Empirical Features and Predictability of Time Series

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Outline

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- In this lecture,
  - we will look into empirical features, the ones coming from sample
  - stylized facts of statistical properties specific to financial domain
  - Also we will see how to check if a time series is predictable at all

Returns vs Prices in Finance

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- Please refer section 1.1 in Tsay book for more detailed explaination of the return calculations

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- A step before that would be to talk about unconditional moments: $E(X)$, $Var(X)$
- Given a return series $\{R_t\}$, we can define following unconditional moments,
  - Mean: $\hat{\mu} = \frac{1}{T}\sum_{t=1}^{T} R_t$
  - Variance : $\hat{\sigma} = \sqrt{\frac{1}{T}\sum_{t=1}^{T}(R_t - \hat{\mu})^2}$
  - Skewness: $\hat{S} = \frac{1}{\hat{\sigma}^3}\frac{1}{T}\sum(R_t - \hat{\mu})^3$
  - Kutosis: $\hat{K} = \frac{1}{\hat{\sigma}^4}\frac{1}{T}\sum(R_t - \hat{\mu})^4$
  - Quantile: For a given α, $\hat{Q}_\alpha = R_{(\lceil \alpha T \rceil)}$

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- The skewness is not a serious problem for both daily and monthly returns
- The descriptive statistics show that the difference between simple and log returns is not substantial

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- Jarque-Bera test can be used to test normality of a given sample of random variable of interest
- JB Statistic is given by:

$$JB = \frac{T}{6}(\hat{S}^2 + \frac{1}{4}(\hat{K} - 3)^2) \tag{2}$$

- Under null hypothesis of data following normal distribution, JB statistica follows χ^2_2 distribution

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- Estimator for Autocorrelation of a returns series $\{R_t\}$ at lag j, is given by:

$$\rho_j = \frac{\frac{1}{T-j}\sum_{t=j+1}^{T}(R_t - \hat{\mu})(R_{t-j} - \hat{\mu})}{\sum (R_t - \hat{\mu})^2} \tag{3}$$

where $\hat{\mu}$ is sample unconditional mean of $\{R_t\}$
- Under the assumption of data being IID, it follows below distribution:

$$\hat{\rho}_j \sim N(0, \frac{1}{T}) \tag{4}$$

- We can also test for autocorrelations at different lags together, using Ljung-Box Q-statistic

"In God we trust, all others bring data."

William Edwards Deming (1900 - 1993).