Modeling Market Volatility

Parthasarathi Edupally

Lec 4, Quantitative Finance,
Russell Square International College, Mumbai

Outline

Outline

Outline

Outline

Lecture 4 So Far ...

- We saw historical progression of ideas around efficient market hypothesis
- Understood how these theories can be translated into empirical studies
  - In particular, how law of iterated expectations can be used to formalise EMH
  - How RWH is a variant of EMH
- Finally, we saw how current research is focused more on comparing efficiencies of markets, than to test for efficiency itself of a particular market

Lecture 4 So Far ...

- We saw historical progression of ideas around efficient market hypothesis
- Understood how these theories can be translated into empirical studies
    - In particular, how law of iterated expectations can be used to formalise EMH
    - How RWH is a variant of EMH
- Finally, we saw how current research is focused more on comparing efficiencies of markets, than to test for efficiency itself of a particular market
- In part B of this lecture, we will try to understand modelling volatility of time series data

Modelling Time Series Data

- We have two parallel ways of looking at time series data
  - Mathematically, that its just a realization of a stochastic process
  - Intutively, that there is certain degree of persistence in properties of time series

Modelling Time Series Data

- We have two parallel ways of looking at time series data
    - Mathematically, that its just a realization of a stochastic process
    - Intutively, that there is certain degree of persistence in properties of time series
- Ultimate goal of modelling has been to say something about X_t, random variable at any point t
    - we could talk about its mean, conditional on the informaiton till t (conditional expectation, as in ARMA models)
    - Nobody stops us from talking about vairance, kurtosis and other moments of a random variable at t, given the information till that point
- In this part of the lecture, we will try to understand modelling variance (conditional on the information till t)

Why volatility modelling ?

- Variance, in particular, has a special meaning
  - in risk management, pricing models etc
  - also there is an intutive understanding about how it behaves in different ranges of values of time series data
- Risk of a financial asset can be quantified by its variance, thus volatility plays an important role in Risk Mangement
- In pricing models of financial instruments, which are based on principles of risk return tradeoff also have volatility as one of the inputs in determining prices
- As we are being very explicit about variance of random variable X_t at any point t, our model for time series data which considers only conditional expectation so far, is more precise and efficient now

What is Volatility ?

- Volatility can be defined as conditional variance of a time series of asset returns
- For a given return series, r_t, we can define its conditional mean and variance as follows

$$\text{conditional mean:} \quad \mu_t = E(r_t|F_{t-1})$$

$$\text{conditional variance:} \quad \sigma_t^2 = Var(r_t|F_{t-1}) = E[(r_t - \mu_t)^2|F_{t-1}]$$

- Some of the important features of volatility are:
  - it is not directly observable
  - we have volatility clusters
  - evolves continuously, there are no sudden jumps
  - doesnt diverge to infinity
  - it reacts differently to big positive and negative changes in return series

Model Structure

- Basic idea behind volatility models is that r_t series is uncorrelated or has minor lower order serial correlation, but is dependent

Model Structure

- Basic idea behind volatility models is that r_t series is uncorrelated or has minor lower order serial correlation, but is dependent
- This behaviour can be understood by observing ACF and PACF of r_t, $|r_t|$ and r_t^2
- We see that there is some kind of persistence in squared residual series, which represents variance

Model Structure

- Basic idea behind volatility models is that r_t series is uncorrelated or has minor lower order serial correlation, but is dependent
- This behaviour can be understood by observing ACF and PACF of r_t, $|r_t|$ and r_t^2
- We see that there is some kind of persistence in squared residual series, which represents variance
- For a given time series $\{r_t\}$, assuming ARMA we have

$$r_t = \mu_t + a_t \tag{1}$$

$$\mu_t = \phi_0 + \sum_{i=0}^{p} \phi_i r_{t-i} - \sum_{i=0}^{q} \theta_i a_{t-i} \tag{2}$$

$$\sigma_t^2 = Var(r_t|F_{t-1}) = Var(a_t|F_{t-1}) \tag{3}$$

Model Structure

- To specify ARCH model, we extend the above model with variance equation below,

$$a_t = \sigma_t \epsilon_t, \sigma_t^2 = \alpha_0 + \alpha_1 a_{t-1}^2 + ... + \alpha_m a_{t-m}^2 \tag{4}$$

Model Structure

- To specify ARCH model, we extend the above model with variance equation below,

$$a_t = \sigma_t \epsilon_t, \sigma_t^2 = \alpha_0 + \alpha_1 a_{t-1}^2 + ... + \alpha_m a_{t-m}^2 \tag{4}$$

- From the structure of the model, it is seen that large past squared shocks $\{a_{t-i}\}_{i=1}^m$ imply a large conditional variance σ_t for the mean-corrected return a_t

Model Structure

- To specify ARCH model, we extend the above model with variance equation below,

$$a_t = \sigma_t \epsilon_t, \sigma_t^2 = \alpha_0 + \alpha_1 a_{t-1}^2 + ... + \alpha_m a_{t-m}^2 \tag{4}$$

- From the structure of the model, it is seen that large past squared shocks $\{a_{t-i}\}_{i=1}^m$ imply a large conditional variance σ_t for the mean-corrected return a_t
- Similarly, we can specify GARCH model for volatility

Please refer section 3.1 and 3.2 from Tsay for more details

MLE

Please use section 8.6 from Study Guide

Alternative Models for Volatility

- GJR-GARCH - models with leverage effect to account for asymmetry in volatility, for positive and negative regions of returns

$$\sigma_{t+1}^2 = \omega + \beta\sigma_t^2 + \alpha\epsilon_t^2 + \delta\epsilon_t^2\mathbf{1}\{\epsilon_t < 0\} \tag{5}$$

Alternative Models for Volatility

- GJR-GARCH - models with leverage effect to account for asymmetry in volatility, for positive and negative regions of returns

$$\sigma_{t+1}^2 = \omega + \beta\sigma_t^2 + \alpha\epsilon_t^2 + \delta\epsilon_t^2 \mathbf{1}\{\epsilon_t < 0\} \tag{5}$$

- ARCH-in-mean - to account for economic rational of higher the risk more will be the return

Alternative Models for Volatility

- GJR-GARCH - models with leverage effect to account for asymmetry in volatility, for positive and negative regions of returns

$$\sigma_{t+1}^2 = \omega + \beta\sigma_t^2 + \alpha\epsilon_t^2 + \delta\epsilon_t^2 \mathbf{1}\{\epsilon_t < 0\} \tag{5}$$

- ARCH-in-mean - to account for economic rational of higher the risk more will be the return
- And others like IGARCH, PARCH etc to account different novel features of financial time series

Choosing a Volatility Model

- As we saw in models for expectation, volatility models also have to deal with balancing data fitting and generatlisation

Choosing a Volatility Model

- As we saw in models for expectation, volatility models also have to deal with balancing data fitting and generatlisation
- Accordingly, we have MSE, QLIKE as measures of data fitting
- AIC, BIC and HQIC as measuring both data fitting and parsimony (generalisation)

Choosing a Volatility Model

- As we saw in models for expectation, volatility models also have to deal with balancing data fitting and generatlisation
- Accordingly, we have MSE, QLIKE as measures of data fitting
- AIC, BIC and HQIC as measuring both data fitting and parsimony (generalisation)
- As the financial time series dont exist in vaccum but have a context - economic decision making
- we can incorporate economic criteria also in choosing a model

Choosing a Volatility Model

- As we saw in models for expectation, volatility models also have to deal with balancing data fitting and generatlisation
- Accordingly, we have MSE, QLIKE as measures of data fitting
- AIC, BIC and HQIC as measuring both data fitting and parsimony (generalisation)
- As the financial time series dont exist in vaccum but have a context - economic decision making
- we can incorporate economic criteria also in choosing a model
- please refer study guide 9.3 for more details

"In God we trust, all others bring data."

William Edwards Deming (1900 - 1993).