

Programming Paradigms

Lecture 5. Introduction to Haskell

Outline

- What is Haskell?
- A program in Haskell
- Numeric types and common operations
- Defining functions with equations
- Boolean types
- Lists and tuples
- User-defined types
- Code.World platform

What is Haskell?



An advanced, purely functional programming language

What is Haskell?



An advanced, purely functional programming language

Simple and solid foundation

Easy* to reason about

Easy* to maintain safe and correct

What is Haskell?



An advanced, purely functional programming language

Very expressive and practical

A program in Haskell

```
x = 2 + 2
```

A program in Haskell

```
x = 2 + 2
```

```
y = 3 * x - 1
```

A program (module) is a collection of declarations

A program in Haskell

```
x = 2 + 2
```

```
y = 3 * x - 1
```

```
z = length "Hello, world!" + 1
```

```
identifier = expression
```

A program (module) is a collection of declarations

A program in Haskell

```
x = 2 + 2
```

```
y = 3 * x - 1
```

```
z = length "Hello, world!" + 1
```

```
isGreater = (x > y)
```

```
identifier = expression
```

A program (module) is a collection of declarations

A program in Haskell

```
x = 2 + 2
```

```
y = 3 * x - 1
```

```
z = length "Hello, world!" + 1
```

```
isGreater = (x > y)
```

```
fivePrimes = [2, 3, 5, 7, 11]
```

```
identifier = expression
```

A program (module) is a collection of declarations

A program in Haskell

```
x = 2 + 2
```

```
y = 3 * x - 1
```

```
z = length "Hello, world!" + 1
```

```
isGreater = (x > y)
```

```
fivePrimes = [2, 3, 5, 7, 11]
```

```
myProgram = print 10 >> print 20
```

```
identifier = expression
```

A program (module) is a collection of declarations

Declarations

`z = 2`

This is declaration, not assignment!

Declarations

`z = 2`

`z = 3`

This is declaration, not assignment!

Declarations

`z = 2` This is declaration, not assignment!

`z = 3`

```
<interactive>:3:1: error:  
  Multiple declarations of 'z'  
  Declared at: <interactive>:2:1  
               <interactive>:3:1
```

Order of declarations

```
someMean = (a + b) / 2
```

```
a = 5.0
```

```
b = 7.0
```

Order of declarations does not matter

Local declarations

```
someMean = (a + b) / 2
```

```
  where
```

```
    a = 5.0
```

```
    b = 7.0
```

Haskell is indentation-sensitive!

Local declarations must be aligned.

Type signatures

$$x = 2 + 2$$

Type signatures

`x :: Int`

`x = 2 + 2`

Type signatures

`x :: Int`

`x = 2 + 2`

`x :: Int`

reads

“x has type `Int`”

Type signatures

```
x :: Int  
x = 2 + 2
```

```
identifier :: type  
identifier = expression
```

```
x :: Int  
reads  
“x has type Int”
```

Type signatures

`x :: Int`

`x = 2 + 2`

`y :: Int`

`y = 3 * x - 1`

`identifier :: type`

`identifier = expression`

Type signatures

```
someMean = (a + b) / 2
```

```
  where
```

```
    a = 5.0
```

```
    b = 7.0
```

Type signatures

```
someMean :: Double  
someMean = (a + b) / 2  
  where  
    a = 5.0  
    b = 7.0
```

Type signatures

```
someMean :: Double
someMean = (a + b) / 2
  where
    a :: Double
    a = 5.0

    b :: Double
    b = 7.0
```


Type signatures

```
someMean :: Double
someMean = (a + b) / 2
  where
    a :: Double
    a = 5.0

    b :: Double
    b = 7.0
```

```
identifier :: type
identifier = expression
  where
    <local declarations>
```

Numeric types: `Int`

```
n :: Int  
n = 42
```

```
minInt, maxInt :: Int  
minInt = -9223372036854775808  
maxInt = 9223372036854775807
```

`Int` is a machine-sized integer.

Very efficient, big enough for most tasks.

Numeric types: `Integer`

```
bigNumber :: Integer  
bigNumber = 3^(4^5)
```

```
373391848741020043532959754184866588225  
409776783734007750636931722079040617265  
251229993688938803977220468765065431475  
158108727054592160858581351336982809187  
314191748594262580938807019951956404285  
571818041046681288797402925517668012340  
617298396574731619152386723046235125934  
896058590588284654793540505936202376547  
807442730582144527058988756251452817793  
413352141920744623027518729185432862375  
737063985485319476416926263819972887006  
907013899256524297198527698749274196276  
811060702333710356481
```

`Integer` is a bignum integer (long arithmetic).
Also efficient and often optimized.
Bounded only by available memory.

Numeric types: Double

```
almostPi :: Double  
almostPi = 3.14
```

Numeric types: Double

```
almostPi :: Double  
almostPi = 3.14
```

```
wholeNumber :: Double  
wholeNumber = 5
```

Integer literals can be interpreted with any numeric type.

Numeric operations: (+), (*), (-)

```
exampleInt :: Int
```

```
exampleInt = 20 + 3 * 47 - 10
```

```
exampleInteger :: Integer
```

```
exampleInteger = 20 + 3 * 47 - 10
```

```
exampleDouble :: Double
```

```
exampleDouble = 2 + 3 * 4.7 - 1
```

```
-- 15.1000000000000001
```

Operations that work for all numeric types.

Numeric operations: (/), sin, cos, sqrt, log, ...

```
almostOne :: Double
almostOne = sin x * sin x + cos x * cos x
  where
    x = log (sqrt 10)

-- 1.0000000000000000002
```

Operations that work for floating point numbers.

Numeric operations: `div`, `mod`

```
six :: Int  
six = 55 `mod` 7
```

```
ten :: Integer  
ten = 1 + (123456 `div` 12346)
```

Operations that work for integers.

Numeric operations: `div`, `mod`

```
six :: Int  
six = 55 `mod` 7
```

```
ten :: Integer  
ten = 1 + (123456 `div` 12346)
```

```
six :: Int  
six = mod 55 7
```

```
ten :: Integer  
ten = 1 + (div 123456 12346)
```

Put backticks around an identifier
to turn it into an infix operator.

Type conversion

```
n :: Int  
n = 42
```

```
almostPi :: Double  
almostPi = 3.14
```

```
bad = n + almostPi
```

Type conversion

```
n :: Int  
n = 42
```

```
almostPi :: Double  
almostPi = 3.14
```

```
bad = n + almostPi
```

```
<interactive>:20:11: error:
```

- Couldn't match expected type 'Int' with actual type 'Double'
- In the second argument of '(+)', namely 'almostPi'
In the expression: n + almostPi
In an equation for 'bad': bad = n + almostPi

Explicit type conversion

```
n :: Int  
n = 42
```

```
almostPi :: Double  
almostPi = 3.14
```

```
good1 = (fromIntegral n) + almostPi
```

fromIntegral converts
from any integer type to any numeric type

Explicit type conversion

```
n :: Int  
n = 42
```

```
almostPi :: Double  
almostPi = 3.14
```

```
good1 = (fromIntegral n) + almostPi
```

```
good2 = n + (floor    almostPi) -- round down  
good3 = n + (ceiling  almostPi) -- round up  
good4 = n + (round    almostPi) -- round to the closest int
```

Defining functions

```
square x = x * x
```

Defining functions

```
square :: Double -> Double  
square x = x * x
```

Defining functions

```
square :: Double -> Double  
square x = x * x
```

```
distance :: Double -> Double -> Double  
distance x y = abs (x - y)
```


Defining functions

```
square :: Double -> Double  
square x = x * x
```

```
distance :: Double -> Double -> Double  
distance x y = abs (x - y)
```

```
factorial :: Integer -> Integer  
factorial 0 = 1  
factorial n = n * factorial (n - 1)
```

Boolean type

```
otherwise :: Bool  
otherwise = True
```

```
lie :: Bool  
lie = False
```

Boolean type

```
otherwise :: Bool  
otherwise = True
```

```
lie :: Bool  
lie = False
```

```
someMax :: Int  
someMax
```

where

```
  x = 5  
  y = 3
```

Boolean type

```
otherwise :: Bool  
otherwise = True
```

```
lie :: Bool  
lie = False
```

```
someMax :: Int  
someMax  
  | x > y      = x
```

where

```
  x = 5  
  y = 3
```

Boolean type

```
otherwise :: Bool  
otherwise = True
```

```
lie :: Bool  
lie = False
```

```
someMax :: Int  
someMax  
  | x > y      = x  
  | otherwise = y  
where  
  x = 5  
  y = 3
```

Boolean type

```
otherwise :: Bool  
otherwise = True
```

```
lie :: Bool  
lie = False
```

```
someMax :: Int  
someMax  
  | x > y      = x  
  | otherwise  = y  
where  
  x = 5  
  y = 3
```

```
identifier :: type  
identifier arg1 ... argN  
  | cond1 = expression1  
  | cond2 = expression2  
  | ...  
where  
  <local declarations>
```

Char and String

```
name :: String  
name = "Anna Karenina"
```

```
initial :: Char  
initial = 'A'
```

Tuples

```
tuple1 = (123, "Hello", False)
```


Tuples

```
tuple1 :: (Int, String, Bool)  
tuple1 = (123, "Hello", False)
```

Tuples

```
tuple1 :: (Int, String, Bool)  
tuple1 = (123, "Hello", False)
```

```
tuple2 :: (Int, String, Bool)  
tuple2 = (678, "Bye", True)
```

Tuples

```
tuple1 :: (Int, String, Bool)
tuple1 = (123, "Hello", False)
```

```
tuple2 :: (Int, String, Bool)
tuple2 = (678, "Bye", True)
```

```
student1 = ("Peter White", 4)
student2 = ("Jane Black", 5)
```

Tuples

```
tuple1 :: (Int, String, Bool)  
tuple1 = (123, "Hello", False)
```

```
tuple2 :: (Int, String, Bool)  
tuple2 = (678, "Bye", True)
```

```
student1 = ("Peter White", 4)  
student2 = ("Jane Black", 5)
```

```
point3D = (1.0, -2.2, 5.3)  
vector3D = (3.0, 0.0, 4.0)
```

A function of a tuple

```
len :: (Double, Double, Double) -> Double  
len (x, y, z) = sqrt (x^2 + y^2 + z^2)
```

A function of a tuple

```
len :: (Double, Double, Double) -> Double  
len (x, y, z) = sqrt (x^2 + y^2 + z^2)
```

```
vector3D = (3.0, 0.0, 4.0)
```

```
example = len vector3D
```

A function of a tuple

```
len :: (Double, Double, Double) -> Double  
len (x, y, z) = sqrt (x^2 + y^2 + z^2)
```

```
vector3D = (3.0, 0.0, 4.0)
```

```
example = len vector3D
```

```
-- example
```

```
-- = len vector3D
```

A function of a tuple

```
len :: (Double, Double, Double) -> Double  
len (x, y, z) = sqrt (x^2 + y^2 + z^2)
```

```
vector3D = (3.0, 0.0, 4.0)
```

```
example = len vector3D
```

```
-- example  
-- = len vector3D  
-- = len (3.0, 0.0, 4.0)
```


A function of a tuple

```
len :: (Double, Double, Double) -> Double  
len (x, y, z) = sqrt (x^2 + y^2 + z^2)
```

```
vector3D = (3.0, 0.0, 4.0)
```

```
example = len vector3D
```

```
-- example  
-- = len vector3D  
-- = len (3.0, 0.0, 4.0)  
-- = sqrt (x^2 + y^2 + z^2)  
--     where x = 3.0, y = 0.0, z = 4.0
```

A function of a tuple

```
len :: (Double, Double, Double) -> Double  
len (x, y, z) = sqrt (x^2 + y^2 + z^2)
```

```
vector3D = (3.0, 0.0, 4.0)
```

```
example = len vector3D
```

```
-- example  
-- = len vector3D  
-- = len (3.0, 0.0, 4.0)  
-- = sqrt (x^2 + y^2 + z^2)  
--     where x = 3.0, y = 0.0, z = 4.0  
-- = sqrt (3.0^2 + 0.0^2 + 4.0^2)
```

A function of a tuple

```
len :: (Double, Double, Double) -> Double  
len (x, y, z) = sqrt (x^2 + y^2 + z^2)
```

```
vector3D = (3.0, 0.0, 4.0)
```

```
example = len vector3D
```

```
-- example  
-- = len vector3D  
-- = len (3.0, 0.0, 4.0)  
-- = sqrt (x^2 + y^2 + z^2)  
--     where x = 3.0, y = 0.0, z = 4.0  
-- = sqrt (3.0^2 + 0.0^2 + 4.0^2)  
-- = ...  
-- = 5.0
```

Empty tuple

```
emptyTuple = ()
```

Empty tuple

```
emptyTuple :: ()  
emptyTuple = ()
```

Lists

```
list1 = [1, 2, 3]
```

Lists

```
list1 :: [Int]  
list1 = [1, 2, 3]
```

Lists

```
list1 :: [Int]  
list1 = [1, 2, 3]
```

```
list2 :: [String]  
list2 = ["Hello", "world"]
```


Lists

```
list1 :: [Int]  
list1 = [1, 2, 3]
```

```
list2 :: [String]  
list2 = ["Hello", "world"]
```

```
list3 :: [Bool]  
list3 = [False, True]
```

Lists

```
list1 :: [Int]  
list1 = [1, 2, 3]
```

```
list2 :: [String]  
list2 = ["Hello", "world"]
```

```
list3 :: [Bool]  
list3 = [False, True]
```

```
list4 :: [(String, Int)]  
list4 = [("Peter White", 4), ("Jane Black", 5)]
```

Lists

```
list1 :: [Int]  
list1 = [1, 2, 3]
```

```
list2 :: [String]  
list2 = ["Hello", "world"]
```

```
list3 :: [Bool]  
list3 = [False, True]
```

```
list4 :: [(String, Int)]  
list4 = [("Peter White", 4), ("Jane Black", 5)]
```

```
list5 :: [[Int]]  
list5 = [[1], [2, 3], []]
```

Lists are homogeneous

```
bad = [1, "two"]
```

Lists are homogeneous

```
bad = [1, "two"]
```

```
<interactive>:21:8: error:
```

```
• No instance for (Num [Char]) arising from the literal  
'1'
```

```
• In the expression: 1
```

```
  In the expression: [1, "two"]
```

```
  In an equation for 'bad': bad = [1, "two"]
```

Lists are homogeneous

```
bad = [1, "two"]
```

One of the items in the list is a number
Another one is a String (which is list of
Chars).

But Strings are not Numbers!

```
<interactive>:21:8: error:
```

```
• No instance for (Num [Char]) arising from the literal  
'1'
```

```
• In the expression: 1
```

```
  In the expression: [1, "two"]
```

```
  In an equation for 'bad': bad = [1, "two"]
```

Lists are homogeneous

```
bad = [False, "two"]
```

```
<interactive>:22:15: error:
```

- Couldn't match expected type 'Bool' with actual type '[Char]
- In the expression: "two"
In the expression: [False, "two"]
In an equation for 'bad': bad = [False, "two"]

Lists are homogeneous

`bad = [False, "two"]` The list started with a `Bool`.
But now we see a `String` (which is a list of `Chars`).
But `String` is not the same as `Bool`.

```
<interactive>:22:15: error:
```

- Couldn't match expected type 'Bool' with actual type '[Char]
 - In the expression: "two"
- In the expression: `[False, "two"]`
In an equation for 'bad': `bad = [False, "two"]`

Constructing lists

```
list0 = []
```

Constructing lists

```
list0 = []  
list1 = 1 : list0    -- [1]
```

Constructing lists

```
list0 = []  
list1 = 1 : list0      -- [1]  
list2 = 2 : list1      -- [2, 1]
```

Constructing lists

```
list0 = []  
list1 = 1 : list0      -- [1]  
list2 = 2 : list1      -- [2, 1]  
list3 = 3 : list2      -- [3, 2, 1]
```

Constructing lists

```
list0 = []  
list1 = 1 : list0      -- [1]  
list2 = 2 : list1      -- [2, 1]  
list3 = 3 : list2      -- [3, 2, 1]  
  
list4 = 4 : (3 : (2 : (1 : [])))  
-- [4, 3, 2, 1]
```

Pattern matching lists

$f \ [] =$
 $f \ (x:xs) =$

Pattern matching lists

```
f [] = something  
f (x:xs) =
```

Pattern matching lists

```
f [] = something  
f (x:xs) = somethingElse x xs
```


Pattern matching lists

```
f [] = something  
f (x:xs) = somethingElse x xs
```

```
list5 = [1, 3, 5]
```

```
example = f list5
```

Pattern matching lists

```
f [] = something  
f (x:xs) = somethingElse x xs
```

```
list5 = [1, 3, 5]
```

```
example = f list5  
-- example  
-- = f list5
```

Pattern matching lists

```
f [] = something  
f (x:xs) = somethingElse x xs
```

```
list5 = [1, 3, 5]
```

```
example = f list5  
-- example  
-- = f list5  
-- = f [1, 3, 5]
```

Pattern matching lists

```
f [] = something  
f (x:xs) = somethingElse x xs
```

```
list5 = [1, 3, 5]
```

```
example = f list5
```

```
-- example  
-- = f list5  
-- = f [1, 3, 5]  
-- = f (1 : [3, 5])
```

Pattern matching lists

```
f [] = something  
f (x:xs) = somethingElse x xs
```

```
list5 = [1, 3, 5]
```

```
example = f list5
```

```
-- example  
-- = f list5  
-- = f [1, 3, 5]  
-- = f (1 : [3, 5])  
-- = somethingElse 1 [3, 5]
```

Pattern matching lists: sum

```
f [] = something  
f (x:xs) = somethingElse x xs
```

```
sum :: [Double] -> Double  
sum [] =  
sum (x:xs) =
```

Pattern matching lists: sum

```
f [] = something  
f (x:xs) = somethingElse x xs
```

```
sum :: [Double] -> Double  
sum [] = 0  
sum (x:xs) =
```

Pattern matching lists: sum

```
f [] = something  
f (x:xs) = somethingElse x xs
```

```
sum :: [Double] -> Double  
sum [] = 0  
sum (x:xs) = x + sum xs
```


Type aliases

```
-- | A 2D point.  
type Point = (Double, Double)
```

Type aliases

```
-- | A 2D point.
```

```
type Point = (Double, Double)
```

```
-- | A distance in kilometers.
```

```
type Kilometers = Double
```

```
-- | A distance in miles.
```

```
type Miles = Double
```

```
milesToKm :: Miles -> Kilometers
```

```
milesToKm miles = 1.609344 * miles
```

User defined types: product types

```
-- | A 2D vector.  
data Vector = MkVector Double Double
```

User defined types: product types

```
-- | A 2D vector.  
data Vector = MkVector Double Double
```



Name of the new type

User defined types: product types

```
-- | A 2D vector.  
data Vector = MkVector Double Double
```

Name of the new type



Name of the **value constructor**



User defined types: product types

-- | A 2D vector.

data Vector = MkVector Double Double

Name of the new type

Name of the **value constructor**

Types of parameters of
the value constructor

User defined types: product types

```
-- | A 2D vector.  
data Vector = MkVector Double Double  
  
someVector :: Vector  
someVector = MkVector 3 4
```

User defined types: product types

```
-- | A 2D vector.
```

```
data Vector = MkVector Double Double
```

```
someVector :: Vector
```

```
someVector = MkVector 3 4
```

```
vectorLen :: Vector -> Double
```

```
vectorLen (MkVector x y) = sqrt (x^2 + y^2)
```


User defined types: product types

```
-- | A 2D vector.
```

```
data Vector = Vector Double Double
```

```
someVector :: Vector
```

```
someVector = Vector 3 4
```

```
vectorLen :: Vector -> Double
```

```
vectorLen (Vector x y) = sqrt (x^2 + y^2)
```

User defined types: product types

```
-- | A 2D vector.
```

```
data Vector = Vector Double Double
```

```
data Student = Student Year Name Grade
```

```
data MenuItem = MenuItem Title Price
```

User defined types: wrapper types

```
data Kilometers = Kilometers Double
```

```
data Miles = Miles Double
```

```
milesToKm :: Miles -> Kilometers
```

```
milesToKm miles  
  = 1.609344 * miles
```

User defined types: wrapper types

```
data Kilometers = Kilometers Double
```

```
data Miles = Miles Double
```

```
milesToKm :: Miles -> Kilometers
```

```
milesToKm miles  
  = 1.609344 * miles
```

```
<interactive>:30:5: error:
```

- Couldn't match expected type 'Kilometers'
with actual type 'Miles'
- In the expression: 1.609344 * miles
In an equation for 'milesToKm': milesToKm miles = 1.609344 * miles

User defined types: wrapper types

```
data Kilometers = Kilometers Double
```

```
data Miles = Miles Double
```

```
milesToKm :: Miles -> Kilometers  
milesToKm miles  
  = Kilometers (1.609344 * miles)
```

User defined types: wrapper types

```
data Kilometers = Kilometers Double
```

```
data Miles = Miles Double
```

```
milesToKm :: Miles -> Kilometers  
milesToKm miles  
  = Kilometers (1.609344 * miles)
```

```
<interactive>:39:17: error:
```

- Couldn't match expected type 'Double' with actual type 'Miles'
- In the first argument of 'Kilometers', namely

```
  '(1.609344 * miles)'
```

```
In the expression: Kilometers (1.609344 * miles)
```

```
In an equation for 'milesToKm':
```

```
  milesToKm miles = Kilometers (1.609344 * miles)
```

User defined types: wrapper types

```
data Kilometers = Kilometers Double
```

```
data Miles = Miles Double
```

```
milesToKm :: Miles -> Kilometers  
milesToKm (Miles miles)  
    = Kilometers (1.609344 * miles)
```

Code.World platform

```
1 import CodeWorld
2
3 myPicture :: Picture
4 myPicture = leftShape <> rightShape
5   where
6     leftShape = solidCircle 1
7     rightShape =
8       translated 3 0 (solidCircle 2)
9
10 main :: IO ()
11 main = drawingOf myPicture
```



<https://code.world/haskell>

Code.World Pictures

```
import CodeWorld
```

```
myPicture :: Picture
```

```
myPicture = leftShape <> rightShape
```

```
  where
```

```
    leftShape = solidCircle 1
```

```
    rightShape =
```

```
      translated 3 0 (solidCircle 2)
```

```
main :: IO ()
```

```
main = drawingOf myPicture
```

Code.World Pictures

```
import CodeWorld
```

```
myPicture :: Picture
```

```
myPicture = leftShape <> rightShape
```

```
  where
```

```
    leftShape = solidCircle 1
```

```
    rightShape =
```

```
      translated 3 0 (solidCircle 2)
```

```
main :: IO ()
```

```
main = drawingOf myPicture
```

```
(<>) :: Picture -> Picture -> Picture
```

Code.World Pictures

```
import CodeWorld
```

```
myPicture :: Picture
```

```
myPicture = leftShape <> rightShape
```

```
  where
```

```
    leftShape = solidCircle 1
```

```
    rightShape =
```

```
      translated 3 0 (solidCircle 2)
```

```
main :: IO ()
```

```
main = drawingOf myPicture
```

```
solidCircle :: Double -> Picture
```

Code.World Pictures

```
import CodeWorld
```

```
myPicture :: Picture
```

```
myPicture = leftShape <> rightShape
```

```
  where
```

```
    leftShape = solidCircle 1
```

```
    rightShape =
```

```
      translated 3 0 (solidCircle 2)
```

```
main :: IO ()
```

```
main = drawingOf myPicture
```

```
translated :: Double -> Double -> Picture -> Picture
```

Code.World Pictures

```
import CodeWorld
```

```
myPicture :: Picture
```

```
myPicture = leftShape <> rightShape
```

```
  where
```

```
    leftShape = solidCircle 1
```

```
    rightShape =
```

```
      translated 3 0 (solidCircle 2)
```

```
main :: IO ()
```

```
main = drawingOf myPicture
```

```
drawingOf :: ???
```

**What what the most
unclear part of the
lecture for you?**

See Moodle

Homework (self-study)

1. Install **Haskell** <https://www.haskell.org/downloads/>
2. Read **Learn you a Haskell for Great Good** Chapters 2, 4, and 5
<http://learnyouahaskell.com/chapters>
3. Test yourself by implementing a program that renders a Koch snowflake of a given rank in Haskell on Code.World platform (<https://code.world/haskell>):

