Basics of Time Series Concepts and ARMA

Parthasarathi Edupally

Quantitative Finance,
Russell Square International College, Mumbai

Outline

Outline

Outline

Outline

Time Series as a sequence of RV

- Time series captures change in value of single variable of interest over time

Time Series as a sequence of RV

- Time series captures change in value of single variable of interest over time
- Mathematically,
  - we can think of it as time indexed set of random variables

$$----, Y_{-2}, Y_{-1}, Y_0, Y_1, Y_2, ----- \tag{1}$$

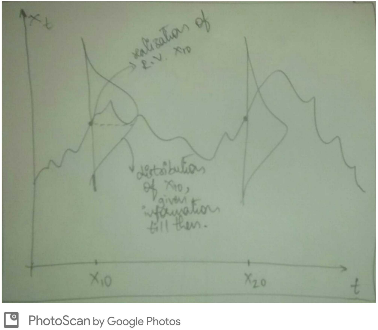
  - stochastic process is a mathematical tool to deal with a set of random variables(indexed by time) together
  - so we can think of time series as a realisation of a stochastic process

Statistical Properties of RVs

- Given that it is a sequence of RVs indexed by time, it helps to pictorially think of time series as below:

Statistical Properties of RVs

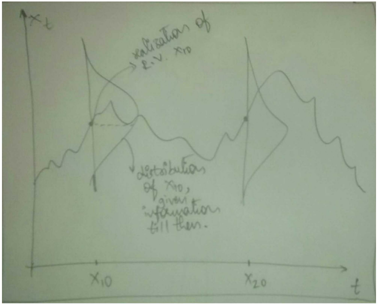
- Given that it is a sequence of RVs indexed by time, it helps to pictorially think of time series as below:



Distribution of RV at X_{10}

Statistical Properties of RVs

- Given that it is a sequence of RVs indexed by time, it helps to pictorially think of time series as below:



Distribution of RV at X_{10}

- Each data point in time series is a realization of random variable defined by its conditional distribution (conditional on information till that point)

RV conditional on Information till t

- Basic measures of a random variable are expectation, variance and covariance

RV conditional on Information till t

- Basic measures of a random variable are expectation, variance and covariance
- So to talk about time series we need to define random variable at every point in time in terms of below quantities:

RV conditional on Information till t

- Basic measures of a random variable are expectation, variance and covariance
- So to talk about time series we need to define random variable at every point in time in terms of below quantities:
  - Expectation - $E(Y_t|\mathcal{F}_{t-1})$
  - Variance - $Var(Y_t|\mathcal{F}_{t-1})$
  - Covariance - $Cov(Y_t, Y_{t-1})$

RV conditional on Information till t

- Basic measures of a random variable are expectation, variance and covariance
- So to talk about time series we need to define random variable at every point in time in terms of below quantities:
  - Expectation - $E(Y_t|\mathcal{F}_{t-1})$
  - Variance - $Var(Y_t|\mathcal{F}_{t-1})$
  - Covariance - $Cov(Y_t, Y_{t-1})$
- Modelling time series involves specifying a function and estimating it for each of the above quantities

RV conditional on Information till t

- Basic measures of a random variable are expectation, variance and covariance
- So to talk about time series we need to define random variable at every point in time in terms of below quantities:
  - Expectation - $E(Y_t|\mathcal{F}_{t-1})$
  - Variance - $Var(Y_t|\mathcal{F}_{t-1})$
  - Covariance - $Cov(Y_t, Y_{t-1})$
- Modelling time series involves specifying a function and estimating it for each of the above quantities
- Some examples of time series
  - White noise : $Corr[\epsilon_t, \epsilon_{t-j}] = 0 \forall j \neq 0$
  - IID White noise : ϵ_t is independent of ϵ_{t-j} $\forall j \neq 0$ and ϵ_t follows F distribution
  - Gaussian White Noise: $\epsilon_t \sim N(0, \sigma^2)$

Specification of Model

- When we model a time series, we are actually specifying a function for random variable at every point in time
- A random variable can be represented in terms of its expectation and variance
- For now we concentrate on expectation/mean value of a random variable(look into variance in volatility models chapter)
- Intutively, we know that points closer to each other tend to be same. We would want to capture this persistence in time series data in our function
- One way to capture persistence is through correlation between random variables at points closer to each other in time, we will see how this will be used in modelling time series data later

Linear Models

- For a given time series, Y_t,

Linear Models

- For a given time series, Y_t,

$$Y_t = \mu_t + \epsilon_t, \tag{2}$$

where μ_t is conditional expectation at t, $E(Y_t)$

Linear Models

- For a given time series, Y_t,

$$Y_t = \mu_t + \epsilon_t, \tag{2}$$

  where μ_t is conditional expectation at t, $E(Y_t)$
- To capture persistence, we define μ_t as some linear function of past values of Y_t

Linear Models

- For a given time series, Y_t,

$$Y_t = \mu_t + \epsilon_t, \tag{2}$$

  where μ_t is conditional expectation at t, $E(Y_t)$
- To capture persistence, we define μ_t as some linear function of past values of Y_t
- Autoregressive model:
  - we model value of time series at any point as sum of its past values
  - AR(1) model: $Y_t = \phi Y_{t-1} + \epsilon_t$, ϵ_t is White noise process
- Moving Average model:
  - here we model time series at any point as sum of its past error terms
  - MA(1) model: $Y_t = \phi_0 + \phi_1 \epsilon_{t-1}$, ϵ_t is White noise process

Covariance Stationarity

- Key assumption on any time series Y_t, for it to be modelled is covariance stationarity:
  - $E[Y_t] = \mu \; \forall t$
  - $Var[Y_t] = \sigma^2 \; \forall t$
  - and Autocovariance, $Cov(Y_t, Y_{t-j}) = \sigma_j \; \forall t, j$

Covariance Stationarity

- Key assumption on any time series Y_t, for it to be modelled is covariance stationarity:
  - $E[Y_t] = \mu \; \forall t$
  - $Var[Y_t] = \sigma^2 \; \forall t$
  - and Autocovariance, $Cov(Y_t, Y_{t-j}) = \sigma_j \; \forall t, j$
- Intutively, we can think of these assumptions as implying no change in statistical properties of the underlying process that generated the time series over the period of observation

Covariance Stationarity

- Key assumption on any time series Y_t, for it to be modelled is covariance stationarity:
  - $E[Y_t] = \mu \; \forall t$
  - $Var[Y_t] = \sigma^2 \; \forall t$
  - and Autocovariance, $Cov(Y_t, Y_{t-j}) = \sigma_j \; \forall t, j$
- Intutively, we can think of these assumptions as implying no change in statistical properties of the underlying process that generated the time series over the period of observation
- Note that we are talking about unconditional expectations, Variance and Covariance (*not* conditional on information at any point in time)

Choosing the best Model

- In statistical modelling, choosing a best model for data is a balance between data fitting and generalisation

Choosing the best Model

- In statistical modelling, choosing a best model for data is a balance between data fitting and generalisation
- Data fitting corresponds to how well my model explains the data used to build the model

Choosing the best Model

- In statistical modelling, choosing a best model for data is a balance between data fitting and generalisation
- Data fitting corresponds to how well my model explains the data used to build the model
- Generalisation, on the other hand sees how does my model work on future data (data not used for model building)

Choosing the best Model

- In statistical modelling, choosing a best model for data is a balance between data fitting and generalisation
- Data fitting corresponds to how well my model explains the data used to build the model
- Generalisation, on the other hand sees how does my model work on future data (data not used for model building)
- Different measures used in choosing a model look at these two aspects in different ways:
  - MSE : It only looks at data fitting aspect
  - AIC, BIC : there is consideration of generalisation, by accounting for parsimony of the model

"In God we trust, all others bring data."

William Edwards Deming (1900 - 1993).