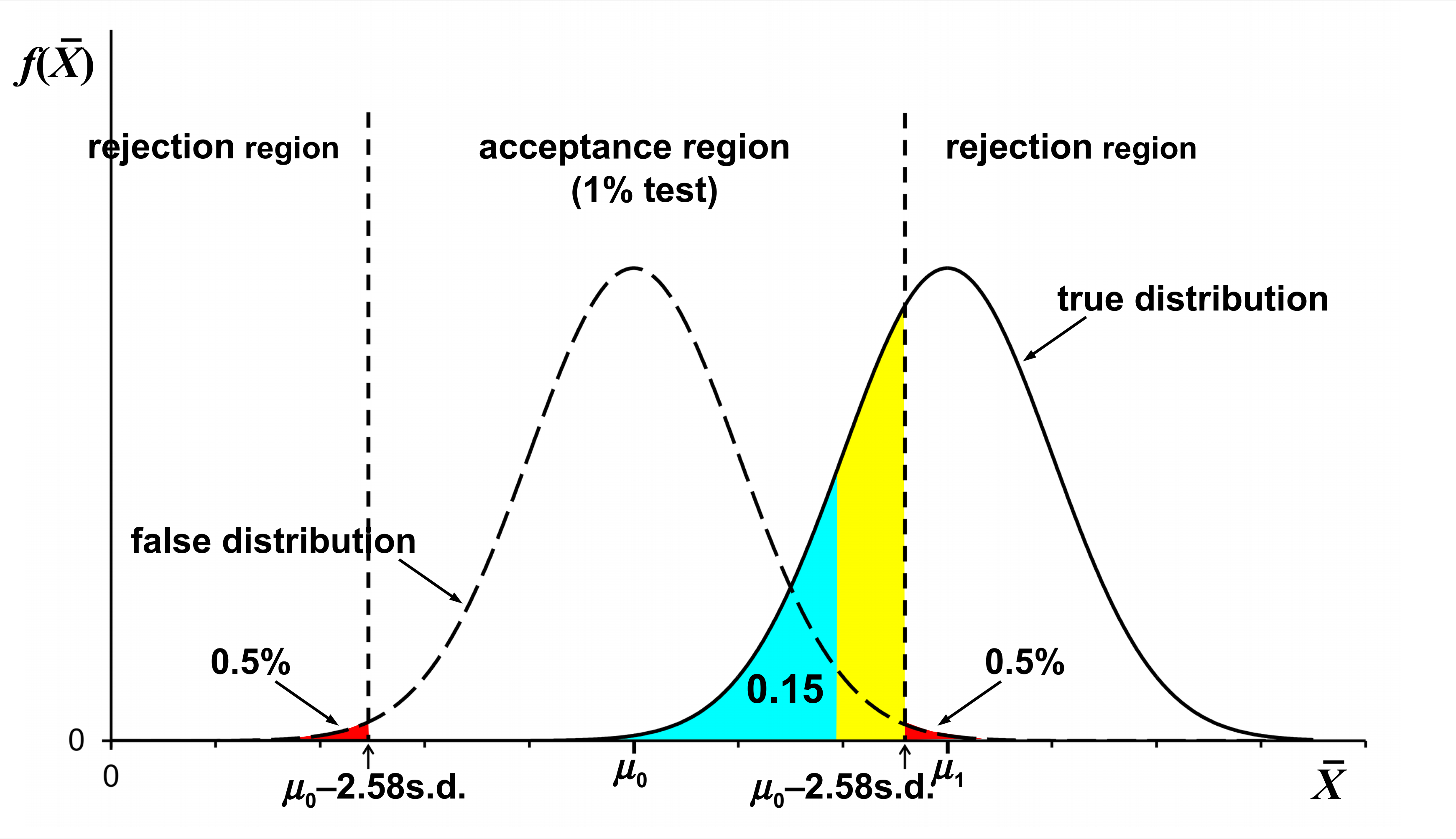

If we perform a 1 percent test instead of a 5 percent test, and H_0 is true, the risk of mistakenly rejecting it (and therefore committing a Type I error) is only 1 percent instead of 5 percent.

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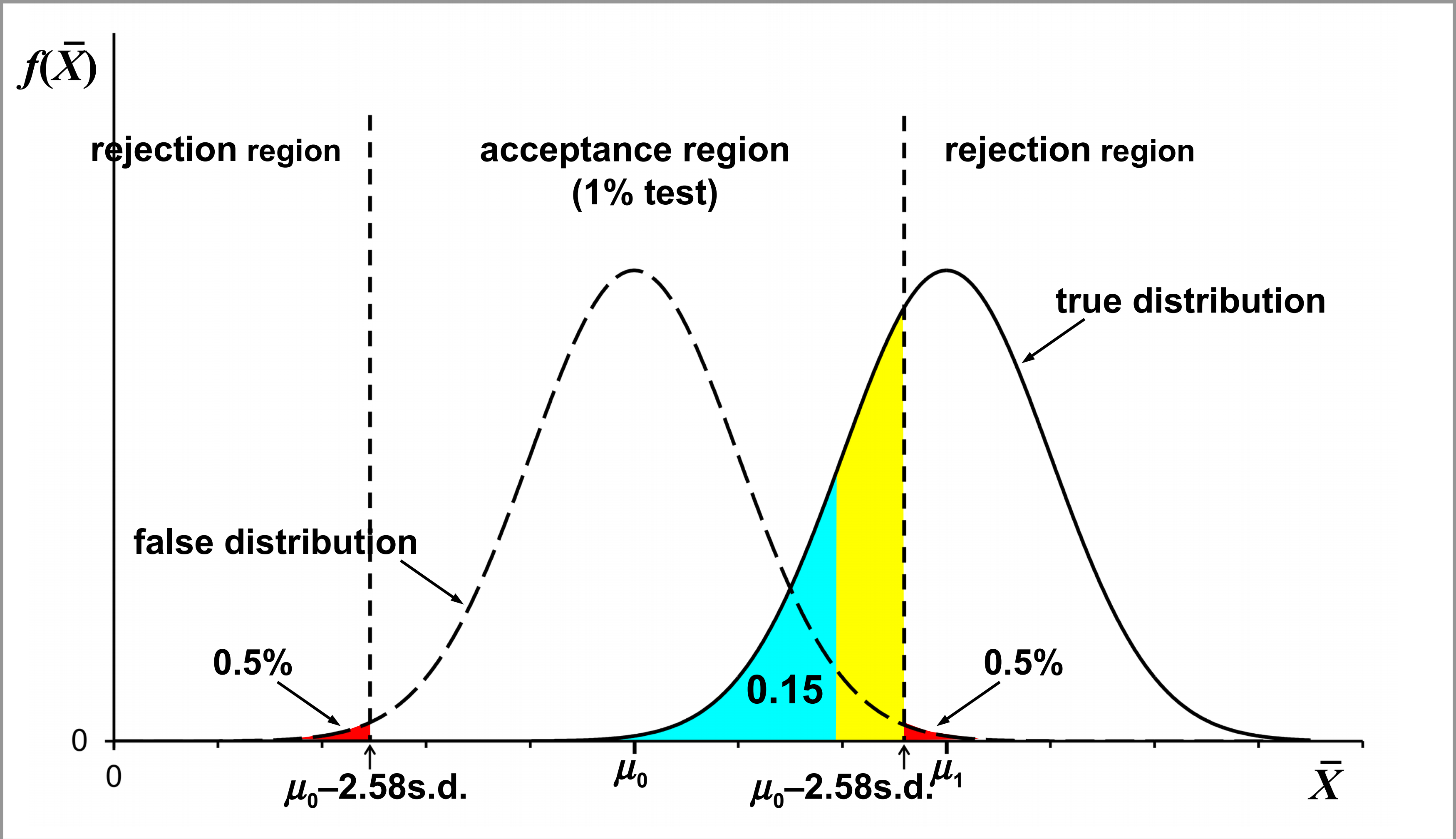

However, if H_0 happens to be false, the probability of not rejecting it (and therefore committing a Type II error) is larger.

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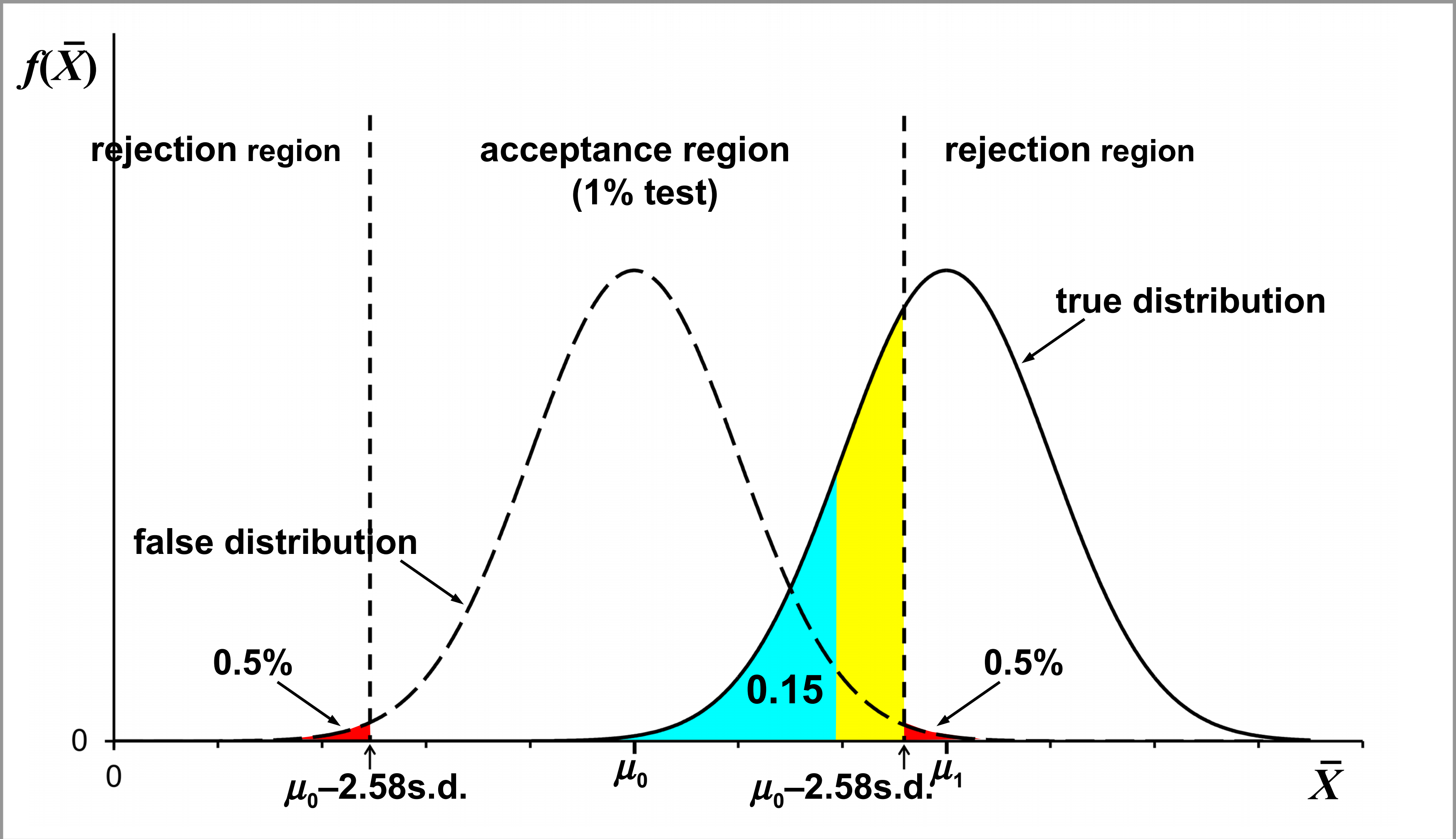

How much larger? This is not fixed. It depends on the distance between μ_0 and μ_1, measured in terms of standard deviations. In this particular case, it has increased from 0.15 to 0.34, so it has about doubled.

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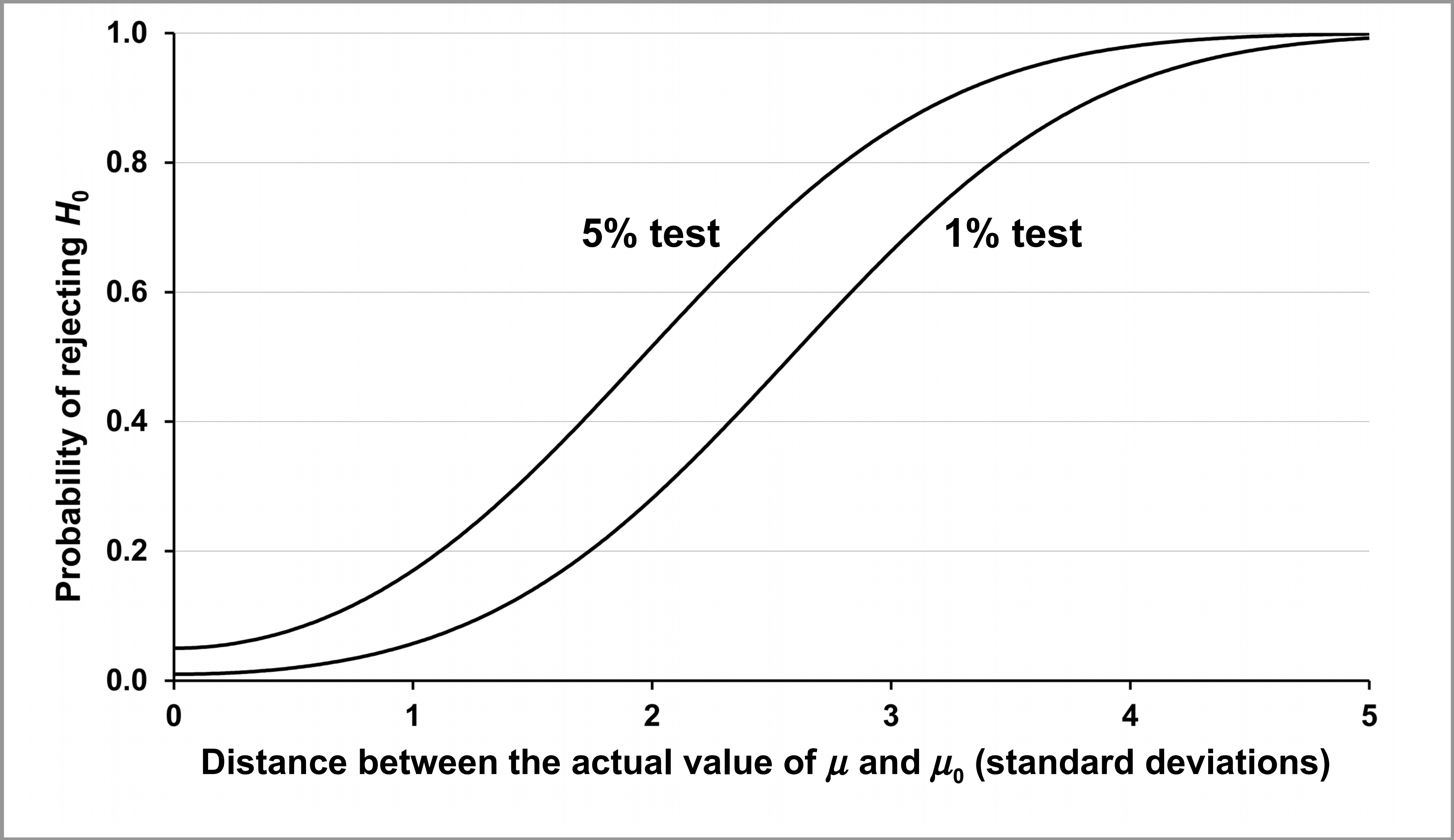

To generalize, we plot the power functions for the 5 percent and 1 percent tests.

The content of this slideshow comes from Section R.10 of C. Dougherty, *Introduction to Econometrics*, fifth edition 2016, Oxford University Press. Additional (free) resources for both students and instructors may be downloaded from the OUP Online Resource Centre
http://www.oxfordtextbooks.co.uk/orc/dougherty5e/

Individuals studying econometrics on their own who feel that they might benefit from participation in a formal course should consider the London School of Economics summer school course
EC212 Introduction to Econometrics
http://www2.lse.ac.uk/study/summerSchools/summerSchool/Home.aspx
or the University of London International Programmes distance learning course
EC2020 Elements of Econometrics
www.londoninternational.ac.uk/lse.