Dougherty

Introduction to Econometrics, 5th edition

Chapter 2: Properties of the Regression Coefficients and Hypothesis Testing

ONE-SIDED t TESTS OF HYPOTHESES RELATING TO REGRESSION COEFFICIENTS

True model

$$Y = \beta_1 + \beta_2 X + u$$

Fitted model

$$\hat{Y} = \hat{\beta}_1 + \hat{\beta}_2 X$$

Null hypothesis $H_0 : \beta_2 = \beta_2^0$

Alternative hypothesis $H_1 : \beta_2 \neq \beta_2^0$

Test statistic

$$t = \frac{\hat{\beta}_2 - \beta_2^0}{\text{s.e.}\left(\hat{\beta}_2\right)}$$

Reject H_0 if $|t| > t_{\text{crit}}$

In the previous sequence, we were performing what are described as two-sided t tests. These are appropriate when we have no information about the alternative hypothesis.

ONE-SIDED t TESTS OF HYPOTHESES RELATING TO REGRESSION COEFFICIENTS

True model	Fitted model
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Null hypothesis $H_0 : \beta_2 = \beta_2^0$

Alternative hypothesis $H_1 : \beta_2 \neq \beta_2^0$

Test statistic
$$t = \frac{\hat{\beta}_2 - \beta_2^0}{\text{s.e.}\left(\hat{\beta}_2\right)}$$

Reject H_0 if $|t| > t_{\text{crit}}$

Under the null, the coefficient is hypothesized to be a certain value. Under the alternative hypothesis, the coefficient could be any value other than that specified by the null. It could be higher or it could be lower.

ONE-SIDED t TESTS OF HYPOTHESES RELATING TO REGRESSION COEFFICIENTS

True model

$$Y = \beta_1 + \beta_2 X + u$$

Fitted model

$$\hat{Y} = \hat{\beta}_1 + \hat{\beta}_2 X$$

Null hypothesis $H_0 : \beta_2 = \beta_2^0$

Alternative hypothesis $H_1 : \beta_2 > \beta_2^0$

Test statistic

$$t = \frac{\hat{\beta}_2 - \beta_2^0}{\text{s.e.}\left(\hat{\beta}_2\right)}$$

Reject H_0 if $|t| > t_{\text{crit}}$

However, sometimes we are in a position to say that, if the null hypothesis is not true, the coefficient cannot be lower than that specified by it. We re-write the null hypothesis as shown and perform a one-sided test.

ONE-SIDED t TESTS OF HYPOTHESES RELATING TO REGRESSION COEFFICIENTS

True model	Fitted model
$Y = \beta_1 + \beta_2 X + u$	$\hat{Y} = \hat{\beta}_1 + \hat{\beta}_2 X$

Null hypothesis $\quad H_0 : \beta_2 = \beta_2^0$

Alternative hypothesis $\quad H_1 : \beta_2 < \beta_2^0$

Test statistic
$$t = \frac{\hat{\beta}_2 - \beta_2^0}{\text{s.e.}\left(\hat{\beta}_2\right)}$$

Reject H_0 if $\quad |t| > t_{\text{crit}}$

On other occasions, we might be in a position to assert that, if the null hypothesis is not true, the coefficient cannot be greater than the value specified by it. The modified null hypothesis for this case is shown.

ONE-SIDED t TESTS OF HYPOTHESES RELATING TO REGRESSION COEFFICIENTS

True model	Fitted model
$Y = \beta_1 + \beta_2 X + u$	$\hat{Y} = \hat{\beta}_1 + \hat{\beta}_2 X$

Null hypothesis $H_0 : \beta_2 = \beta_2^0$

Alternative hypothesis $H_1 : \beta_2 < \beta_2^0$

Test statistic

$$t = \frac{\hat{\beta}_2 - \beta_2^0}{\text{s.e.}\left(\hat{\beta}_2\right)}$$

Reject H_0 if $|t| > t_{\text{crit}}$

The theory behind one-sided tests, in particular, the gain in the trade-off between the size (significance level) and power of a test, is non-trivial and an understanding requires a careful study of section R.13 of the Review chapter.

ONE-SIDED t TESTS OF HYPOTHESES RELATING TO REGRESSION COEFFICIENTS

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Null hypothesis $H_0 : \beta_2 = \beta_2^0$

Alternative hypothesis $H_1 : \beta_2 < \beta_2^0$

Test statistic

$$t = \frac{\hat{\beta}_2 - \beta_2^0}{\text{s.e.}\left(\hat{\beta}_2\right)}$$

Reject H_0 if $|t| > t_{\text{crit}}$

This sequence assumes a good understanding of that material.

ONE-SIDED t TESTS OF HYPOTHESES RELATING TO REGRESSION COEFFICIENTS

Example: $p = \beta_1 + \beta_2 w + u$

Null hypothesis: $H_0\!: \beta_2 = 1.0$

Alternative hypothesis: $H_1\!: \beta_2 \neq 1.0$

$$\hat{p} = \underset{(0.05)}{1.21} + \underset{(0.10)}{0.82}w$$

$$t = \frac{\hat{\beta}_2 - \beta_2^0}{\text{s.e.}\left(\hat{\beta}_2\right)} = \frac{0.82 - 1.00}{0.10} = -1.80$$

$$n = 20 \qquad \text{degrees of freedom} = 18 \qquad t_{\text{crit, 5\%}} = 2.101 \left(\text{two-sided test}\right)$$

Returning to the price inflation/wage inflation model, we saw that we could not reject the null hypothesis β_2 = 1, even at the 5% significance level. That was using a two-sided test.

ONE-SIDED t TESTS OF HYPOTHESES RELATING TO REGRESSION COEFFICIENTS

Example: $p = \beta_1 + \beta_2 w + u$

Null hypothesis: $H_0\!: \beta_2 = 1.0$

Alternative hypothesis: $H_1\!: \beta_2 \neq 1.0$

$$\hat{p} = 1.21 + 0.82w$$
$$\quad\;(0.05)\;(0.10)$$

$$t = \frac{\hat{\beta}_2 - \beta_2^0}{\text{s.e.}\left(\hat{\beta}_2\right)} = \frac{0.82 - 1.00}{0.10} = -1.80$$

$$n = 20 \qquad \text{degrees of freedom} = 18 \qquad t_{\text{crit},5\%} = 2.101\ (\text{two-sided test})$$

However, in practice, improvements in productivity may cause the rate of cost inflation, and hence that of price inflation, to be lower than that of wage inflation.

Example: $p = \beta_1 + \beta_2 w + u$

Null hypothesis: $H_0\colon \beta_2 = 1.0$

Alternative hypothesis: $H_1\colon \beta_2 \neq 1.0$

$$\underset{}{\hat{p}} = \underset{(0.05)}{1.21} + \underset{(0.10)}{0.82}w$$

$$t = \frac{\hat{\beta}_2 - \beta_2^0}{\text{s.e.}\left(\hat{\beta}_2\right)} = \frac{0.82 - 1.00}{0.10} = -1.80$$

$$n = 20 \qquad \text{degrees of freedom} = 18 \qquad t_{\text{crit},5\%} = 2.101\ (\text{two-sided test})$$

Certainly, improvements in productivity will not cause price inflation to be greater than wage inflation and so in this case we are justified in ruling out β_2 > 1. We are left with H_0: β_2 = 1 and H_1: β_2 < 1.

Example: $p = \beta_1 + \beta_2 w + u$

Null hypothesis: $H_0: \beta_2 = 1.0$

Alternative hypothesis: $H_1: \beta_2 \neq 1.0$

$$\hat{p} = \underset{(0.05)}{1.21} + \underset{(0.10)}{0.82}w$$

$$t = \frac{\hat{\beta}_2 - \beta_2^0}{\text{s.e.}\left(\hat{\beta}_2\right)} = \frac{0.82 - 1.00}{0.10} = -1.80$$

$$n = 20 \qquad \text{degrees of freedom} = 18 \qquad t_{\text{crit, 5\%}} = 1.734 \left(\text{one-sided test}\right)$$

Thus we can perform a one-sided test, for which the critical value of t with 18 degrees of freedom at the 5% significance level is 1.73. Now we *can* reject the null hypothesis and conclude that price inflation is significantly lower than wage inflation, at the 5% significance level.

ONE-SIDED t TESTS OF HYPOTHESES RELATING TO REGRESSION COEFFICIENTS

Model $$Y = \beta_1 + \beta_2 X + u$$

Null hypothesis: $$H_0: \beta_2 = 0$$

Alternative hypothesis: $$H_1: \beta_2 \neq 0$$

Now we will consider the special, but very common, case H_0: $\beta_2 = 0$.

Model $$Y = \beta_1 + \beta_2 X + u$$

Null hypothesis: $$H_0: \beta_2 = 0$$

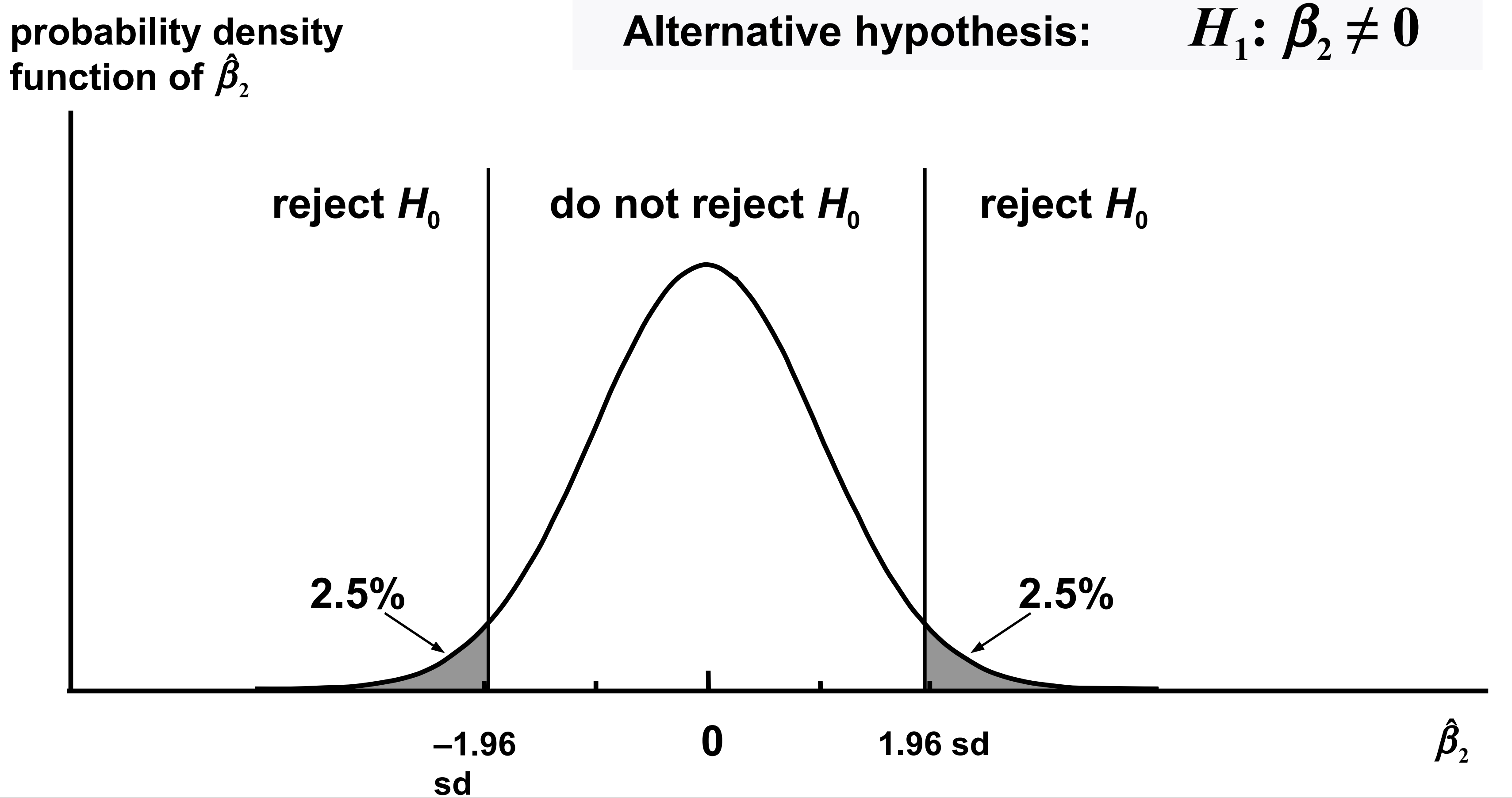
Alternative hypothesis: $$H_1: \beta_2 \neq 0$$

It occurs when you wish to demonstrate that a variable *X* influences another variable *Y*. You set up the null hypothesis that *X* has no effect (β_2 = 0) and try to reject H_0.

ONE-SIDED t TESTS OF HYPOTHESES RELATING TO REGRESSION COEFFICIENTS

Null hypothesis: $H_0: \beta_2 = 0$

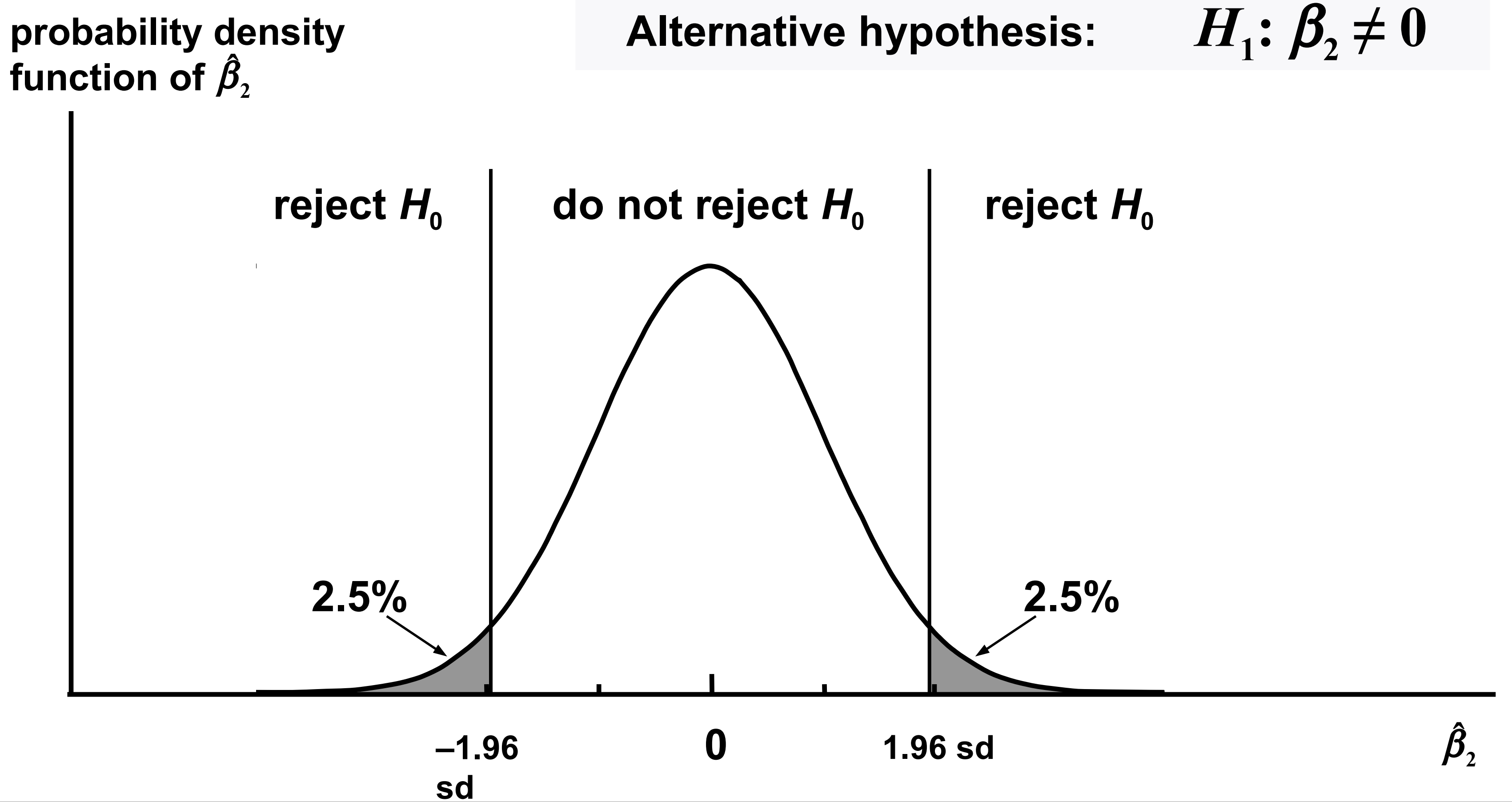
Alternative hypothesis: $H_1: \beta_2 \neq 0$



The figure shows the distribution of $\hat{\beta}_2$, conditional on H_0: $\beta_2 = 0$ being true. For simplicity, we initially assume that we know the standard deviation.

Null hypothesis: $H_0: \beta_2 = 0$

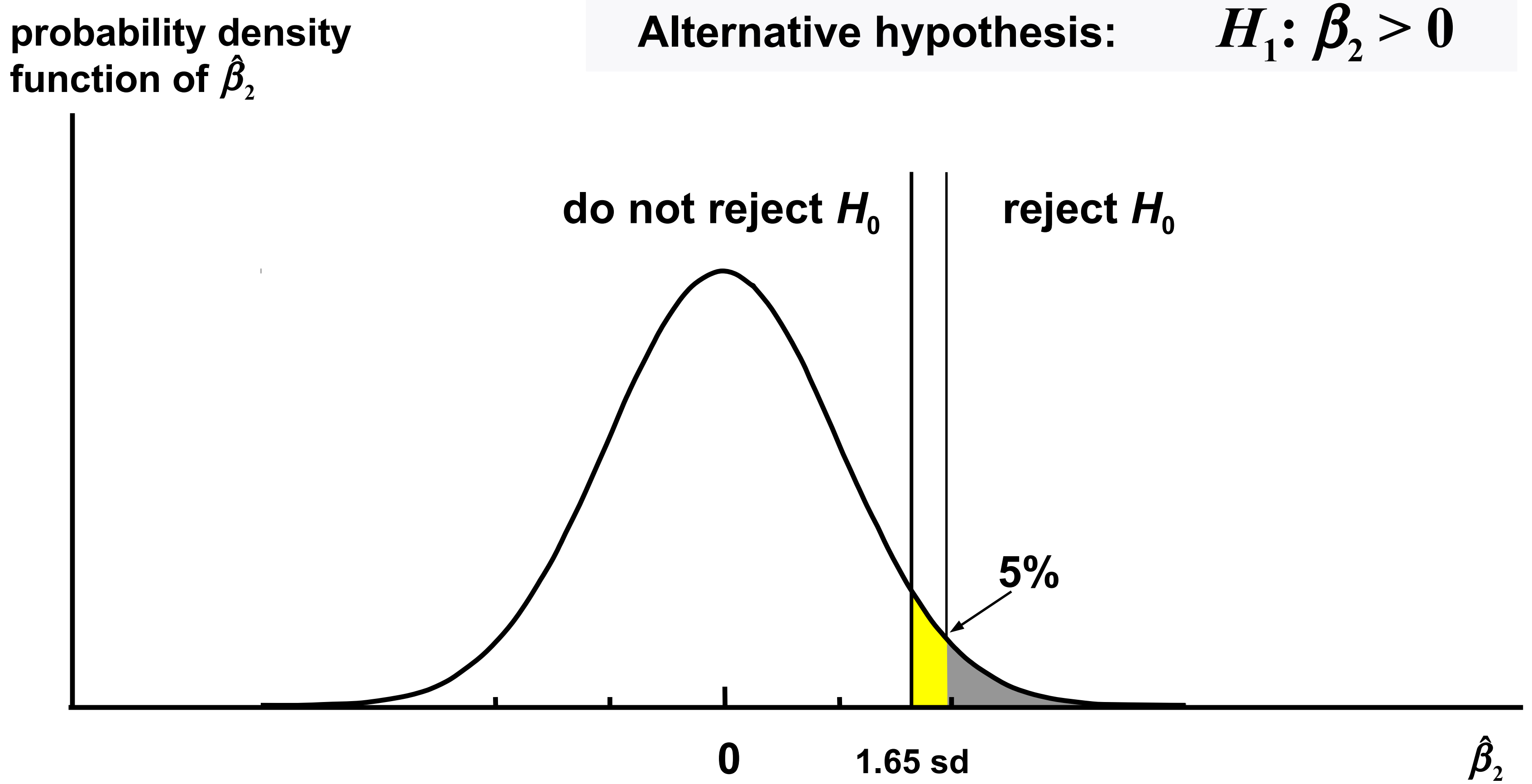
Alternative hypothesis: $H_1: \beta_2 \neq 0$



If you use a two-sided 5% significance test, your estimate must be 1.96 standard deviations above or below 0 if you are to reject H_0.

Null hypothesis: $H_0: \beta_2 = 0$

Alternative hypothesis: $H_1: \beta_2 > 0$

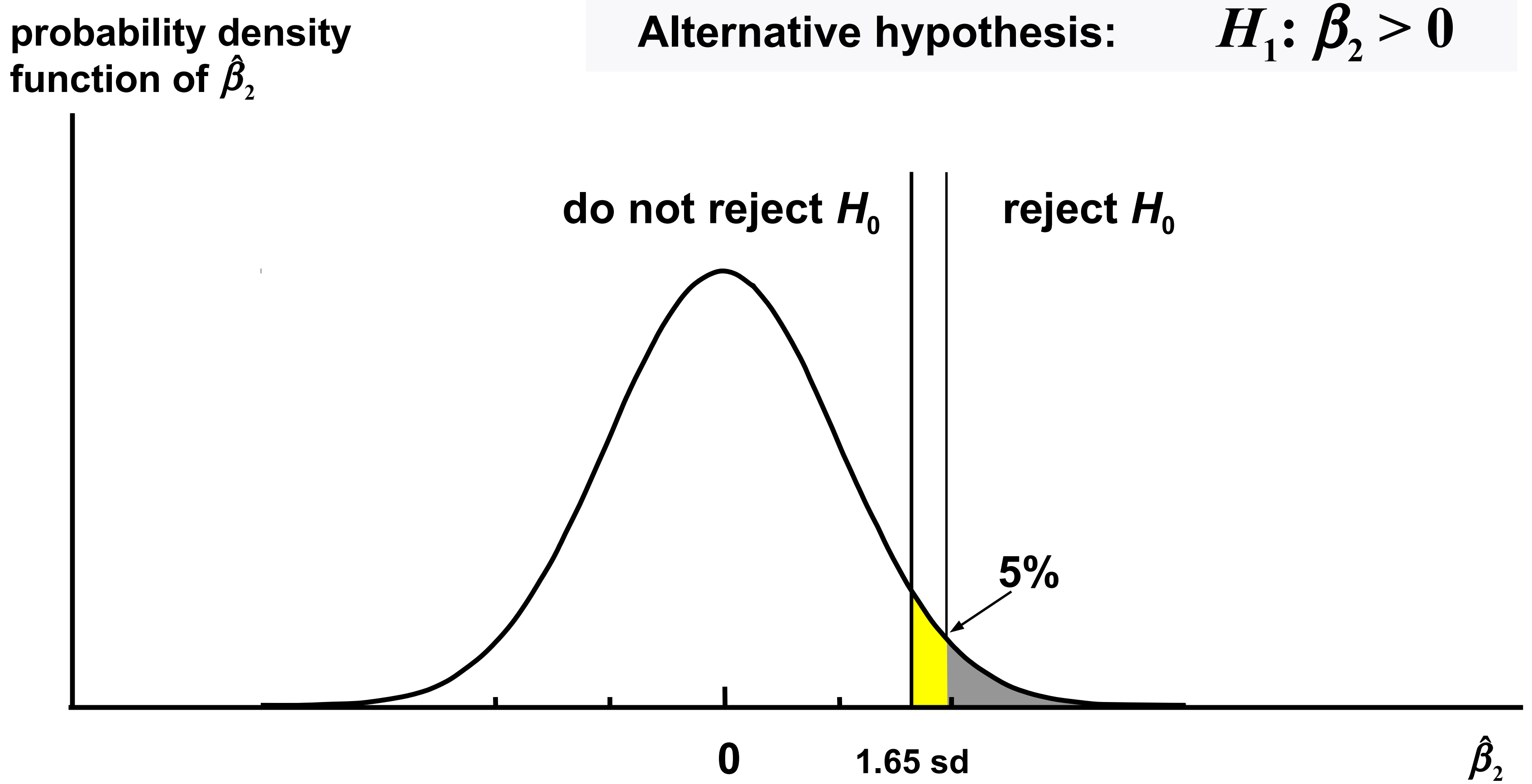


However, if you can justify the use of a one-sided test, for example with H_0: $\beta_2 > 0$, your estimate has to be only 1.65 standard deviations above 0.

ONE-SIDED *t* TESTS OF HYPOTHESES RELATING TO REGRESSION COEFFICIENTS

Null hypothesis: $H_0: \beta_2 = 0$

Alternative hypothesis: $H_1: \beta_2 > 0$

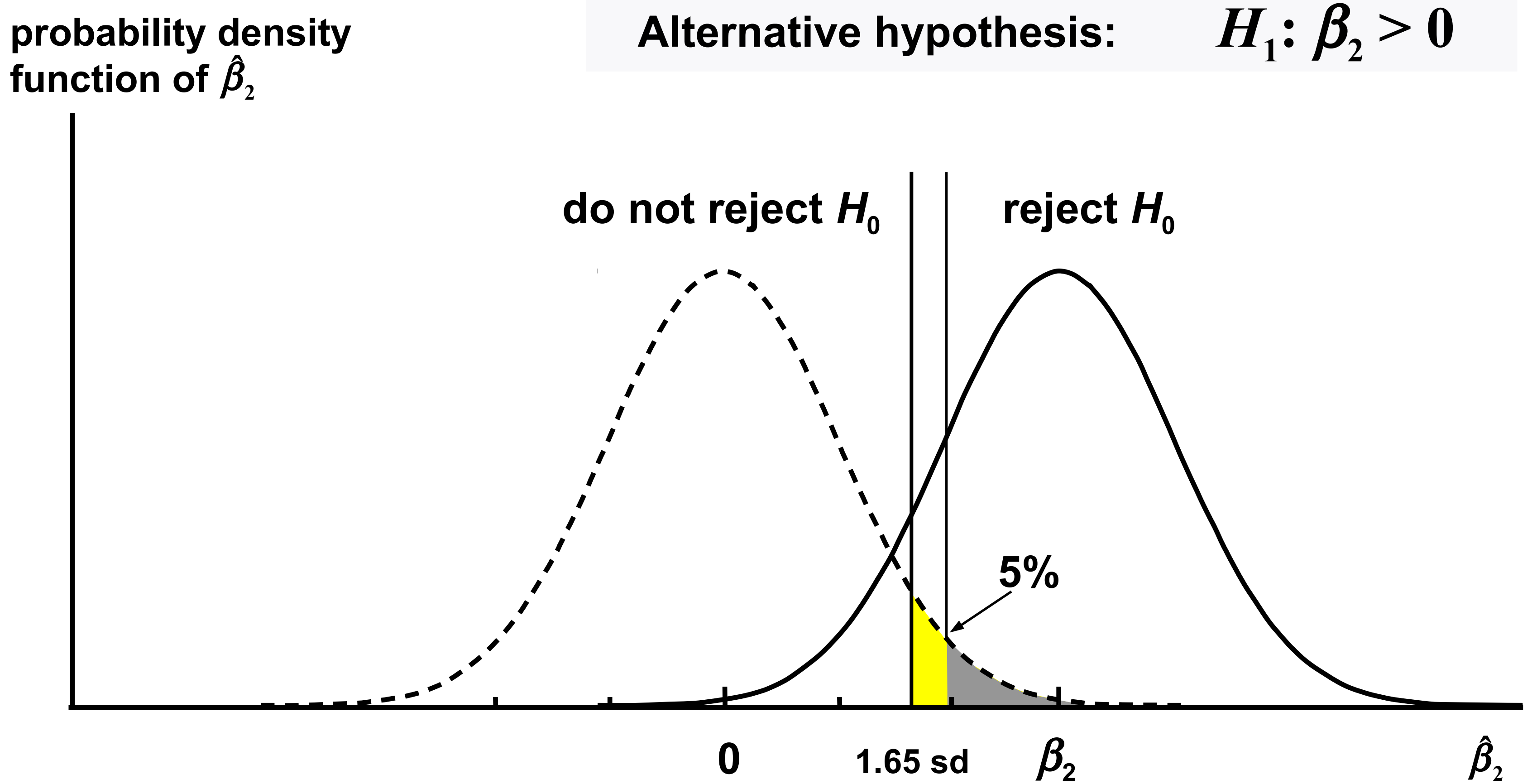


This makes it easier to reject H_0 and thereby demonstrate that Y really is influenced by X (assuming that your model is correctly specified).

ONE-SIDED t TESTS OF HYPOTHESES RELATING TO REGRESSION COEFFICIENTS

Null hypothesis: $H_0: \beta_2 = 0$

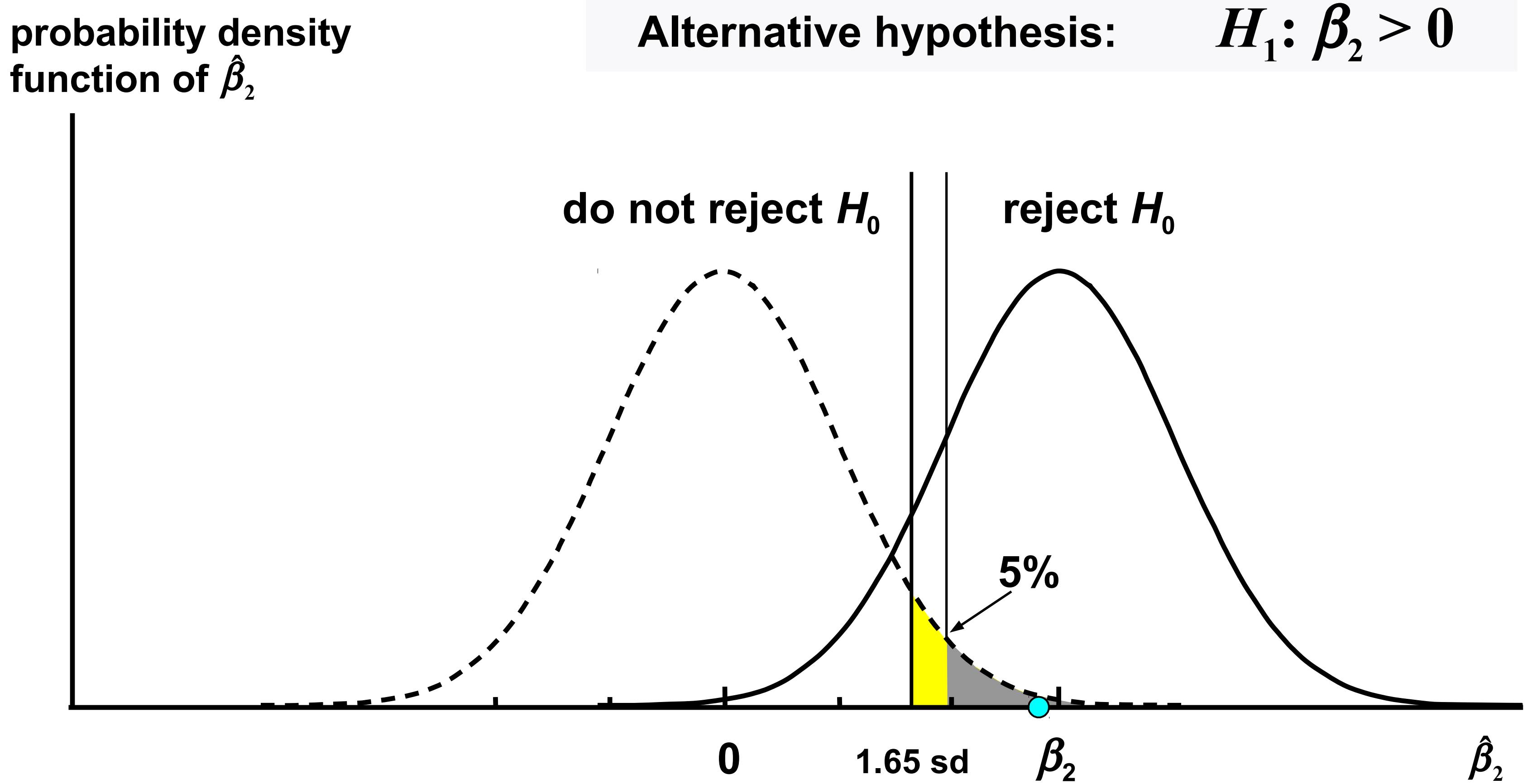
Alternative hypothesis: $H_1: \beta_2 > 0$



Suppose that Y is genuinely determined by X and that the true (unknown) coefficient is β_2, as shown.

Null hypothesis: $H_0: \beta_2 = 0$

Alternative hypothesis: $H_1: \beta_2 > 0$

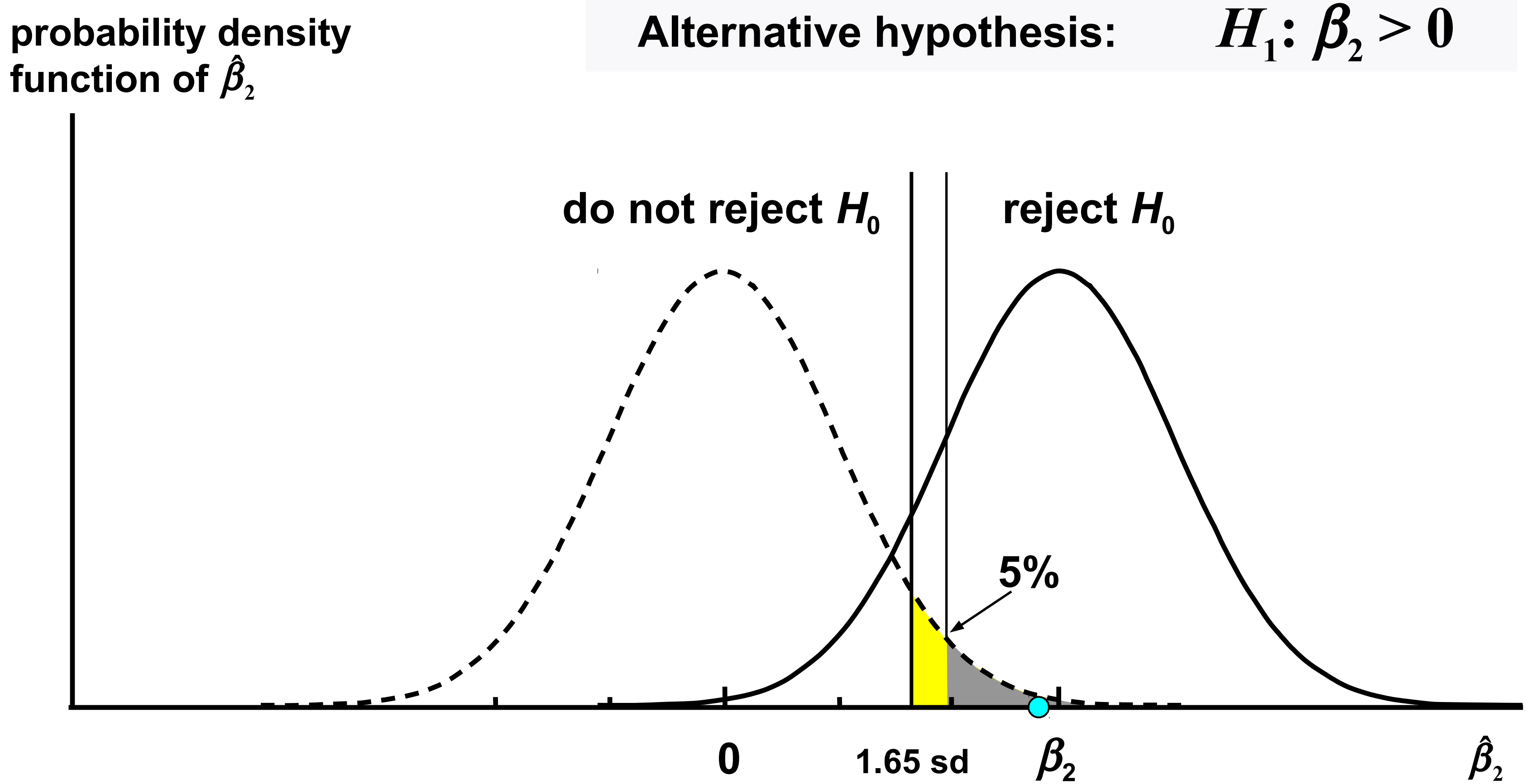


Suppose that we have a sample of observations and calculate the estimated slope coefficient, $\hat{\beta}_2$. If it is as shown in the diagram, what do we conclude when we test H_0?

ONE-SIDED t TESTS OF HYPOTHESES RELATING TO REGRESSION COEFFICIENTS

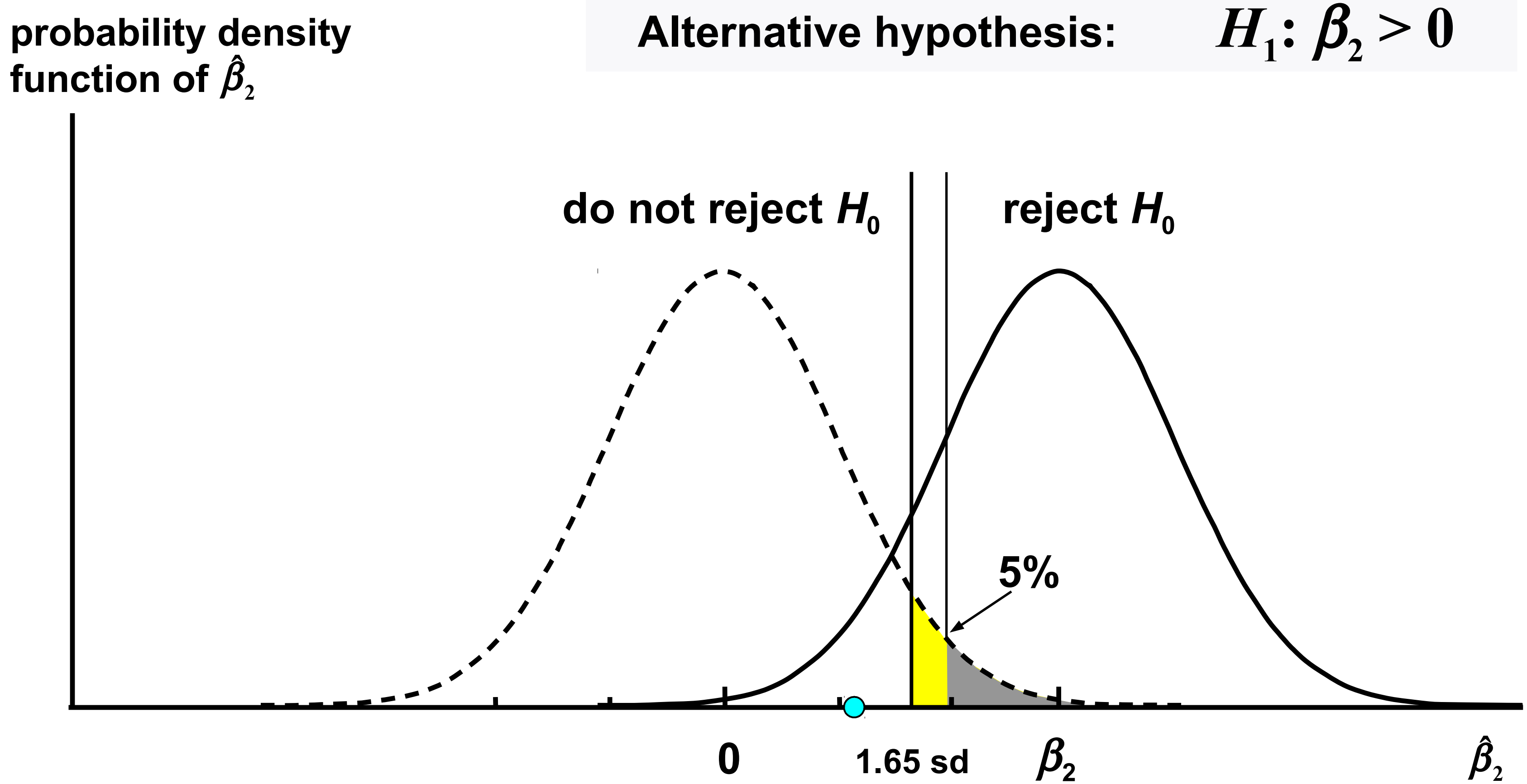
Null hypothesis: $H_0: \beta_2 = 0$

Alternative hypothesis: $H_1: \beta_2 > 0$



The answer is that $\hat{\beta}_2$ lies in the rejection region. It makes no difference whether we perform a two-sided test or a one-sided test. We come to the correct conclusion.

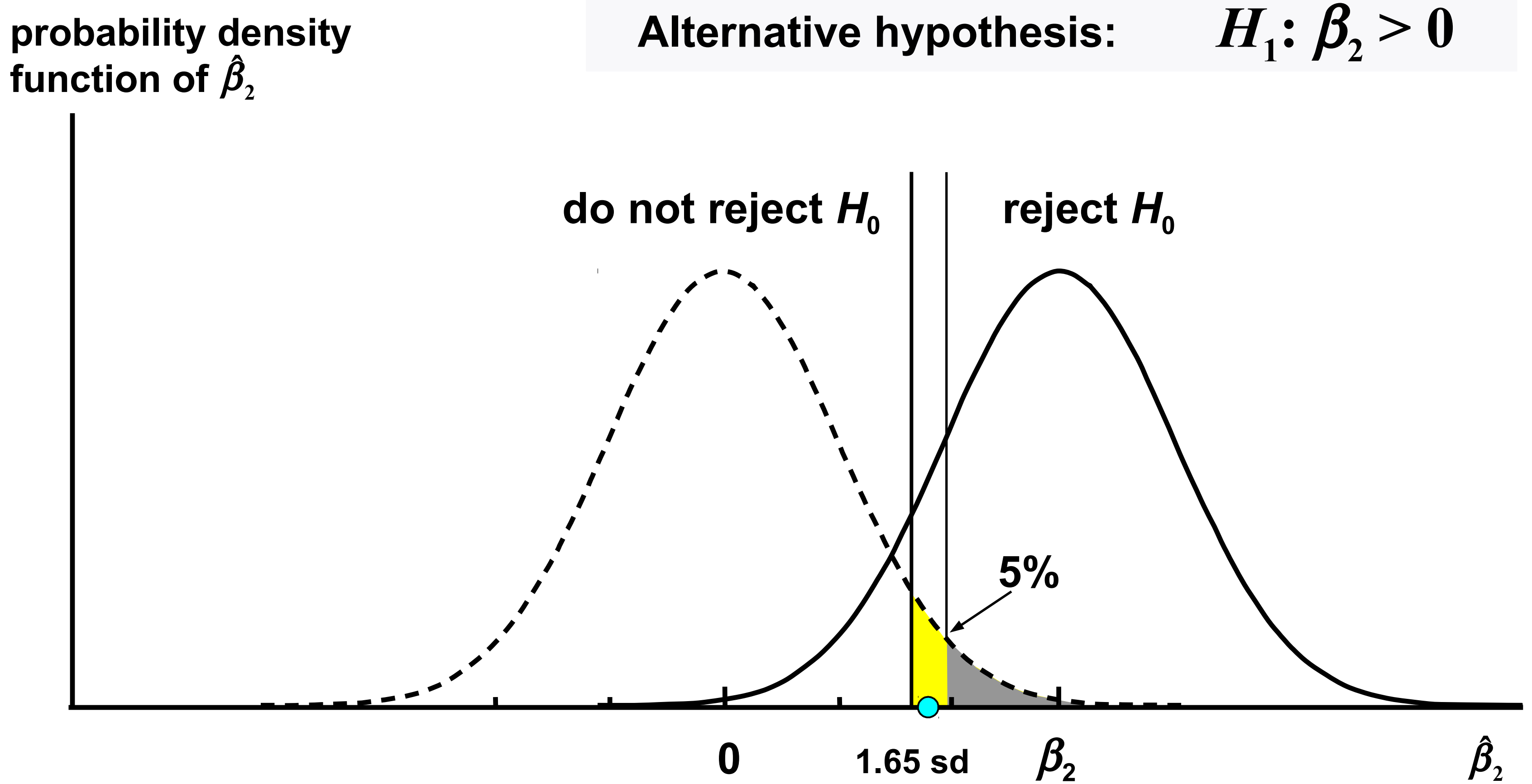
Null hypothesis:	$H_0: \beta_2 = 0$
Alternative hypothesis:	$H_1: \beta_2 > 0$



What do we conclude if $\hat{\beta}_2$ is as shown? We fail to reject H_0, irrespective of whether we perform a two-sided test or a two-sided test. We would make a Type II error in either case.

ONE-SIDED t TESTS OF HYPOTHESES RELATING TO REGRESSION COEFFICIENTS

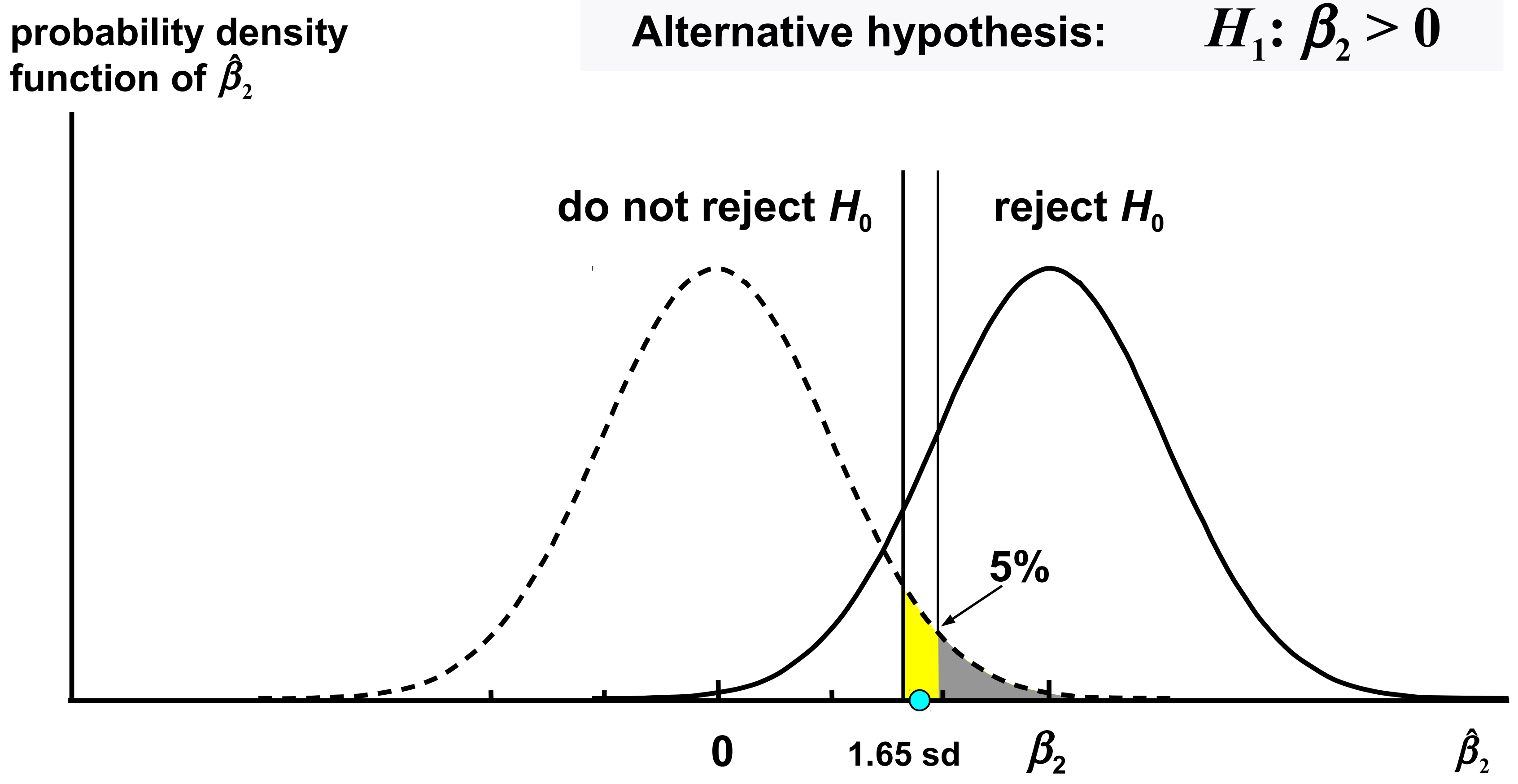
Null hypothesis:	$H_0: \beta_2 = 0$
Alternative hypothesis:	$H_1: \beta_2 > 0$



What do we conclude if $\hat{\beta}_2$ is as shown here? In the case of a two-sided test, $\hat{\beta}_2$ is not in the rejection region. We are unable to reject H_0.

Null hypothesis: $H_0: \beta_2 = 0$

Alternative hypothesis: $H_1: \beta_2 > 0$

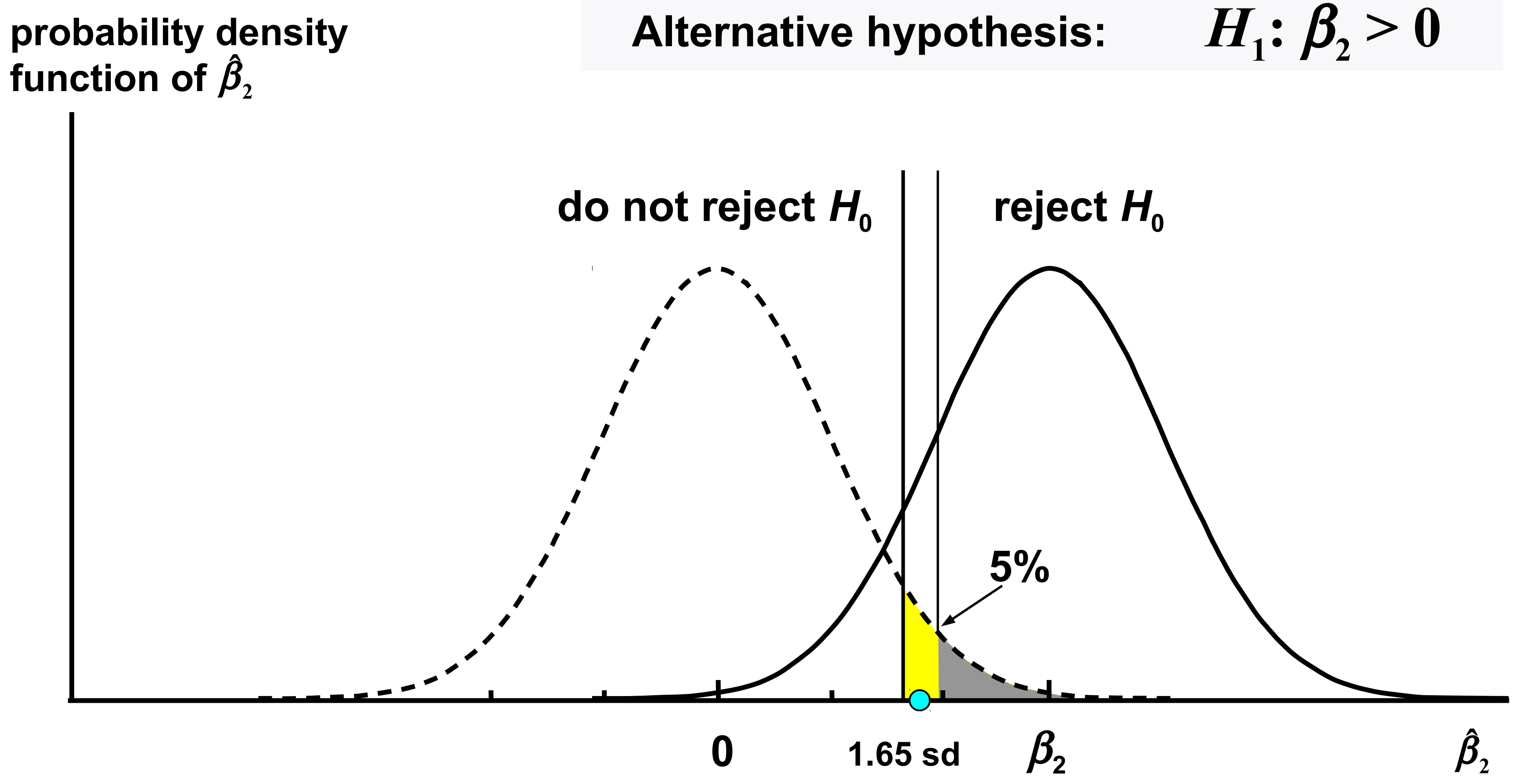


This means that we are unable to demonstrate that X has a significant effect on Y. This is disappointing, because we were hoping to demonstrate that X is a determinant of Y.

ONE-SIDED t TESTS OF HYPOTHESES RELATING TO REGRESSION COEFFICIENTS

Null hypothesis: $H_0: \beta_2 = 0$

Alternative hypothesis: $H_1: \beta_2 > 0$

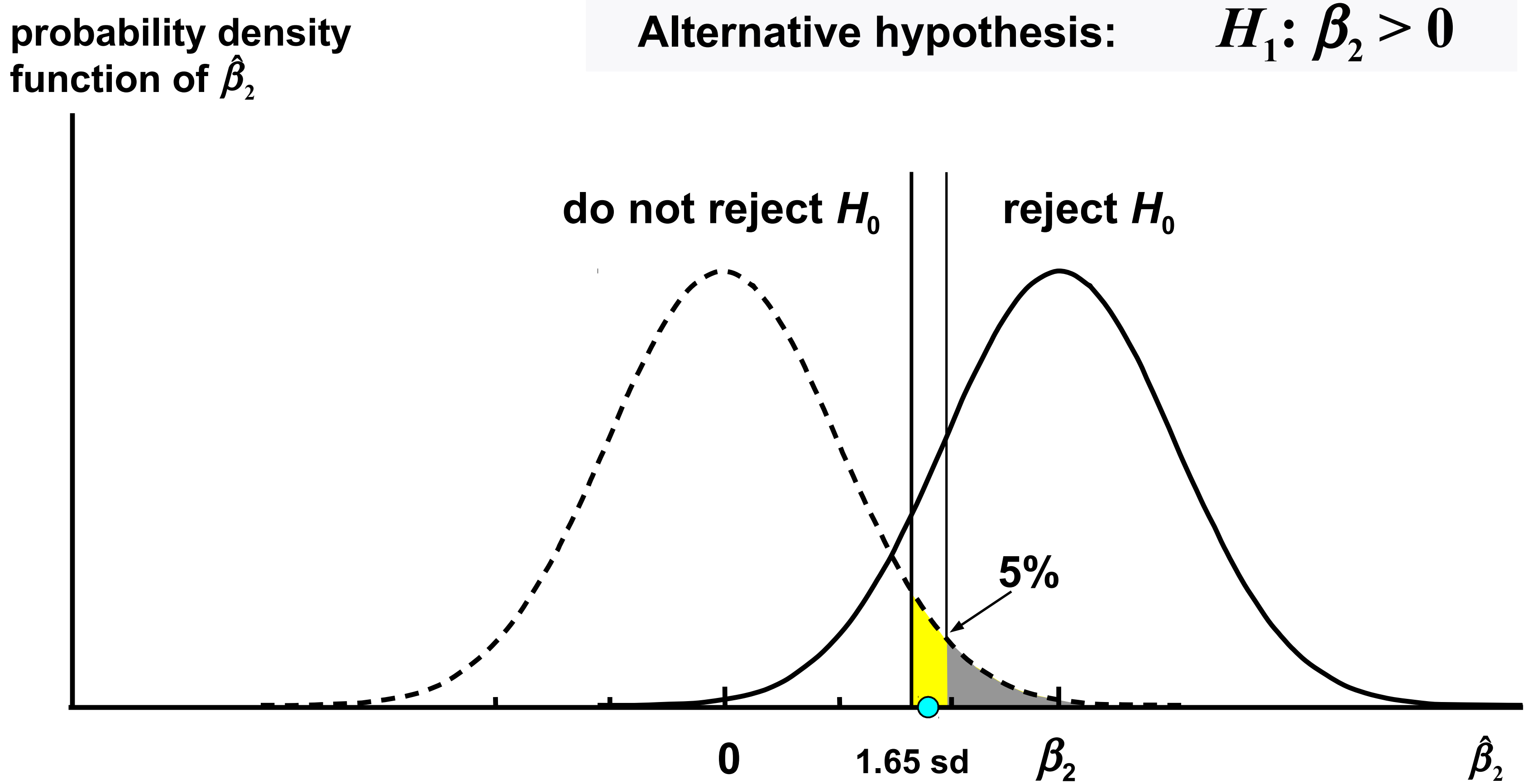


However, if we are in a position to perform a one-sided test, $\hat{\beta}_2$ does lie in the rejection region and so we have demonstrated that X has a significant effect on Y (at the 5% significance level, of course).

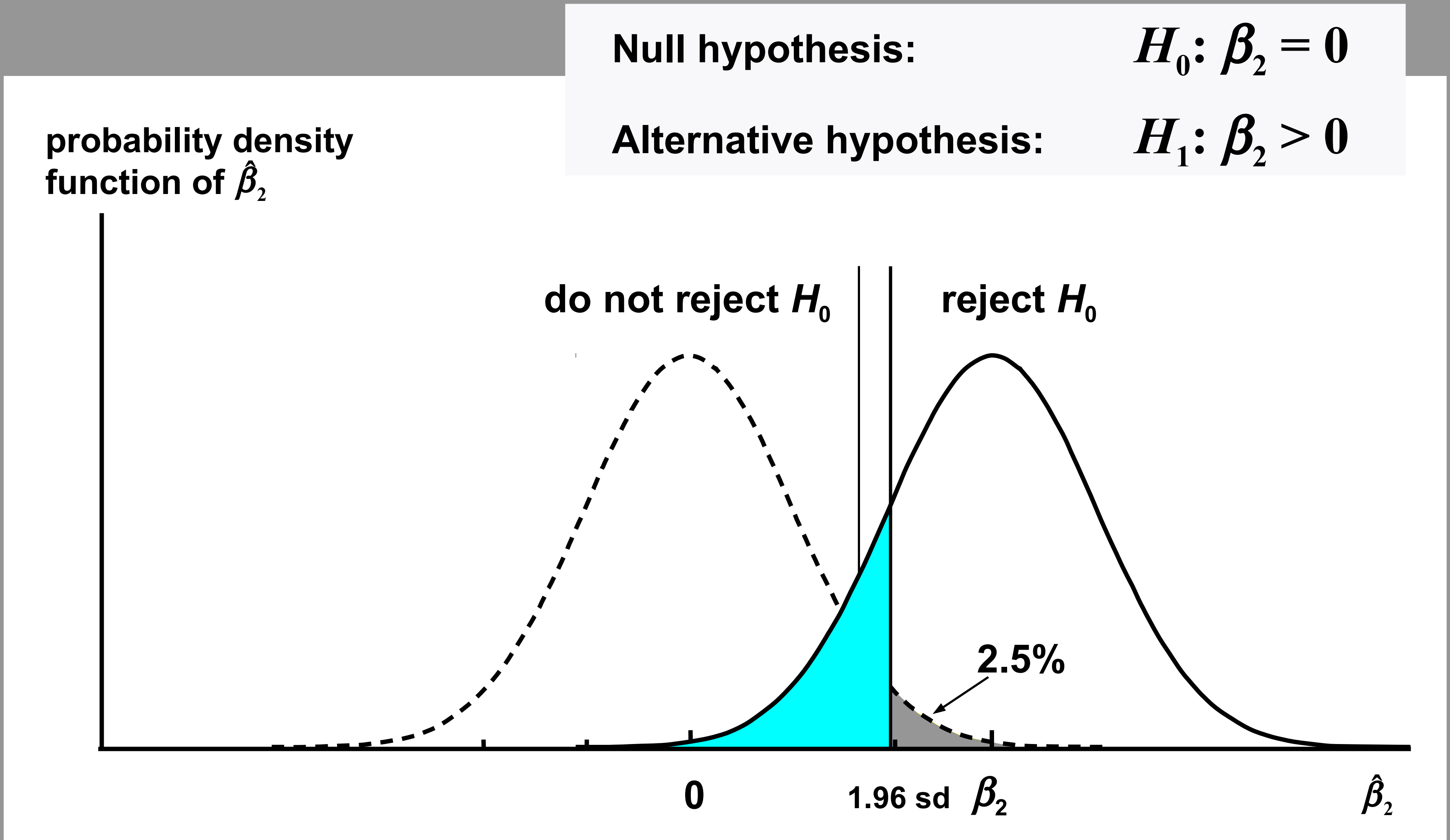
ONE-SIDED t TESTS OF HYPOTHESES RELATING TO REGRESSION COEFFICIENTS

Null hypothesis: $H_0: \beta_2 = 0$

Alternative hypothesis: $H_1: \beta_2 > 0$



Thus we get a positive finding that we could not get with a two-sided test.



To put this reasoning more formally, the power of a one-sided test is greater than that of a two-sided test. The blue area shows the probability of making a Type II error using a two-sided test. It is the area under the true curve to the left of the rejection region.

ONE-SIDED *t* TESTS OF HYPOTHESES RELATING TO REGRESSION COEFFICIENTS

Null hypothesis: $H_0: \beta_2 = 0$

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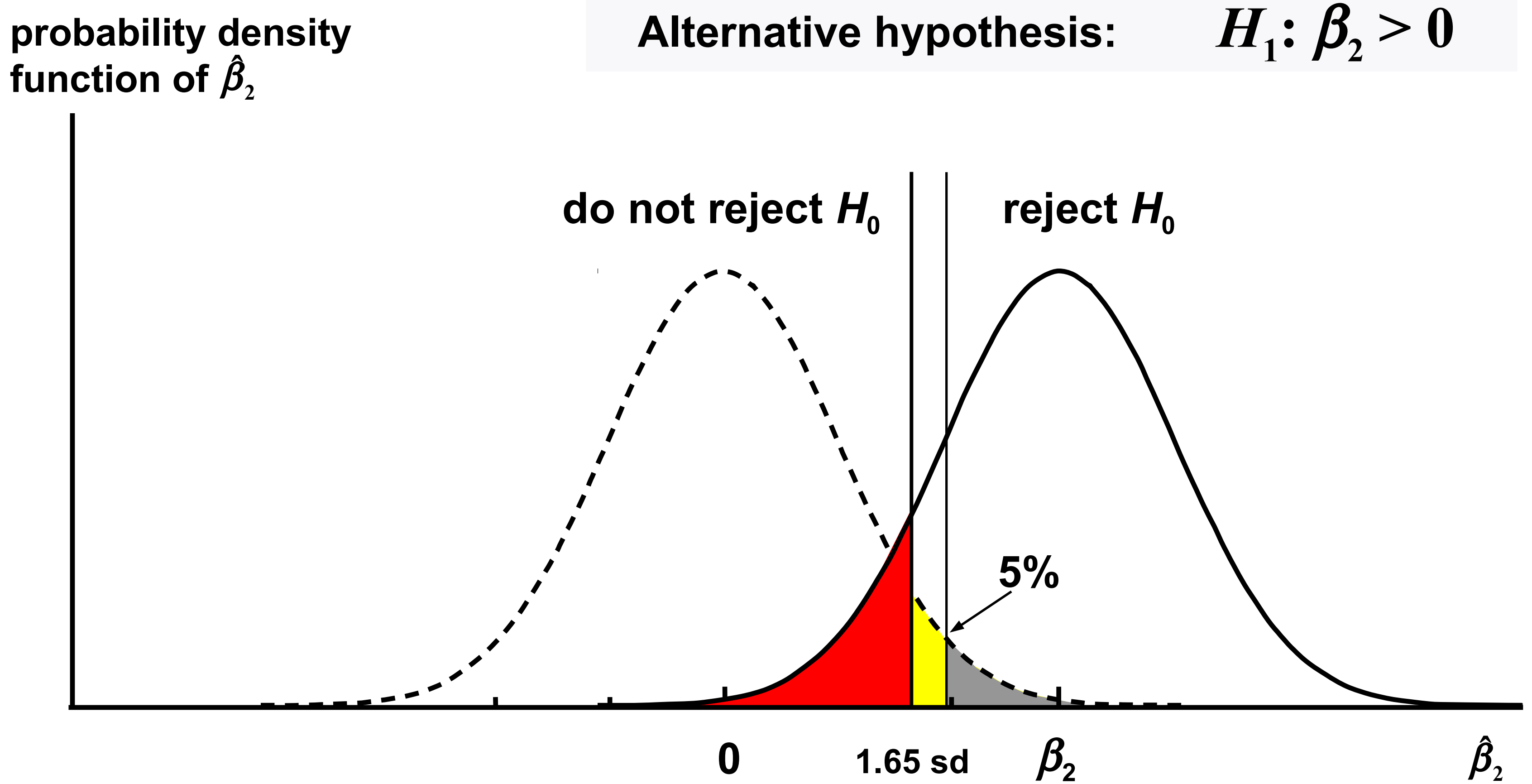
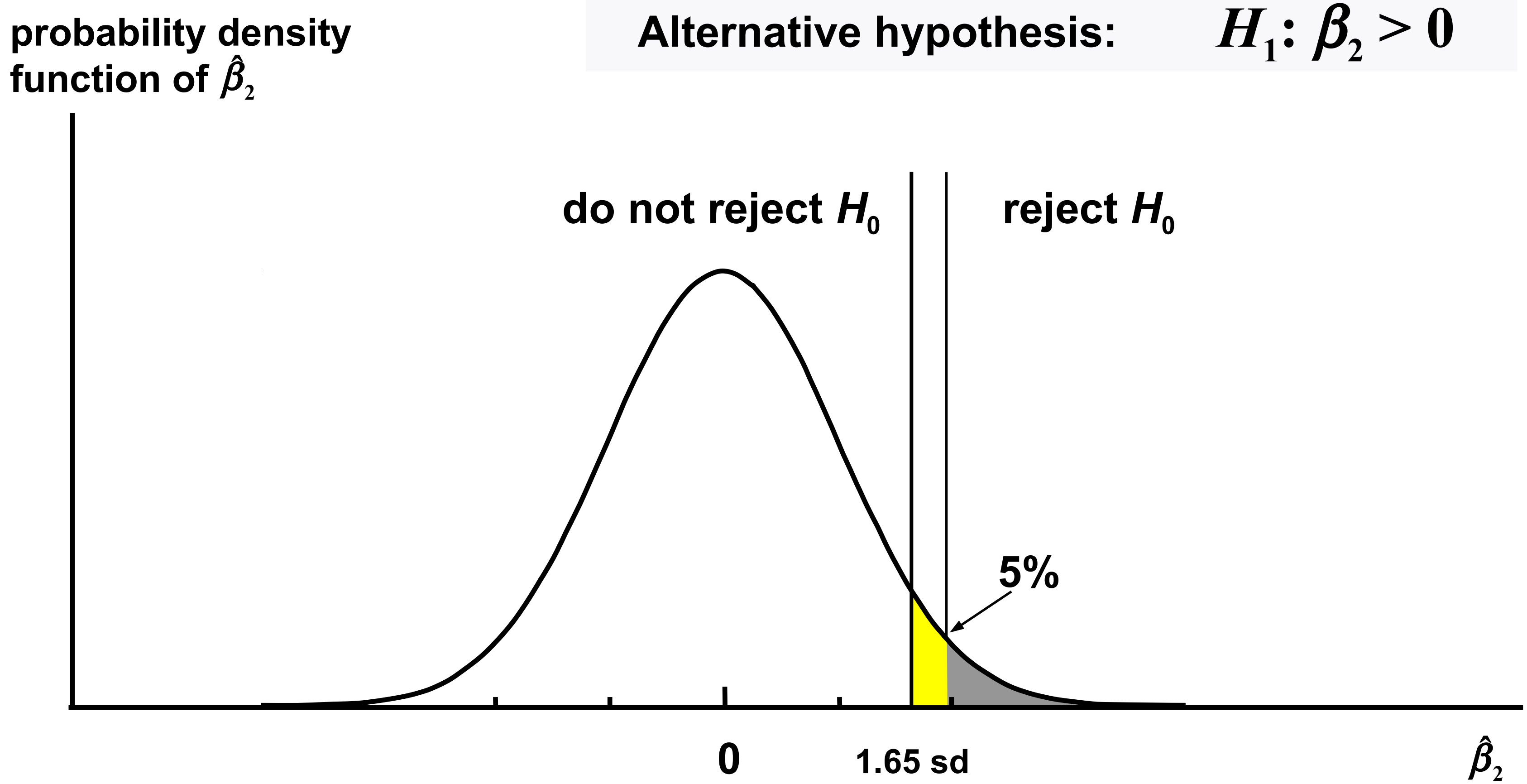


The red area shows the probability of making a Type II error using a one-sided test. It is smaller. Since the power of a test is (1 – probability of making a Type II error when H_0 is false), the power of a one-sided test is greater than that of a two-sided test.

ONE-SIDED t TESTS OF HYPOTHESES RELATING TO REGRESSION COEFFICIENTS

Null hypothesis: $H_0: \beta_2 = 0$

Alternative hypothesis: $H_1: \beta_2 > 0$

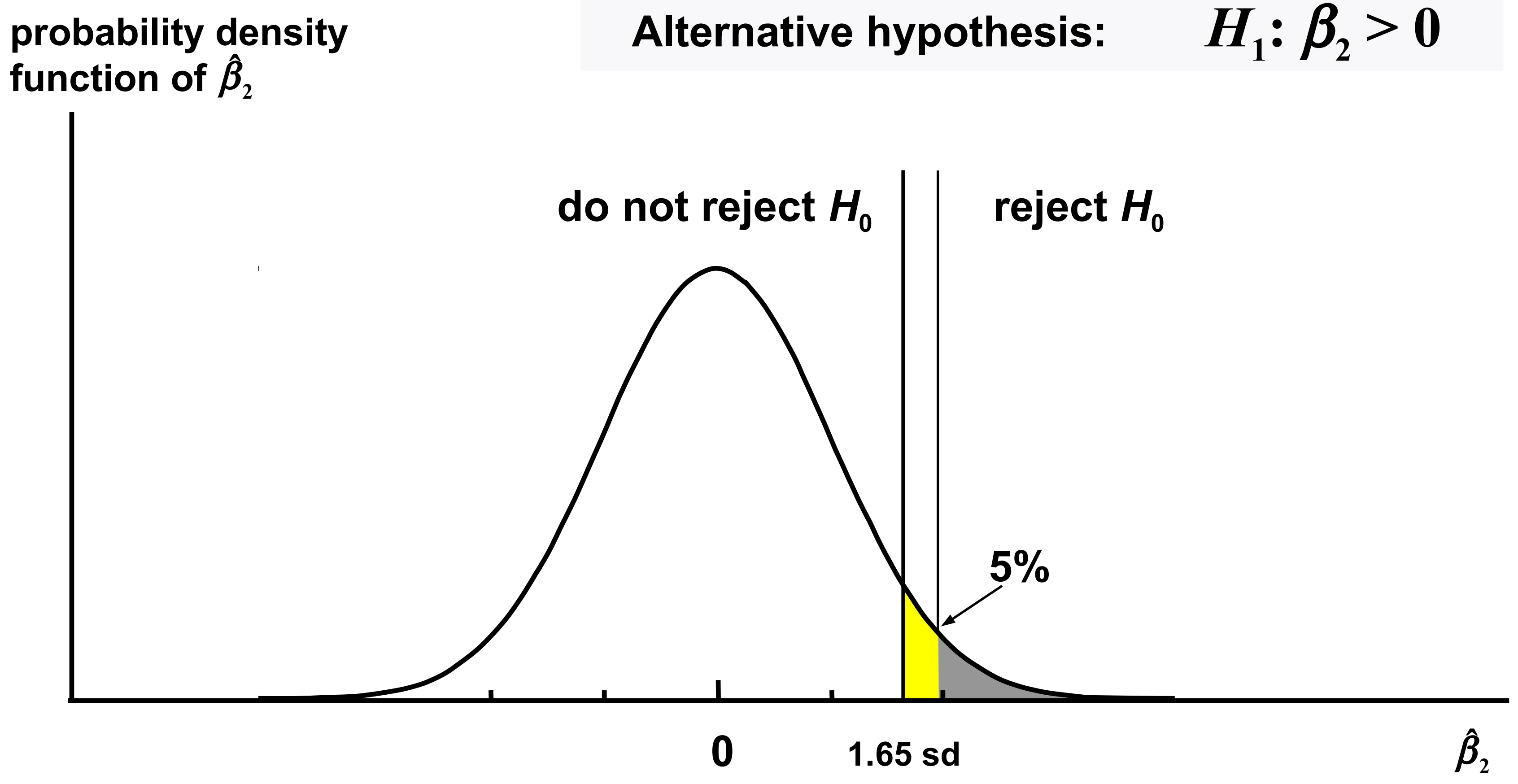


In all of this, we have assumed that we knew the standard deviation of the distribution of $\hat{\beta}_2$. In practice, of course, the standard deviation has to be estimated as the standard error, and the t distribution is the relevant distribution. However, the logic is exactly the same.

ONE-SIDED t TESTS OF HYPOTHESES RELATING TO REGRESSION COEFFICIENTS

Null hypothesis: $H_0: \beta_2 = 0$

Alternative hypothesis: $H_1: \beta_2 > 0$

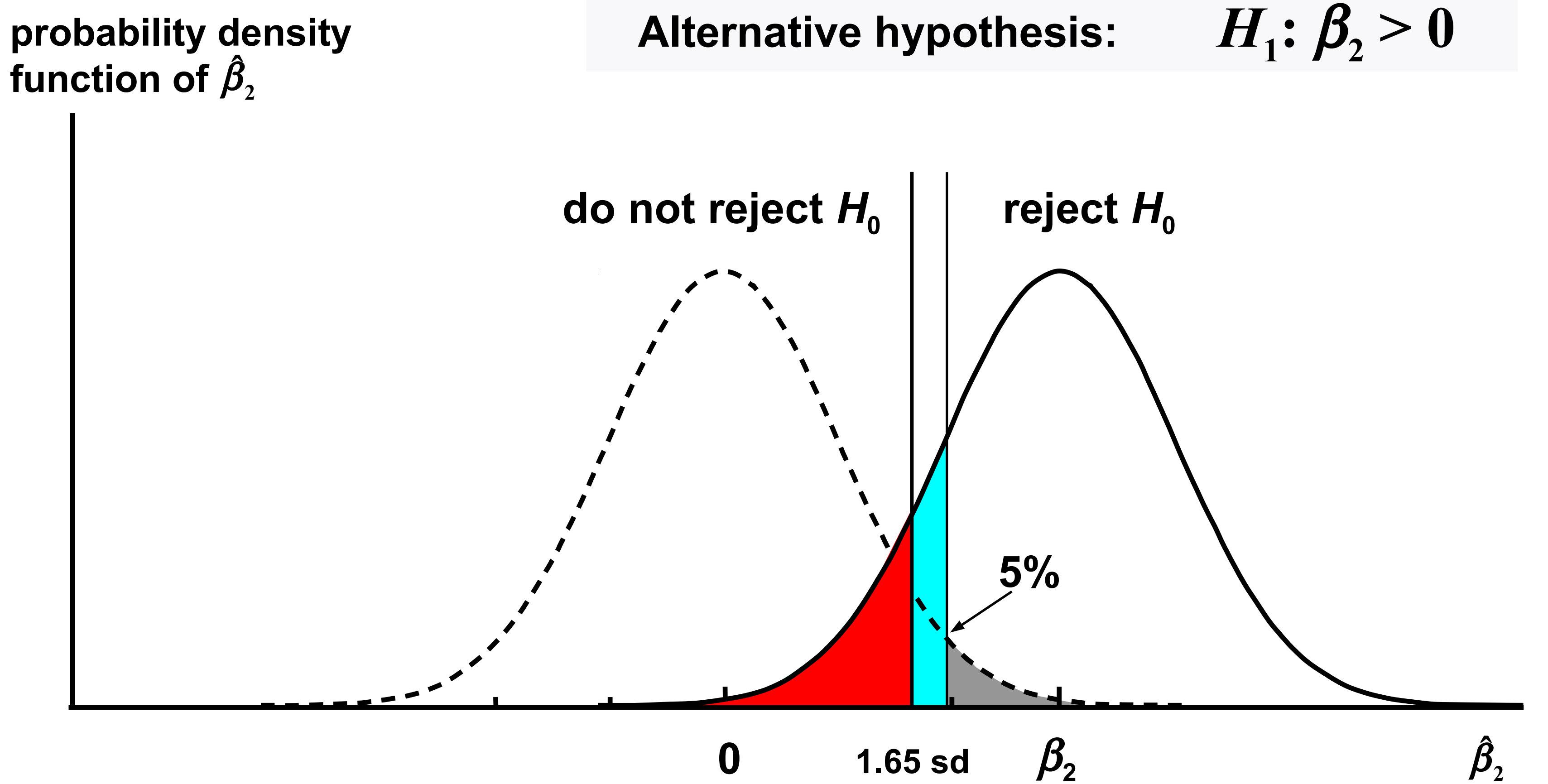


At any given significance level, the critical value of t for a one-sided test is lower than that for a two-sided test.

ONE-SIDED *t* TESTS OF HYPOTHESES RELATING TO REGRESSION COEFFICIENTS

Null hypothesis: $H_0: \beta_2 = 0$

Alternative hypothesis: $H_1: \beta_2 > 0$



Hence, if H_0 is false, the risk of not rejecting it, thereby making a Type II error, is smaller, and so the power of a one-sided test is greater than that of a two-sided test.

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EC212 Introduction to Econometrics
http://www2.lse.ac.uk/study/summerSchools/summerSchool/Home.aspx
or the University of London International Programmes distance learning course
EC2020 Elements of Econometrics
www.londoninternational.ac.uk/lse.