

Programming Paradigms

Lecture 3. Higher-order functions and lists

Test N°2

See Moodle

Clarification: cons vs append

```
(cons 1 (list 2 3))
```

```
; (list 1 2 3)
```

Clarification: cons vs append

```
(cons 1 (list 2 3))      ; (list 1 2 3)  
(append (list 1) (list 2 3)) ; (list 1 2 3)
```

Clarification: cons vs append

```
(cons 1 (list 2 3))           ; (list 1 2 3)  
(append (list 1) (list 2 3)) ; (list 1 2 3)
```

```
(cons (list 1) (list 2 3))
```

Clarification: cons vs append

<code>(cons 1 (list 2 3))</code>	<code>; (list 1 2 3)</code>
<code>(append (list 1) (list 2 3))</code>	<code>; (list 1 2 3)</code>
<code>(cons (list 1) (list 2 3))</code>	<code>; (list (list 1) 2 3)</code>

Clarification: cons vs append

<code>(cons 1 (list 2 3))</code>	<code>; (list 1 2 3)</code>
<code>(append (list 1) (list 2 3))</code>	<code>; (list 1 2 3)</code>
<code>(cons (list 1) (list 2 3))</code>	<code>; (list (list 1) 2 3)</code>
<code>(append 1 (list 2 3))</code>	

Clarification: cons vs append

<code>(cons 1 (list 2 3))</code>	<code>; (list 1 2 3)</code>
<code>(append (list 1) (list 2 3))</code>	<code>; (list 1 2 3)</code>
<code>(cons (list 1) (list 2 3))</code>	<code>; (list (list 1) 2 3)</code>

```
(append 1 (list 2 3))
```

append: contract violation
expected: list?
given: 1

Clarification: tail recursion

```
(define (factorial n)
  (cond
    [(<= n 1) 1]
    [else (* n (factorial (- n 1)))]))
```

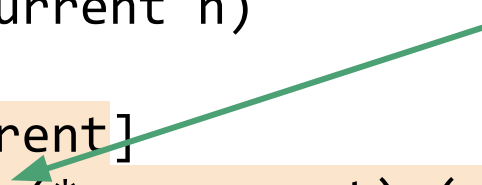
```
(define (factorial n)
  (define (helper current n)
    (cond
      [(<= n 1) current]
      [else (helper (* n current) (- n 1))]))
  (helper 1 n))
```

Clarification: tail recursion

```
(define (factorial n)
  (cond
    [(<= n 1) 1]
    [else (* n (factorial (- n 1)))]))
```

```
(define (factorial n)
  (define (helper current n)
    (cond
      [(<= n 1) current]
      [else (helper (* n current) (- n 1))]))
  (helper 1 n))
```

Tail recursive call



Outline

- Pairs and Lists in Racket
- Quotation
- Higher-order functions
- Generators in Racket

Pairs in Racket

```
(cons 1 2)           ; '(1 . 2)  
(cons "hi" "world") ; '("hi" . "world")
```

Pairs in Racket

```
(cons 1 2)           ; '(1 . 2)  
(cons "hi" "world") ; '("hi" . "world")
```

```
; cons? = pair?  
(pair? (cons 1 2))
```

Pairs in Racket

```
(cons 1 2)           ; '(1 . 2)  
(cons "hi" "world") ; '("hi" . "world")
```

```
; cons? = pair?  
(pair? (cons 1 2))
```

```
(cons 1 empty)       ; '(1)  
(cons 1 (list 2 3)) ; '(1 2 3)
```

Pairs in Racket

```
(cons 1 2)           ; '(1 . 2)
(cons "hi" "world") ; '("hi" . "world")
```

```
; cons? = pair?
(pair? (cons 1 2))
```

```
(cons 1 empty)       ; '(1)
(cons 1 (list 2 3))  ; '(1 2 3)
(cons 1 (cons 2 3))  ; '(1 2 . 3)
```

Pairs in Racket: accessors

```
(first (list 1 2 3)) ; 1
```

```
(second (list 1 2 3)) ; 2
```

```
(third (list 1 2 3)) ; 3
```

```
(rest (list 1 2 3)) ; '(2 3)
```


Pairs in Racket: accessors

```
(first (list 1 2 3)) ; 1
```

```
(second (list 1 2 3)) ; 2
```

```
(third (list 1 2 3)) ; 3
```

```
(rest (list 1 2 3)) ; '(2 3)
```

```
(car (cons 3 4)) ; 3
```

```
(cdr (cons 3 4)) ; 4
```

Pairs in Racket: accessors

```
(first (list 1 2 3)) ; 1
```

```
(second (list 1 2 3)) ; 2
```

```
(third (list 1 2 3)) ; 3
```

```
(rest (list 1 2 3)) ; '(2 3)
```

```
(car (cons 3 4)) ; 3
```

```
(cdr (cons 3 4)) ; 4
```

```
(car (list 1 2 3))
```

```
(cdr (list 1 2 3))
```

Pairs in Racket: accessors

```
(first (list 1 2 3)) ; 1
```

```
(second (list 1 2 3)) ; 2
```

```
(third (list 1 2 3)) ; 3
```

```
(rest (list 1 2 3)) ; '(2 3)
```

```
(car (cons 3 4)) ; 3
```

```
(cdr (cons 3 4)) ; 4
```

```
(car (list 1 2 3))
```

```
(cadr (list 1 2 3))
```

```
(caddr (list 1 2 3))
```

```
(cdr (list 1 2 3))
```

Quotation

```
(list (list 1) (list 2 3) (list 4)) ; '((1) (2 3) (4))
```

Quotation

```
(list (list 1) (list 2 3) (list 4)) ; '((1) (2 3) (4))
```

```
(quote ((1) (2 3) (4))) ; '((1) (2 3) (4))
```

Quotation

```
(list (list 1) (list 2 3) (list 4)) ; '((1) (2 3) (4))
```

```
(quote ((1) (2 3) (4))) ; '((1) (2 3) (4))
```

```
'((1) (2 3) (4))
```

Quotation

```
(list (list 1) (list 2 3) (list 4)) ; '((1) (2 3) (4))
```

```
(quote ((1) (2 3) (4))) ; '((1) (2 3) (4))
```

```
'((1) (2 3) (4))
```

```
(first '((1) (2 3) (4))) ; '(1)
```

```
(rest '((1) (2 3) (4))) ; '((2 3) (4))
```

Conditionals with lists

```
(define (fib n)
  (cond
    [(<= n 2) 1]
    [else (+ (fib (- n 1))
              (fib (- n 2)))]))
```

```
(or (> 3 2) (> (fib 100) 1000)) ; #t
```

```
(and (> 2 3) (> (fib 100) 1000)) ; #f
```


Higher-order functions: twice

```
(define (twice f x)  
  (f (f x)))
```

```
(twice sqrt 16) ; 2
```

Higher-order functions: map

```
(map (lambda (x) (* x x))  
     '(1 2 3 4 5))  
; '(1 4 9 16 25)
```

Higher-order functions: map

```
(map (lambda (x) (* x x))  
      '(1 2 3 4 5))  
; '(1 4 9 16 25)
```

```
(ormap odd? '(2 4 6 8 10))  
; #f
```

Higher-order functions: map

```
(map (lambda (x) (* x x))  
      '(1 2 3 4 5))  
; '(1 4 9 16 25)
```

```
(ormap odd? '(2 4 6 8 10))  
; #f
```

```
(andmap even? '(2 4 6 8 10))  
; #t
```

Higher-order functions: ormap

```
(define (divides? n m)
  (= 0 (remainder m n)))
```

```
(define (prime? n)
  (not (ormap
        (lambda (k) (divides? k n))
        (range 2 n))))
```

Higher-order functions: ormap vs or+map

```
(define (my-ormap f lst)  
  (apply or (map f lst)))
```

Higher-order functions: ormap vs or+map

```
(define (my-ormap f lst)
  (apply or (map f lst)))
```

or: bad syntax in: or

Higher-order functions: filter

```
(filter even? '(1 2 3 4 5))  
; '(2 4)
```

```
(filter prime? (range 2 50))  
; '(2 3 5 7 11 13 17 19 23 29 31 37 41 43 47)
```


Higher-order functions: `filter`

```
(define students  
  (list  
    (cons "Anna"    4.3)  
    (cons "Boris"   3.6)  
    (cons "Charles" 4.1)  
    (cons "Daria"   2.9)))
```

Higher-order functions: `filter`

```
(define students
  (list
    (cons "Anna"    4.3)
    (cons "Boris"   3.6)
    (cons "Charles" 4.1)
    (cons "Daria"   2.9)))

(map car
  (filter (lambda (student)
            (> (cdr student) 4.0))
    students))
; '("Anna" "Charles")
```

Higher-order functions: `filter`

```
(define students  
  (list  
    (cons "Anna"      4.3)  
    (cons "Boris"     3.6)  
    (cons "Charles"   4.1)  
    (cons "Daria"     2.9)))
```

```
'("Anna"      . 4.3)  
'("Boris"     . 3.6)  
'("Charles"   . 4.1)  
'("Daria"     . 2.9)
```

```
(map car  
  (filter (lambda (student)  
           (> (cdr student) 4.0))  
    students))  
; '("Anna" "Charles")
```

Higher-order functions: `filter`

```
(define students
  (list
    (cons "Anna"      4.3)
    (cons "Boris"     3.6)
    (cons "Charles"   4.1)
    (cons "Daria"     2.9)))
```

'("Anna" . 4.3)

'("Charles" . 4.1)

```
(map car
  (filter (lambda (student)
    (> (cdr student) 4.0))
  students))
```

; '("Anna" "Charles")

Higher-order functions: `filter`

```
(define students  
  (list  
    (cons "Anna"    4.3)  
    (cons "Boris"   3.6)  
    (cons "Charles" 4.1)  
    (cons "Daria"   2.9)))
```

"Anna"

"Charles"

```
(map car  
  (filter (lambda (student)  
           (> (cdr student) 4.0))  
    students))  
; '("Anna" "Charles")
```

Higher-order functions: foldl

```
(define (f x current) ...)
```

```
(foldl f z '(a b c d))  
; f (f (f (f z a) b) c) d
```

```
(foldl + 0 '(1 2 3 4)) ; 10  
(foldl * 1 '(1 2 3 4)) ; 24
```

Higher-order functions: foldl

```
(foldl
  (lambda (student current)
    (cond
      [(> (cdr student) 4.0)
       (cons (car student) current)]
      [else current]))
  empty
  students)
```

students	
'("Anna"	. 4.3)
'("Boris"	. 3.6)
'("Charles"	. 4.1)
'("Daria"	. 2.9)

current
empty

Higher-order functions: foldl

	students
(foldl	student = '("Anna" . 4.3)
(lambda (student current)	'("Boris" . 3.6)
(cond	'("Charles" . 4.1)
[(> (cdr student) 4.0)	'("Daria" . 2.9)
(cons (car student) current)]	
[else current]))	
empty	
students)	current
	empty

Higher-order functions: foldl

(foldl		students
(lambda (student current)	student =	'("Anna" . 4.3)
(cond		'("Boris" . 3.6)
[(> (cdr student) 4.0)		'("Charles" . 4.1)
(cons (car student) current)]		'("Daria" . 2.9)
[else current]))		
empty		
students)		current
		'("Anna")

Higher-order functions: foldl

```
(foldl
  (lambda (student current)
    (cond
      [(> (cdr student) 4.0)
       (cons (car student) current)]
      [else current]))
  empty
  students)
```

students

```
'("Anna"      . 4.3)
'("Boris"      . 3.6)
'("Charles"    . 4.1)
'("Daria"      . 2.9)
```

current

```
'("Anna")
```

Higher-order functions: foldl

```
(foldl
  (lambda (student current)
    (cond
      [(> (cdr student) 4.0) student =
        (cons (car student) current)]
      [else current]))
  empty
  students)
```

students

```
'("Anna"      . 4.3)
'("Boris"      . 3.6)
'("Charles"    . 4.1)
'("Daria"      . 2.9)
```

current

```
'("Charles" "Anna")
```

Implementing insertion sort with higher-order functions

Generators: generator and yield

```
(define (loop students)
  (cond
    [(empty? students)
     "no more 4.0+ students"]
    [(> (cdar students) 4.0)
     (begin
      (yield (car students))
      (loop (cdr students)))]
    [else (loop (cdr students))]))
```

```
(define students-with-4+
  (generator () (loop students)))
```

Generators: generator and yield

```
(define (loop students)
  (cond
    [(empty? students)
     "no more 4.0+ students"]
    [(> (cdr (first students)) 4.0)
     (begin
      (yield (car (first
students))))
      (loop (rest students)))]
    [else (loop (rest students))]))
```

```
(define students-with-4+
  (generator () (loop students)))
```

Generators: generator and yield

```
(students-with-4+) ; "Anna"
```

```
(define (loop students)
  (cond
    [(empty? students)
     "no more 4.0+ students"]
    [(> (cdr (first students)) 4.0)
     (begin
        (yield (car (first
students))))
      (loop (rest students)))]
    [else (loop (rest students))]))
```

```
(define students-with-4+
  (generator () (loop students)))
```

Generators: generator and yield

```
(define (loop students)
  (cond
    [(empty? students)
     "no more 4.0+ students"]
    [(> (cdr (first students)) 4.0)
     (begin
      (yield (car (first
students))))
      (loop (rest students)))]
    [else (loop (rest students))]))

(define students-with-4+
  (generator () (loop students)))

(students-with-4+) ; "Anna"
(students-with-4+) ; "Charles"
```


Generators: generator and yield

```
(define (loop students)
  (cond
    [(empty? students)
     "no more 4.0+ students"]
    [(> (cdr (first students)) 4.0)
     (begin
      (yield (car (first
students))))
      (loop (rest students)))]
    [else (loop (rest students))]))

(define students-with-4+
  (generator () (loop students)))

(students-with-4+) ; "Anna"
(students-with-4+) ; "Charles"
(students-with-4+) ; "no more 4.0+ students"
```

**What what the most
unclear part of the
lecture for you?**

See Moodle

References

1. [Racket Essentials 2.4](#) — Pairs, Lists, and Racket Syntax
2. [Racket Essentials 2.2.5](#) — Conditionals with `if`, `and`, `or`, and `cond`
3. [Racket Essentials 2.3.1](#) — Predefined List Loops