Prelude

Modelling Time Series

Introduction

Why volatility

modelling?

what is voiatility

Model Structure

Estimation of GARCH mode

GARCH mode

Alternative Models for Volatility

Choosing a Volatility Model

Thankyou

Modeling Market Volatility

Parthasarathi Edupally

Lec 4, Quantitative Finance, Russell Square International College, Mumbai

Dealude

Lecture 4 So Far ... Modelling Time Series

Introduction to

Why volatility

What is Volatilit

Model Structur

Estimation of

GARCH mode

MIE

Alternative Mode

Choosing a Volatility

Thankyou

Outline

Prelude

Lecture 4 So Far ... Modelling Time Series data

Outline

Prelude

Lecture 4 So Far ... Modelling Time Series data

Introduction t Volatility

modelling? What is Volatility

Estimation of GARCH mode

MLE MLE

Choosing a Volatility

Thankyou

Prelude

Lecture 4 So Far ... Modelling Time Series data

2 Introduction to Volatility

Why volatility modelling?
What is Volatility?
Model Structure

Outline

Prelude

Prelude

Lecture 4 So Far ... Modelling Time Series data

2 Introduction to Volatility

Why volatility modelling? What is Volatility?

Model Structure

3 Estimation of GARCH model

MLE Alternative Models for Volatility Choosing a Volatility Model

data
Introduction to

Why volatility modelling? What is Volatility

Estimation of GARCH mode

Alternative Models for Volatility

Choosing a Volatility Model

Outline

Prelude

Prelude

Lecture 4 So Far ... Modelling Time Series data

2 Introduction to Volatility

Why volatility modelling? What is Volatility?

Model Structure

3 Estimation of GARCH model

MLE Alternative Models for Volatility Choosing a Volatility Model

4 Thankyou

data

Introduction to

Why volatility modelling? What is Volatility

Estimation of GARCH mode

Alternative Models for Volatility

Choosing a Volatility

Choosing a Volatility Model

Prelude

Lecture 4 So Far ...

Modelling Time Series data

Volatility Volatility

modelling ? What is Volatility

Estimation of

GARCH mode

Alternative Models for Volatility

Choosing a Volatility Model

Thankyou

Lecture 4 So Far ...

- We saw historical progression of ideas around efficient market hypothesis
- Understood how these theories can be translated into empirical studies
 - In particular, how law of iterated expectations can be used to formalise EMH
 - How RWH is a variant of EMH
- Finally, we saw how current research is focused more on comparing efficiencies of markets, than to test for efficiency itself of a particular market

Prelude

Lecture 4 So Far ...

Modelling Time Serie data

Introduction to Volatility

modelling?
What is Volatility
Model Structure

Estimation of GARCH mode

GARCH mode

Alternative Models for Volatility

Choosing a Volatility Model

Thankyou

Lecture 4 So Far ...

- We saw historical progression of ideas around efficient market hypothesis
- Understood how these theories can be translated into empirical studies
 - In particular, how law of iterated expectations can be used to formalise EMH
 - How RWH is a variant of EMH
- Finally, we saw how current research is focused more on comparing efficiencies of markets, than to test for efficiency itself of a particular market
- In part B of this lecture, we will try to understand modelling volatility of time series data

Prelude

Lecture 4 So Far ...

Modelling Time Series

data

Introduction to Volatility

Why volatilit modelling?

What is Volatility

Estimation o

GARCH mode

MLE Alternative Models (

Volatility Choosing a Volatility

Modelling Time Series Data

- We have two parallel ways of looking at time series data
 - Mathematically, that its just a realization of a stochastic process
 - Intutively, that there is certain degree of persistence in properties of time series

Prelude

Modelling Time Series data

Introduction to Volatility

modelling?
What is Volatility

Estimation of GARCH mode

Alternative Models for Volatility Choosing a Volatility

Thankyou

Modelling Time Series Data

- We have two parallel ways of looking at time series data
 - Mathematically, that its just a realization of a stochastic process
 - Intutively, that there is certain degree of persistence in properties of time series
- Ultimate goal of modelling has been to say something about X_t , random variable at any point t
 - we could talk about its mean, conditional on the information till t (conditional expectation, as in ARMA models)
 - Nobody stops us from talking about vairance, kurtosis and other moments of a random variable at t, given the information till that point
- In this part of the lecture, we will try to understand modelling variance (conditional on the information till t)

Proludo

Modelling Time Series data

Volatility
Why volatility

modelling?

What is Volatility

Model Structure

Estimation of GARCH mode

Alternative Models for Volatility Choosing a Volatility

Thankyou

Why volatility modelling?

- Variance, in particular, has a special meaning
 - in risk management, pricing models etc
 - also there is an intutive understanding about how it behaves in different ranges of values of time series data
- Risk of a financial asset can be quantified by its variance, thus volatility plays an important role in Risk Mangement
- In pricing models of financial instruments, which are based on principles of risk return tradeoff also have volatility as one of the inputs in determining prices
- As we are being very explicit about variance of random variable X_t at any point t, our model for time series data which considers only conditional expectation so far, is more precise and efficient now

Droludo

Lecture 4 So Far ... Modelling Time Series data

Volatility

What is Volatility ?

Model Structure

Estimation of GARCH mod

Alternative Models for Volatility Choosing a Volatility

Thankyou

What is Volatility?

- Volatility can be defined as conditional variance of a time series of asset returns
- For a given return series, r_t , we can define its conditional mean and variance as follows

conditional mean:
$$\mu_t = E(r_t|F_{t-1})$$

conditional variance:
$$\sigma_t^2 = Var(r_t|F_{t-1}) = E[(r_t - \mu_t)^2|F_{t-1}]$$

- Some of the important features of volatility are:
 - it is not directly observable
 - we have volatility clusters
 - evolves continuously, there are no sudden jumps
 - doesnt diverge to infinity
 - it reacts differently to big positive and negative changes in return series

Prelude

1101440

Modelling Time Serie

Volatility

Why volatilit

What is Volatility

Trinicis Tollining

Model Structure

Estimation of GARCH model

....

MLE

Volatility

Choosing a Volatility Model

Thankyou

Model Structure

• Basic idea behind volatility models is that r_t series is uncorrelated or has minor lower order serial correlation, but is dependent

Prelude

Lecture 4 So Far ... Modelling Time Series data

Volatility Volatility

Why volatility modelling?

Model Structure

Model Structure

Estimation of GARCH mode

Alternative Models for Volatility

Choosing a Volatility Model

Thankyou

Model Structure

- Basic idea behind volatility models is that r_t series is uncorrelated or has minor lower order serial correlation, but is dependent
- This behaviour can be understood by observing ACF and PACF of r_t , $|r_t|$ and r_t^2
- We see that there is some kind of persistence in squared residual series, which represents variance

Proludo

Lecture 4 So Far ... Modelling Time Series data

Introduction to Volatility

What is Volatility

Model Structure

Estimation of GARCH mod

Alternative Models for Volatility Choosing a Volatility

Thankyou

Model Structure

- Basic idea behind volatility models is that r_t series is uncorrelated or has minor lower order serial correlation, but is dependent
- This behaviour can be understood by observing ACF and PACF of r_t , $|r_t|$ and r_t^2
- We see that there is some kind of persistence in squared residual series, which represents variance
- For a given time series $\{r_t\}$, assuming ARMA we have

$$r_t = \mu_t + a_t \tag{1}$$

$$\mu_t = \phi_0 + \sum_{i=0}^p \phi_i r_{t-i} - \sum_{i=0}^q \theta_i a_{t-i}$$
 (2)

$$\sigma_t^2 = Var(r_t|\mathcal{F}_{t-1}) = Var(a_t|\mathcal{F}_{t-1})$$
 (3)

Prelude

Lecture 4 So Far ... Modelling Time Series data

Introduction Volatility

Volatility

modelling?

What is Volatility

Model Structure

GARCH mode

MIE

Alternative Models

Choosing a Volatili

Thankyou

Model Structure

• To specify ARCH model, we extend the above model with variance equation below,

$$a_t = \sigma_t \epsilon_t, \sigma_t^2 = \alpha_0 + \alpha_1 a_{t-1}^2 + \dots + \alpha_m a_{t-m}^2$$
 (4)

Danbada

Lecture 4 So Far ... Modelling Time Ser

Introduction to

Volatility
Why volatili

modelling ? What is Volatility

Model Structure

GARCH mode

Alternative Models for Volatility Choosing a Volatility

Choosing a Volatility Model

Thonlaron

Model Structure

• To specify ARCH model, we extend the above model with variance equation below,

$$a_t = \sigma_t \epsilon_t, \sigma_t^2 = \alpha_0 + \alpha_1 a_{t-1}^2 + \dots + \alpha_m a_{t-m}^2$$
 (4)

• From the structure of the model, it is seen that large past squared shocks $\{a_{t-i}\}_{i=1}^m$ imply a large conditional variance σ_t for the mean-corrected return a_t

Lecture 4 So Far ... Modelling Time Series

Introduction to

Why volatili modelling?

What is Volati

Model Structure

Estimation of

GARCH mode

Alternative Models for Volatility Choosing a Volatility

Thankwai

Model Structure

 To specify ARCH model, we extend the above model with variance equation below,

$$a_t = \sigma_t \epsilon_t, \sigma_t^2 = \alpha_0 + \alpha_1 a_{t-1}^2 + \dots + \alpha_m a_{t-m}^2$$
 (4)

- From the structure of the model, it is seen that large past squared shocks $\{a_{t-i}\}_{i=1}^m$ imply a large conditional variance σ_t for the mean-corrected return a_t
- Similarly, we can specify GARCH model for volatility

Please refer section 3.1 and 3.2 from Tsay for more details

MLE

Prende

ecture 4 So Far ... Iodelling Time Series

Introduction Volatility

Why volatility

What is Volatility

Model Structure

GARCH me

MLE

Alternative Models fo Volatility

Choosing a Volatility Model

Thankyou

Please use section 8.6 from Study Guide

Prelude

Lecture 4 So Far ... Modelling Time Series

Introduction Volatility

Volatility

What is Volatilit

Model Structure

Estimation of GARCH mod

MLE
Alternative Models for

Volatility Choosing a Volatility

Alternative Models for Volatility

 GJR-GARCH - models with leverage effect to account for asymmetry in volatility, for positive and negative regions of returns

$$\sigma_{t+1}^2 = \omega + \beta \sigma_t^2 + \alpha \epsilon_t^2 + \delta \epsilon_t^2 \mathbf{1} \{ \epsilon_t < 0 \}$$
 (5)

Prelude

Lecture 4 So Far ... Modelling Time Series data

Introduction t Volatility

modelling ? What is Volatility

Estimation of GARCH mode

MLE
Alternative Models for

Volatility
Choosing a Volatility

Model Model

Thankyo

Alternative Models for Volatility

 GJR-GARCH - models with leverage effect to account for asymmetry in volatility, for positive and negative regions of returns

$$\sigma_{t+1}^2 = \omega + \beta \sigma_t^2 + \alpha \epsilon_t^2 + \delta \epsilon_t^2 \mathbf{1} \{ \epsilon_t < 0 \}$$
 (5)

 ARCH-in-mean - to account for economic rational of higher the risk more will be the return

Prelude

Lecture 4 So Far ... Modelling Time Serie data

Why volatility

modelling?
What is Volatility?
Model Structure

GARCH mode

MLE Alternative Models for Volatility

Choosing a Volatilit Model

Thankyo

Alternative Models for Volatility

 GJR-GARCH - models with leverage effect to account for asymmetry in volatility, for positive and negative regions of returns

$$\sigma_{t+1}^2 = \omega + \beta \sigma_t^2 + \alpha \epsilon_t^2 + \delta \epsilon_t^2 \mathbf{1} \{ \epsilon_t < 0 \}$$
 (5)

- ARCH-in-mean to account for economic rational of higher the risk more will be the return
- And others like IGARCH, PARCH etc to account different novel features of financial time series

Prelude

Lecture 4 So Far ... Modelling Time Se

Introduction t

Volatility

Why volatility

modelling?

What is Volatility

Model Structure

Estimation of GARCH model

GARCH mode

MLE

Volatility

Choosing a Volatility

Model

Thankyou

Choosing a Volatility Model

 As we saw in models for expectation, volatility models also have to deal with balancing data fitting and generalisation

Prelude

Modelling Time Series data

Introduction to Volatility

Why volatility modelling? What is Volatility?

Estimation of

GARCH mode

Alternative Models fo Volatility

Choosing a Volatility Model

Thankyou

Choosing a Volatility Model

- As we saw in models for expectation, volatility models also have to deal with balancing data fitting and generallisation
- Accordingly, we have MSE, QLIKE as measures of data fitting
- AIC, BIC and HQIC as measuring both data fitting and parsimony (generalisation)

Prelude

Modelling Time Series data

Introduction to Volatility

modelling?
What is Volatility
Model Structure

Estimation of GARCH model

Alternative Models for Volatility

Choosing a Volatility Model

Thankyou

Choosing a Volatility Model

- As we saw in models for expectation, volatility models also have to deal with balancing data fitting and generallisation
- Accordingly, we have MSE, QLIKE as measures of data fitting
- AIC, BIC and HQIC as measuring both data fitting and parsimony (generalisation)
- As the financial time series dont exist in vaccum but have a context - economic decision making
- we can incorporate economic criteria also in choosing a model

Proludo

Modelling Time Series data

Introduction to Volatility

modelling ?
What is Volatility
Model Structure

Estimation of GARCH mode

Alternative Models fo Volatility

Choosing a Volatility Model

Thankyou

Choosing a Volatility Model

- As we saw in models for expectation, volatility models also have to deal with balancing data fitting and generallisation
- Accordingly, we have MSE, QLIKE as measures of data fitting
- AIC, BIC and HQIC as measuring both data fitting and parsimony (generalisation)
- As the financial time series dont exist in vaccum but have a context - economic decision making
- we can incorporate economic criteria also in choosing a model
- please refer study guide 9.3 for more details

Prelude

Lecture 4 So Far ... Modelling Time Series

Introduction to Volatility

Why volatility modelling?

What is Volatility

GARCH mode

MLE

Choosing a Volatility

Thankyou

"In God we trust, all others bring data."

William Edwards Deming (1900 - 1993).