Basics of Time Series Concepts and ARMA

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Outline

What is Time Series?

Time Series as a sequence of RV

How do we ta about Time Series

Statistical Properties RVs

RV conditional o

information till t

Modelling Tim Series

Specification of Mo

Covariance Stationar

Choosing the be

Thankyou

1 What is Time Series?

Time Series as a sequence of RV

Outline

What is Time Series?

sequence of RV

How do we tal about Time Series

RVs RV conditional on

Modelling Tim

Series Series

Linear Models

Choosing the bes

Pl. . . . 1....

1 What is Time Series?
Time Series as a sequence of RV

2 How do we talk about Time Series Statistical Properties of RVs RV conditional on Information till t

Outline

What is Time Series?

sequence of RV

How do we tal about Time Series

RVs
RV conditional on
Information till t

Modelling Tim

Specification of Mode Linear Models Covariance Stationarit

Choosing the best Model

Thankyou

- What is Time Series?
 Time Series as a sequence of RV
- 2 How do we talk about Time Series Statistical Properties of RVs RV conditional on Information till t
- 3 Modelling Time Series
 Specification of Model
 Linear Models
 Covariance Stationarity
 Choosing the best Model

Outline

What is Time Series?

sequence of RV

How do we tal about Time Series

RVs
RV conditional on
Information till t

Modelling Tim

Linear Models

Covariance Stationarity

Thankyou

What is Time Series?
Time Series as a sequence of RV

- 2 How do we talk about Time Series Statistical Properties of RVs RV conditional on Information till t
- 3 Modelling Time Series
 Specification of Model
 Linear Models
 Covariance Stationarity
 Choosing the best Model
- 4 Thankyou

What is Tim

Time Series as a sequence of RV

How do we ta about Time Series

Statistical Properties of RVs

RV conditional or Information till t

Modelling Tir

Series Specification of Mo

Covariance Stationar

Choosing the best Model

Thankyou

Time Series as a sequence of RV

 Time series captures change in value of single variable of interest over time

What is Time

Time Series as a sequence of RV

How do we tal about Time

RVs
RV conditional on
Information till t

Modelling Tin

Series Series

Specification of Mode Linear Models

Choosing the best Model

Thankyou

Time Series as a sequence of RV

- Time series captures change in value of single variable of interest over time
- Mathematically,
 - we can think of it as time indexed set of random variables

$$----, Y_{-2}, Y_{-1}, Y_0, Y_1, Y_2, ----$$
 (1)

- stochastic process is a mathematical tool to deal with a set of random variables(indexed by time) together
- so we can think of time series as a realisation of a stochastic process

What is Time Series?

Time Series as a

How do we tal about Time Series

Statistical Properties of RVs

RV conditional on

Modelling Tir

Series

Linear Models

Covariance Stationari

Choosing the best Model

Thankyou

Statistical Properties of RVs

• Given that it is a sequence of RVs indexed by time, it helps to pictorially think of time series as below:

What is Time Series?

Time Series as a

How do we tal about Time Series

Statistical Properties of RVs

Information till t

Modelling Tin Series

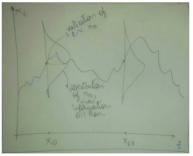
Specification of Mod Linear Models

Choosing the best

Thankyou

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PhotoScan by Google Photos

Distribution of RV at X_{10}

What is Time Series?

Time Series as a sequence of RV

How do we tal about Time Series

Statistical Properties of RVs

Information till t

Modelling Tin Series

Linear Models

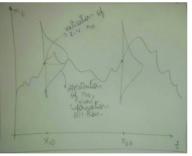
Covariance Stationarity

Model

Thankyou

Statistical Properties of RVs

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PhotoScan by Google Photos

Distribution of RV at X_{10}

• Each data point in time series is a realization of random variable defined by its conditional distribution (conditional on information till that point)

What is Time Series?

Time Series as a sequence of RV

about Time

Statistical Properties RVs

RV conditional on Information till t

Modelling Tin

Series
Specification of Moo

Covariance Stationari

Choosing the bes

Thankyou

RV conditional on Information till t

 Basic measures of a random variable are expectation, variance and covariance

What is Time Series?

Time Series as a sequence of RV

about Time Series

Statistical Properties of RVs

RV conditional on Information till t

Modelling Tin

Series

Linear Models

Covariance Stationari

Choosing the best Model

Thankyou

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What is Time Series?

Time Series as a sequence of RV

about Time Series

RVs RV conditional on

Information till t

Modelling Tim

Specification of Mod

Linear Models

Covariance Stationary

Choosing the best Model

Thankyou

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- So to talk about time series we need to define random variable at every point in time in terms of below quantities:
 - Expectation $E(Y_t|\mathcal{F}_{t-1})$
 - Variance $Var(Y_t|\mathcal{F}_{t-1})$
 - Covariance $Cov(Y_t, Y_{t-1})$

What is Time Series?

sequence of RV

Series

RVs RV conditional on

Information till t

Modelling Tim Series

Linear Models
Covariance Stationaria
Choosing the best

Thankyou

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- Modelling time series involves specifying a function and estimating it for each of the above quantities

What is Time

Time Series as a sequence of RV

about Time Series

RVs
RV conditional on

Information till t

Modelling Tim Series

Linear Models
Covariance Stationarity
Choosing the best

Thankyou

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- Modelling time series involves specifying a function and estimating it for each of the above quantities
- Some examples of time series
 - White noise : $Corr[\epsilon_t, \epsilon_{t-j}] = 0 \forall j \neq 0$
 - IID White noise : ϵ_t is independent of $\epsilon_{t-j} \, \forall j \neq 0$ and ϵ_t follows F distribution
 - Gaussian White Noise: $\epsilon_t \sim N(0, \sigma^2)$

What is Time

Time Series as a sequence of RV

How do we tal about Time Series

RVs RV conditional on

Information till t

Modelling Tin Series

Specification of Model Linear Models

Covariance Stationar Choosing the best Model

Thankyou

Specification of Model

- When we model a time series, we are actually specifying a function for random variable at every point in time
- A random variable can be represented in terms of its expectation and variance
- For now we concentrate on expectation/mean value of a random variable(look into variance in volatility models chapter)
- Intutively, we know that points closer to each other tend to be same. We would want to capture this persistence in time series data in our function
- One way to capture persistence is through correlation between random variables at points closer to each other in time, we will see how this will be used in modelling time series data later

Linear Models

What is Time Series?

Time Series as a sequence of RV

How do we to about Time Series

Statistical Properties (RVs

Information till

Modelling Tir

Series

Specification of Mod Linear Models

Covariance Stationar

Choosing the be Model • For a given time series, Y_t ,

Linear Models

Linear Models

• For a given time series, Y_t ,

$$Y_t = \mu_t + \epsilon_t, \tag{2}$$

where μ_t is conditional expectation at t, $E(Y_t)$

Linear Models

What is Tim Series?

Time Series as a sequence of RV

How do we tal about Time Series

Statistical Properties RVs

RV conditional on Information till t

Modelling Time Series

Specification of Mod

Linear Models

Choosing the best Model

Thankyou

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• To capture persistence, we define μ_t as some linear function of past values of Y_t

Linear Models

What is Tim

Time Series as a sequence of RV

How do we tall about Time Series

Statistical Properties RVs

RV conditional on Information till t

Modelling Tin Series

Specification of Mo Linear Models

Covariance Stationar Choosing the best

Thankyou

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- To capture persistence, we define μ_t as some linear function of past values of Y_t
- Autoregressive model:
 - we model value of time series at any point as sum of its past values
 - AR(1) model: $Y_t = \phi Y_{t-1} + \epsilon_t$, ϵ_t is White noise process
- Moving Average model:
 - here we model time series at any point as sum of its past error terms
 - MA(1) model: $Y_t = \phi_0 + \phi_1 \epsilon_{t-1}$, ϵ_t is White noise process

What is Time Series?

Time Series as a sequence of RV

How do we tal about Time

Series Series

Statistical Properties of RVs

RV conditional on Information till t

Modelling Tim

Series Specification of Mod

Linear Models

Covariance Stationarity

Choosing the best

Thankyou

Covariance Stationarity

- Key assumption on any time series Y_t , for it to be modelled is covariance stationarity:
 - $E[Y_t] = \mu \ \forall t$
 - $Var[Y_t] = \sigma^2 \ \forall t$
 - and Autocovariance, $Cov(Y_t, Y_{t-j}) = \sigma_j \ \forall t, j$

What is Tim

Time Series as a sequence of RV

How do we tal

Statistical Properties

RVs RV conditional on

Information till t

Modelling Tim Series

Series Specification of Mod

Covariance Stationarity

Choosing the best Model

Thankyou

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What is Tim

Time Series as a sequence of RV

How do we tal

Statistical Properties RVs

RV conditional on

Modelling Tin

Series

Linear Models

Covariance Stationarity

Choosing the best Model

Thankyou

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- Intutively, we can think of these assumptions as implying no change in statistical properties of the underlying process that generated the time series over the period of observation
- Note that we are talking about unconditional expectations,
 Variance and Covariance (not conditional on information at any point in time)

What is Time Series?

Time Series as a sequence of RV

How do we ta about Time

Statistical Properties of RVs

RV conditional o

Information till t

Series 1111

Linear Models

Covariance Stationar

Choosing the best Model

Thankyou

Choosing the best Model

• In statistical modelling, choosing a best model for data is a balance between data fitting and generalisation

What is Tim Series?

sequence of RV

How do we tal about Time Series

RVs
RV conditional on

Modelling Tim

Specification of Mod Linear Models

Covariance Stationar

Choosing the best Model

Thankyou

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What is Tim Series?

Time Series as a sequence of RV

How do we tal about Time

RVs
RV conditional on

Modelling Tim

Series

Linear Models

Choosing the best Model

Thankyou

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What is Tim Series?

Time Series as a sequence of RV

How do we tal about Time Series

Statistical Properties o RVs RV conditional on

Modelling Tim

Specification of Mode

Linear Models

Covariance Stationarit

Choosing the best Model

Thankyou

Choosing the best Model

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- Data fitting corresponds to how well my model explains the data used to build the model
- Generalisation, on the other hand sees how does my model work on future data (data not used for model building)
- Different measures used in choosing a model look at these two aspects in different ways:
 - MSE: It only looks at data fitting aspect
 - AIC, BIC: there is consideration of generalisation, by accounting for parsimony of the model

What is Tim Series?

Time Series as a

How do we t about Time

Statistical Properties

RVS PV conditional on

Information till

Modelling Time

Series Specification of Mode

Linear Models

Choosing the best

Thankyou

"In God we trust, all others bring data."

William Edwards Deming (1900 - 1993).