

# Programming Paradigms

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Lecture 2. Functional programming in Racket

# Test N°1

See Moodle

# Outline

- Recap: Simple definitions and expressions
- The Substitution Model
- Lists and recursion
- Tail recursion

Recap: definitions in Racket

```
(define a-disk  
  (colorize (disk 90) "blue"))
```

Recap: definitions in Racket

special form



```
(define a-disk  
  (colorize (disk 90) "blue"))
```

## Recap: definitions in Racket

special form

identifier



```
(define a-disk  
  (colorize (disk 90) "blue"))
```

The diagram illustrates the components of a Racket definition. A red arrow points from the text 'special form' to the keyword **define** in the code. A blue arrow points from the text 'identifier' to the identifier `a-disk` in the code. The identifier `a-disk` is highlighted with a light blue background.

## Recap: definitions in Racket

special form

identifier



The diagram illustrates the components of a Racket definition form. The text `(define a-disk (colorize (disk 90) "blue"))` is shown. The word `define` is highlighted in black. The identifier `a-disk` is highlighted in a light blue box. The entire expression `(colorize (disk 90) "blue")` is highlighted in a light red box. A red arrow points from the text "special form" to `define`. A blue arrow points from the text "identifier" to `a-disk`. Another red arrow points from the text "expression" to the `(colorize (disk 90) "blue")` sub-expression.

```
(define a-disk (colorize (disk 90) "blue"))
```

expression

Recap: function definitions in Racket

```
(define (square width color)
  (colorize
    (filled-rectangle width width)
    color))
```



Recap: function definitions in Racket

identifier

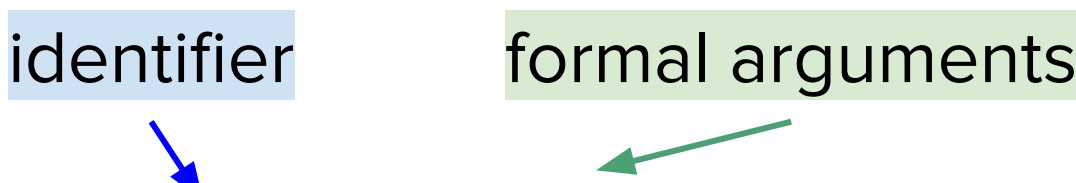


```
(define (square width color)  
  (colorize  
    (filled-rectangle width width)  
    color))
```

## Recap: function definitions in Racket

identifier

formal arguments



```
(define (square width color)  
  (colorize  
    (filled-rectangle width width)  
    color))
```

## Recap: function definitions in Racket

identifier

formal arguments

(**define** (square width color)

(colorize

(filled-rectangle width width)

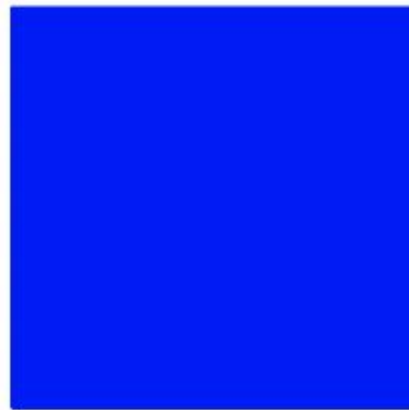
color))

body (expression)

Recap: function calls in Racket

```
(square 100 "blue")
```

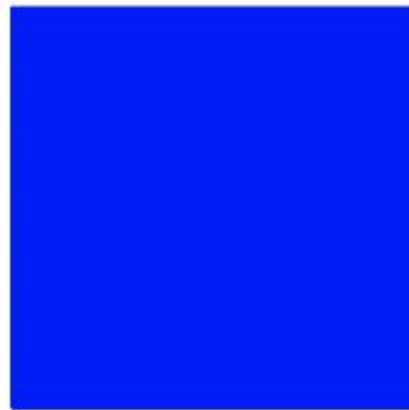
```
(square (+ 50 50) "blue")
```



## Recap: function calls in Racket

```
(square 100 "blue")
```

```
(square (+ 50 50) "blue")
```



Recap: conditionals

```
(define (quantity-to-text word n)
  (cond
    [(= n 1)
     (string-append "one " word)]
    [(<= 2 n 3)
     (string-append
      "a couple of " word "s")]
    [else
     (string-append
      "many " word "s")]))
```

Recap: conditionals

```
(define (quantity-to-text word n)
  (cond
    [(= n 1)
     (string-append "one " word)]
    [(<= 2 n 3)
     (string-append
      "a couple of " word "s")]
    [else
     (string-append
      "many " word "s")]))
```

## Recap: conditionals

```
(define (quantity-to-text word n)
  (cond
    [(= n 1)
     (string-append "one " word)]
    [(<= 2 n 3)
     (string-append
      "a couple of " word "s")]
    [else
     (string-append
      "many " word "s")]))
```



Recap: conditionals

```
(quantity-to-text "apple" 1)  
; "one apple"
```

```
(quantity-to-text "orange" 2)  
; "a couple of oranges"
```

```
(quantity-to-text "banana" 4)  
; "many bananas"
```

Recap: anonymous functions

```
(define (twice f x)  
  (f (f x)))
```

Recap: anonymous functions

```
(define (twice f x)  
  (f (f x)))
```

```
(twice sqrt 16) ; 2
```

Recap: anonymous functions

```
(define (twice f x)  
  (f (f x)))
```

```
(twice (lambda (x) (* x x)) 2) ; 16
```

Recap: anonymous functions

```
(define (twice f x)  
  (f (f x)))
```

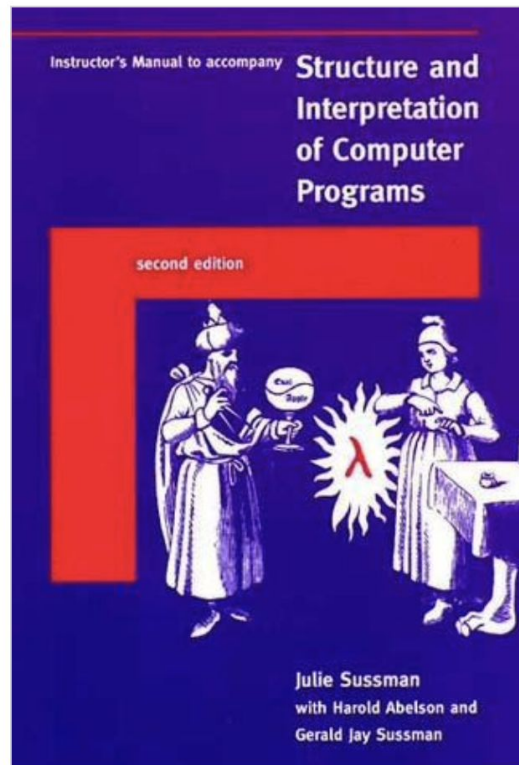
formal arguments

```
(twice (lambda (x) (* x x)) 2) ; 16
```

body (expression)

SICP, the wizard book

# Structure and Interpretation of Computer Programs



<https://mitpress.mit.edu/sites/default/files/sicp/index.html>

# The Substitution Model

```
(define (square x) (* x x))
```

## The Substitution Model

```
(define (square x) (* x x))
```

```
(define (sum-of-squares x y)  
  (+ (square x) (square y)))
```



## The Substitution Model

```
(define (square x) (* x x))
```

```
(define (sum-of-squares x y)  
  (+ (square x) (square y)))
```

```
(define (f z)  
  (sum-of-squares (+ z 2) (* z 3)))
```

## The Substitution Model

```
(define (square x) (* x x))
```

```
(define (sum-of-squares x y)  
  (+ (square x) (square y)))
```

```
(define (f z)  
  (sum-of-squares (+ z 2) (* z 3)))
```

```
(f 5)
```

## The Substitution Model

```
(define (square x) (* x x))
```

```
(define (sum-of-squares x y)  
  (+ (square x) (square y)))
```

```
(define (f z)  
  (sum-of-squares (+ z 2) (* z 3)))
```

```
(f 5)
```

```
(sum-of-squares (+ z 2) (* z 3))
```

## The Substitution Model

```
(define (square x) (* x x))
```

```
(define (sum-of-squares x y)  
  (+ (square x) (square y)))
```

```
(define (f z)  
  (sum-of-squares (+ z 2) (* z 3)))
```

```
(f 5)
```

```
(sum-of-squares (+ 5 2) (* 5 3))
```

## The Substitution Model

```
(define (square x) (* x x))
```

```
(define (sum-of-squares x y)  
  (+ (square x) (square y)))
```

```
(define (f z)  
  (sum-of-squares (+ z 2) (* z 3)))
```

```
(sum-of-squares (+ 5 2) (* 5 3))
```

## The Substitution Model

```
(define (square x) (* x x))
```

```
(define (sum-of-squares x y)  
  (+ (square x) (square y)))
```

```
(define (f z)  
  (sum-of-squares (+ z 2) (* z 3)))
```

```
(sum-of-squares 7 15)
```

## The Substitution Model

```
(define (square x) (* x x))
```

```
(define (sum-of-squares x y)  
  (+ (square x) (square y)))
```

```
(define (f z)  
  (sum-of-squares (+ z 2) (* z 3)))
```

```
(sum-of-squares 7 15)
```

```
(+ (square x) (square y))
```

## The Substitution Model

```
(define (square x) (* x x))
```

```
(define (sum-of-squares x y)  
  (+ (square x) (square y)))
```

```
(define (f z)  
  (sum-of-squares (+ z 2) (* z 3)))
```

```
(sum-of-squares 7 15)
```

```
(+ (square 7) (square 15))
```



## The Substitution Model

```
(define (square x) (* x x))
```

```
(define (sum-of-squares x y)  
  (+ (square x) (square y)))
```

```
(define (f z)  
  (sum-of-squares (+ z 2) (* z 3)))
```

```
(+ (square 7) (square 15))
```

## The Substitution Model

```
(define (square x) (* x x))
```

```
(define (sum-of-squares x y)  
  (+ (square x) (square y)))
```

```
(define (f z)  
  (sum-of-squares (+ z 2) (* z 3)))
```

```
(+ (* 7 7) (square 15))
```

## The Substitution Model

```
(define (square x) (* x x))
```

```
(define (sum-of-squares x y)  
  (+ (square x) (square y)))
```

```
(define (f z)  
  (sum-of-squares (+ z 2) (* z 3)))
```

```
(+ 49 (square 15))
```

## The Substitution Model

```
(define (square x) (* x x))
```

```
(define (sum-of-squares x y)  
  (+ (square x) (square y)))
```

```
(define (f z)  
  (sum-of-squares (+ z 2) (* z 3)))
```

```
(+ 49 (* 15 15))
```

## The Substitution Model

```
(define (square x) (* x x))
```

```
(define (sum-of-squares x y)  
  (+ (square x) (square y)))
```

```
(define (f z)  
  (sum-of-squares (+ z 2) (* z 3)))
```

```
(+ 49 225)
```

## The Substitution Model

```
(define (square x) (* x x))
```

```
(define (sum-of-squares x y)  
  (+ (square x) (square y)))
```

```
(define (f z)  
  (sum-of-squares (+ z 2) (* z 3)))
```

## The Substitution Model: anonymous functions

```
(define (twice f x)  
  (f (f x)))
```

```
(twice (lambda (x) (* x x)) 2)
```

## The Substitution Model: anonymous functions

```
(define (twice f x)  
  (f (f x)))
```

```
(twice (lambda (x) (* x x)) 2)  
((lambda (x) (* x x)) ((lambda (x) (* x x)) 2))
```



## The Substitution Model: anonymous functions

```
(define (twice f x)  
  (f (f x)))
```

```
(twice (lambda (x) (* x x)) 2)  
( (lambda (x) (* x x)) ( (lambda (x) (* x x)) 2) )  
( (lambda (x) (* x x)) (* 2 2) )
```

## The Substitution Model: anonymous functions

```
(define (twice f x)
  (f (f x)))
```

```
(twice (lambda (x) (* x x)) 2)
((lambda (x) (* x x)) ((lambda (x) (* x x)) 2))
((lambda (x) (* x x)) (* 2 2))
((lambda (x) (* x x)) 4)
```

## The Substitution Model: anonymous functions

```
(define (twice f x)  
  (f (f x)))
```

```
(twice (lambda (x) (* x x)) 2)  
( (lambda (x) (* x x)) ( (lambda (x) (* x x)) 2) )  
( (lambda (x) (* x x)) (* 2 2) )  
( (lambda (x) (* x x)) 4 )  
(* 4 4)
```

## The Substitution Model: anonymous functions

```
(define (twice f x)  
  (f (f x)))
```

```
(twice (lambda (x) (* x x)) 2)  
(  
  (lambda (x) (* x x))  
  (  
    (lambda (x) (* x x))  
    (* 2 2)  
  )  
(  
  (lambda (x) (* x x))  
  4)  
(* 4 4)
```

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## Lists

```
(list 1 2 3 4 5)
```

```
(list "apples" "bananas" "oranges")
```

```
(list 1 (list 2 3) (list (list)))
```

## Function on lists

```
(define example  
  (list "apples" "bananas" "oranges"))
```

```
(length example)
```

```
(list-ref example 1)
```

```
(reverse example)
```

```
(append example example example)
```

## Deconstructing lists

```
(define example  
  (list "apples" "bananas" "oranges"))
```

```
(first example) ; "apples"  
(rest example) ; '("bananas" "oranges")
```

```
(car example) ; "apples"  
(cdr example) ; '("bananas" "oranges")
```

## Constructing lists

empty

; '()



## Constructing lists

```
empty ; '()
```

```
(cons 1 (list 2 3)) ; '(1 2 3)
```

## Constructing lists

`empty`

`; '()`

`(cons 1 (list 2 3)) ; '(1 2 3)`

`(cons 1 (cons 2 (cons 3 empty)))`

## Checking structure of a list

```
(empty? (list 2 3)) ; #f
```

```
(empty? (rest (rest (list 2 3)))) ; #t
```

```
(cons? (list 2 3)) ; #t
```

## Checking structure of a list

```
(empty? (list 2 3)) ; #f
```

```
(empty? (rest (rest (list 2 3)))) ; #t
```

```
(cons? (list 2 3)) ; #t
```

## Recursive functions over lists

```
(define (my-sum lst)  
  ...)
```

## Recursive functions over lists

```
(define (my-sum lst)
  (cond
    [(empty? lst) ...]
    [else ...]))
```

## Recursive functions over lists

```
(define (my-sum lst)
  (cond
    [(empty? lst) 0]
    [else ...]))
```

## Recursive functions over lists

```
(define (my-sum lst)
  (cond
    [(empty? lst) 0]
    [else (+ (first lst)
              (my-length (rest lst)))]))
```



## Recursive functions over lists

```
(define (my-sum lst)
  (cond
    [(empty? lst) 0]
    [else (+ (first lst)
              (my-length (rest lst)))]))
```

```
(my-sum (list 1 2 3)) ; 6
```

## Tail recursion

```
(define (my-sum lst)
  (cond
    [(empty? lst) 0]
    [else (+ (first lst)
              (my-length (rest lst)))]))
```

```
(my-sum (list 1 2 3))
```

## Tail recursion

```
(define (my-sum lst)
  (cond
    [(empty? lst) 0]
    [else (+ (first lst)
              (my-length (rest lst)))])
```

```
(my-sum (list 1 2 3))
(+ 1 (my-sum (list 2 3)))
```

## Tail recursion

```
(define (my-sum lst)
  (cond
    [(empty? lst) 0]
    [else (+ (first lst)
              (my-length (rest lst)))]))
```

```
(my-sum (list 1 2 3))
(+ 1 (my-sum (list 2 3)))
(+ 1 (+ 2 (my-sum (list 3))))
```

## Tail recursion

```
(define (my-sum lst)
  (cond
    [(empty? lst) 0]
    [else (+ (first lst)
              (my-length (rest lst)))]))
```

```
(my-sum (list 1 2 3))
(+ 1 (my-sum (list 2 3)))
(+ 1 (+ 2 (my-sum (list 3))))
(+ 1 (+ 2 (+ 3 (my-sum empty))))
```

## Tail recursion

```
(define (my-sum lst)
  (cond
    [(empty? lst) 0]
    [else (+ (first lst)
              (my-length (rest lst)))]))
```

```
(my-sum (list 1 2 3))
(+ 1 (my-sum (list 2 3)))
(+ 1 (+ 2 (my-sum (list 3))))
(+ 1 (+ 2 (+ 3 (my-sum empty))))
(+ 1 (+ 2 (+ 3 0)))
```

## Tail recursion

```
(define (my-sum lst)
  (define (helper lst current)
    (cond
      [(empty? lst) current]
      [else (helper (rest lst)
                    (+ current (first lst)))])
    (helper lst 0))
```

## Tail recursion

```
(define (my-sum lst)
  (define (helper lst current)
    (cond
      [(empty? lst) current]
      [else (helper (rest lst)
                    (+ current (first lst)))]))
  (helper lst 0))

(my-sum (list 1 2 3))
```



## Tail recursion

```
(define (my-sum lst)
  (define (helper lst current)
    (cond
      [(empty? lst) current]
      [else (helper (rest lst)
                    (+ current (first lst)))])
    (helper lst 0))
```

```
(my-sum (list 1 2 3))
(helper (list 1 2 3) 0)
```

## Tail recursion

```
(define (my-sum lst)
  (define (helper lst current)
    (cond
      [(empty? lst) current]
      [else (helper (rest lst)
                    (+ current (first lst)))]))
  (helper lst 0))
```

```
(my-sum (list 1 2 3))
(helper (list 1 2 3) 0)
(helper (list 2 3) (+ 0 1))
```

# Tail recursion

```
(define (my-sum lst)
  (define (helper lst current)
    (cond
      [(empty? lst) current]
      [else (helper (rest lst)
                    (+ current (first lst)))])
    (helper lst 0))
```

```
(my-sum (list 1 2 3))
(helper (list 1 2 3) 0)
(helper (list 2 3) (+ 0 1))
(helper (list 2 3) 1)
```

...

Using apply

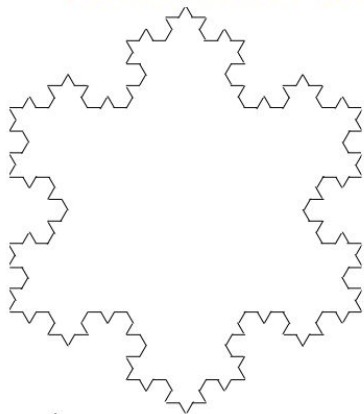
```
(define (my-sum lst)  
  (apply + lst))
```

```
(my-sum (list 1 2 3 4))
```

# Homework

1. Read **SICP 1.2 Procedures and the Processes They Generate**
2. Solve **exercises 1.11, 1.14, 1.16, 1.26** from **SICP**
3. Implement a function in Racket that renders a Koch snowflake of given rank:

```
> (koch-snowflake 4)
```



# References

1. The Substitution Model — SICP 1.1.5
2. Racket essentials 2.3–2.4

# Mud cards