

# Programming Paradigms

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Lecture 12. Cut and negation as failure in Prolog

# Outline

- The Cut
- Using cut for efficiency
- Using cut for fixing overlapping branches
- Negation as failure

## Overlapping solutions

`p(X) :- a(X), b(X).`

`p(X) :- c(X).`

`a(1). b(1). b(2). c(1). c(2).`

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p(X) :- c(X).
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```

```
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```

```
X = 1
```

The cut operator (!):

1. Always succeeds.
2. Commits variable substitutions made by pattern matching in the head.
3. Commits variable substitutions made by all prior predicates in the body.

## Branching without cut

?- p(X)

p(X) :- a(X), b(X).

p(X) :- c(X).

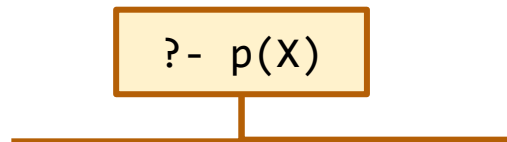
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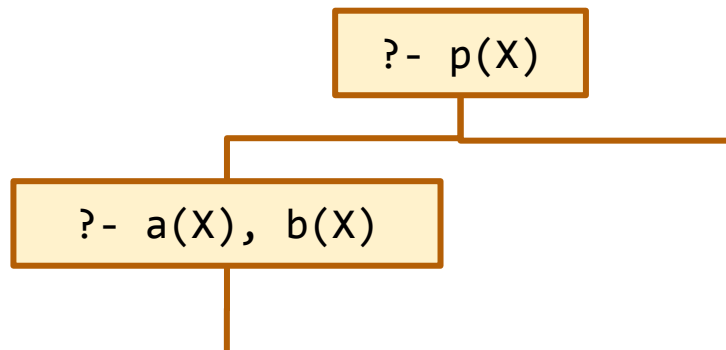


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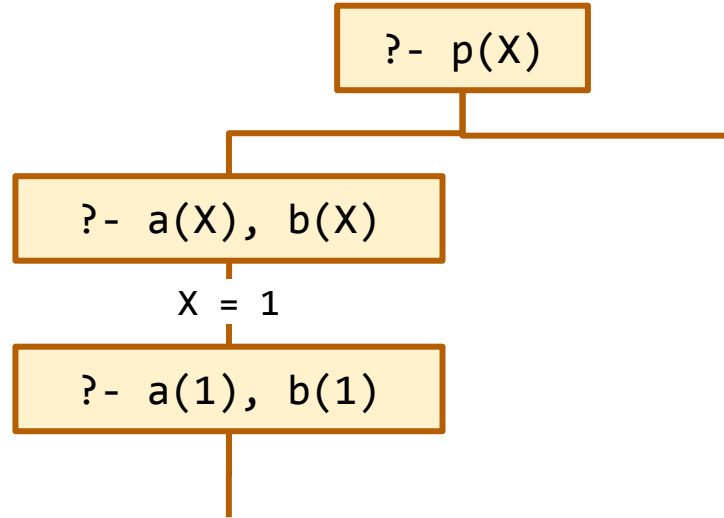


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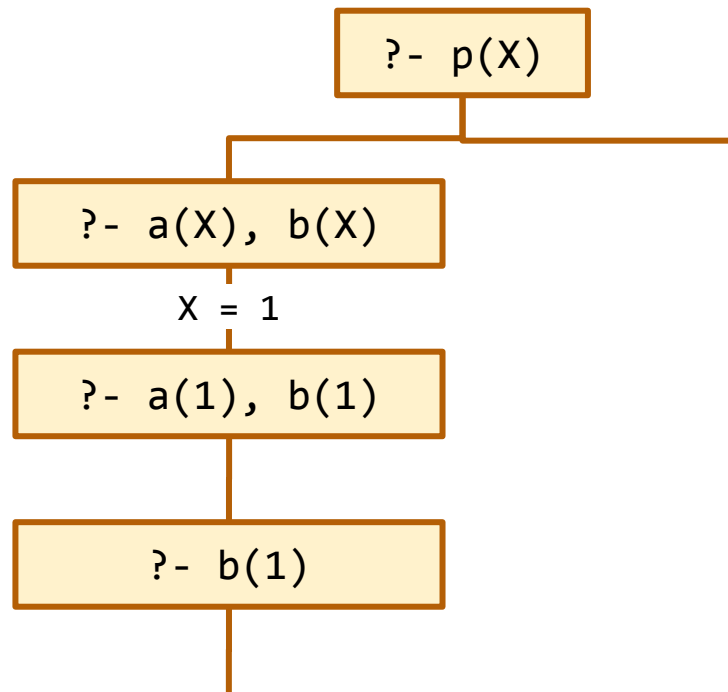


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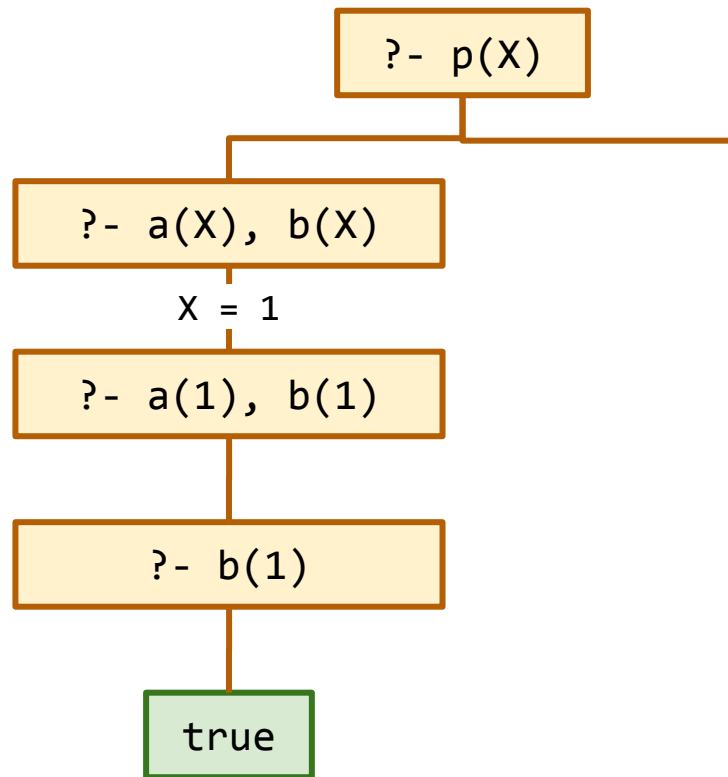


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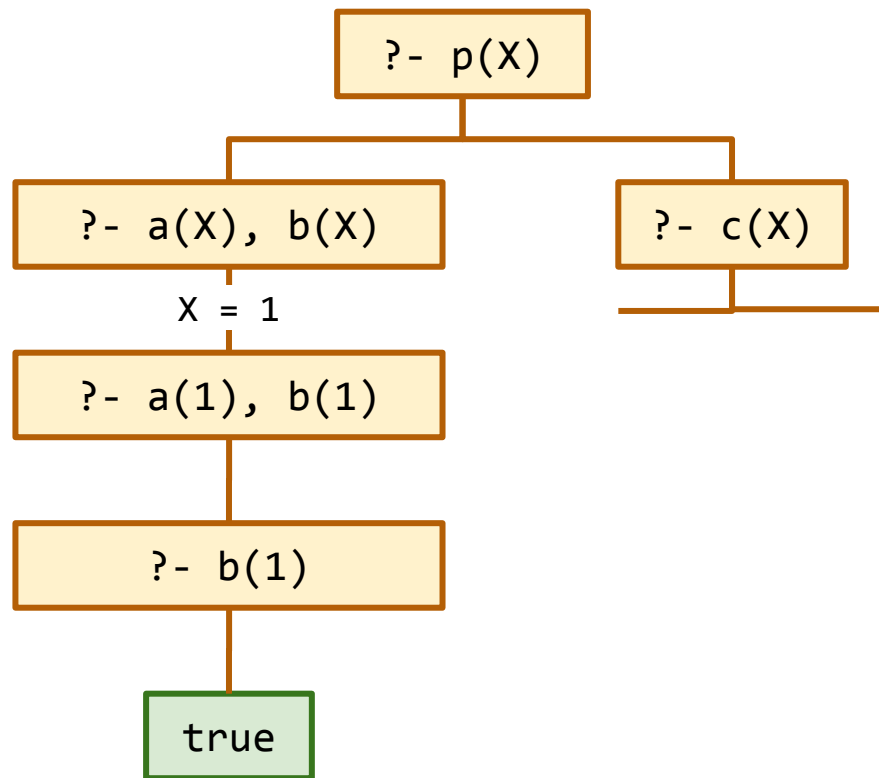


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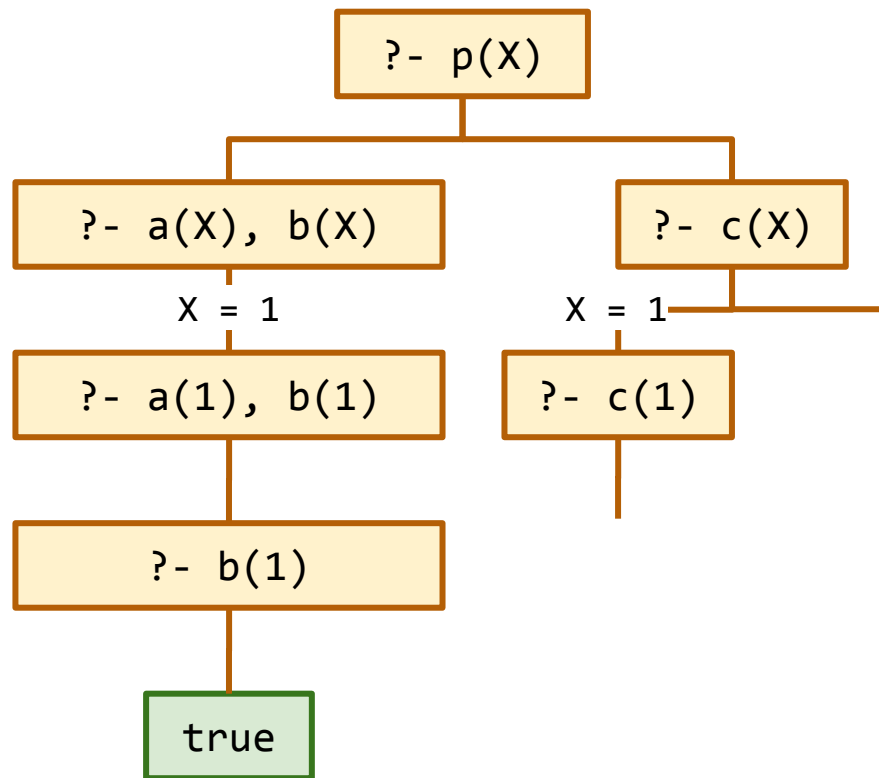


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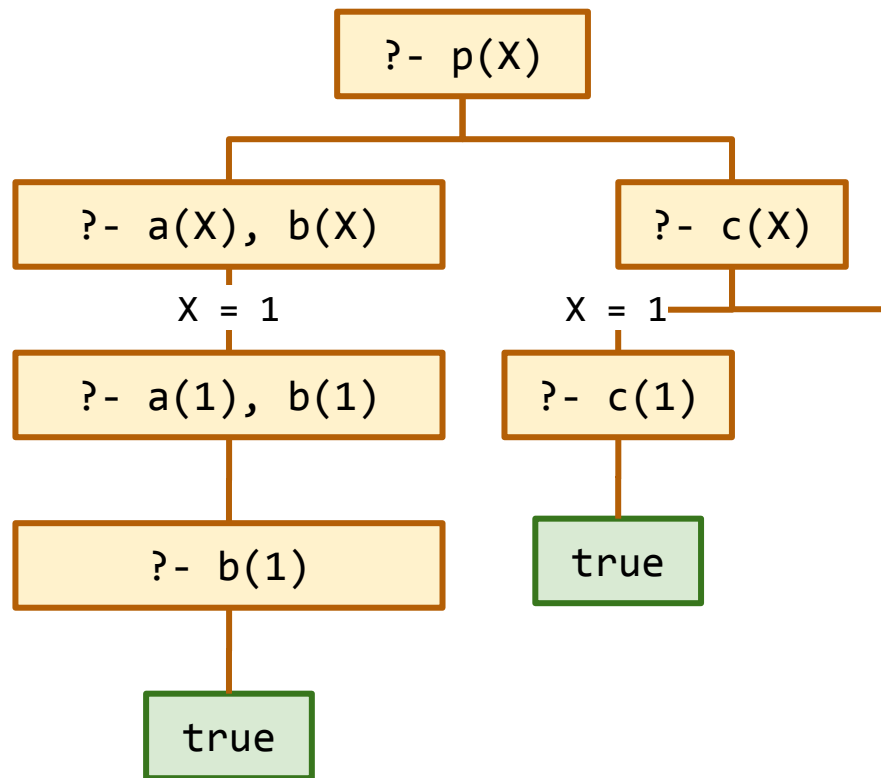


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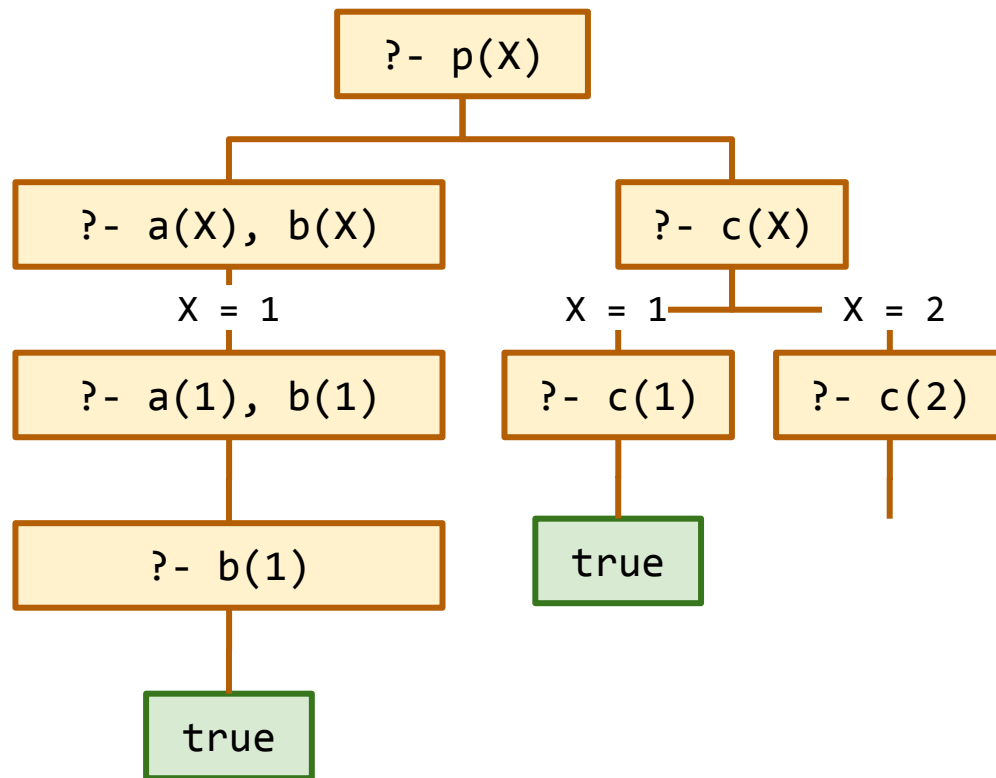


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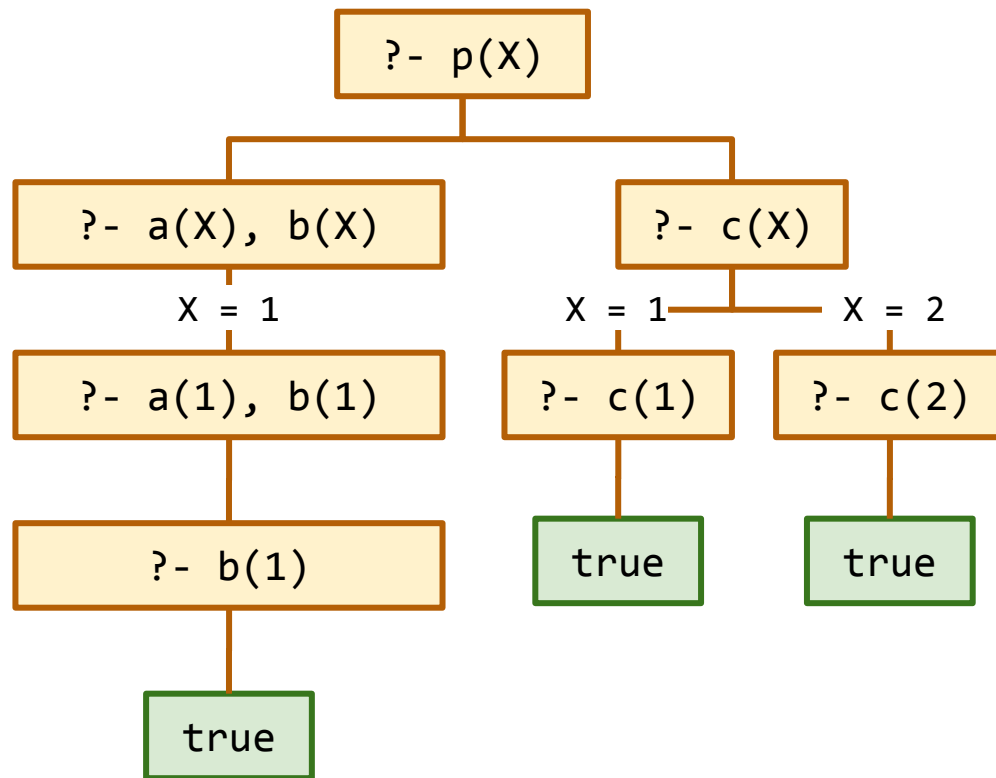


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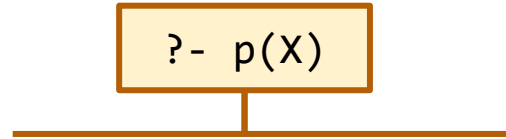
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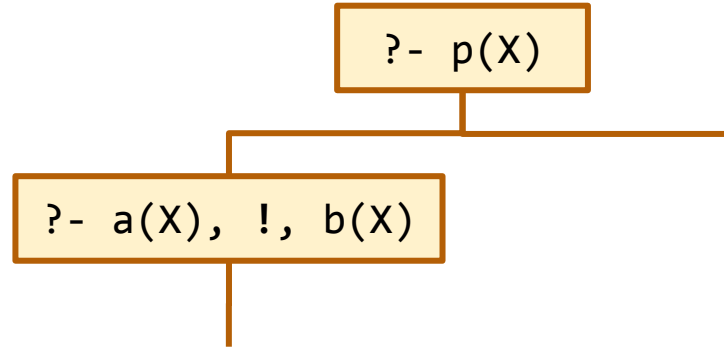


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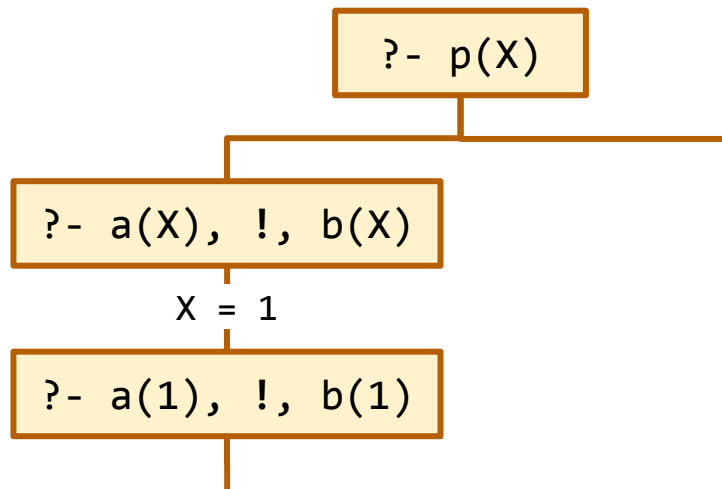


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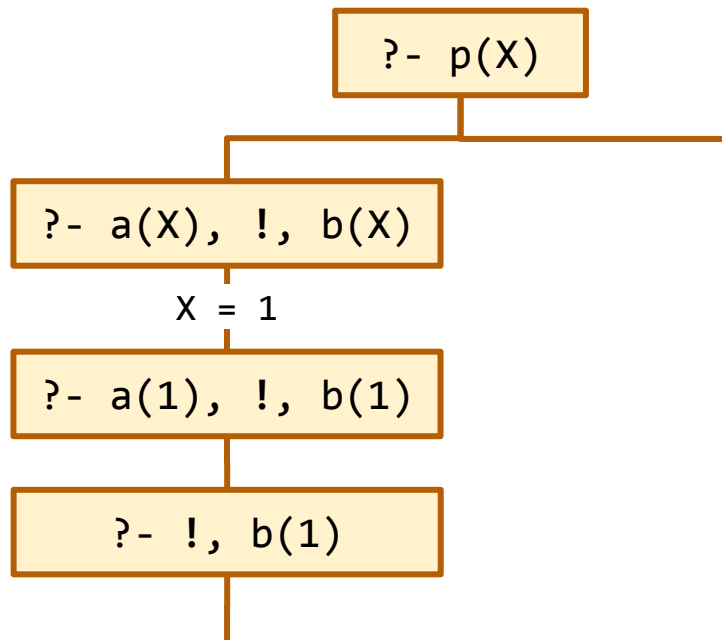


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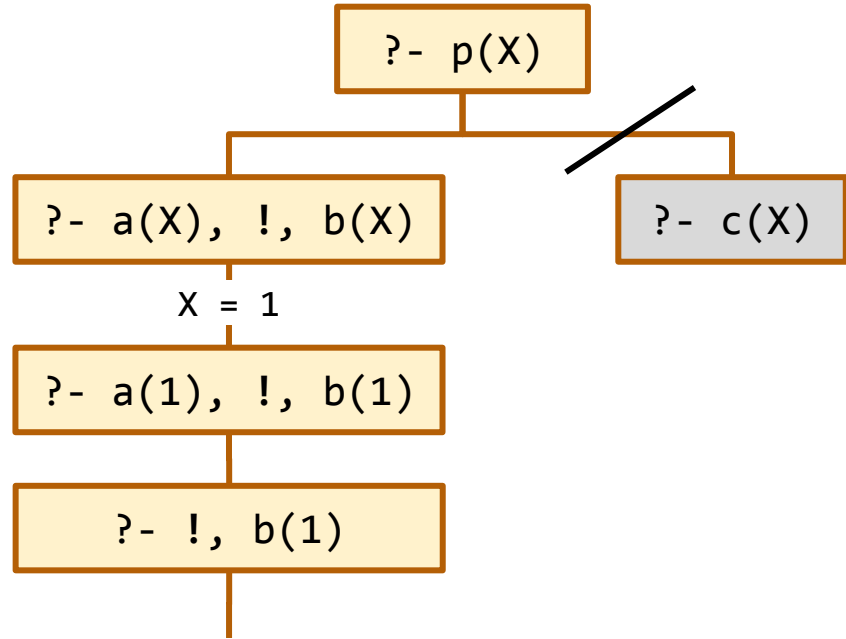


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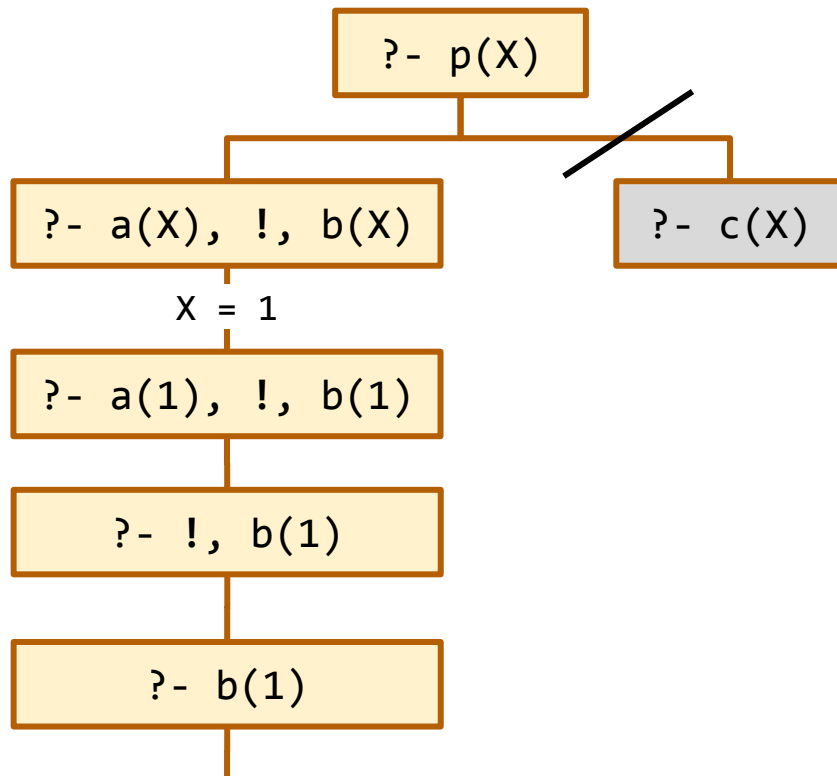


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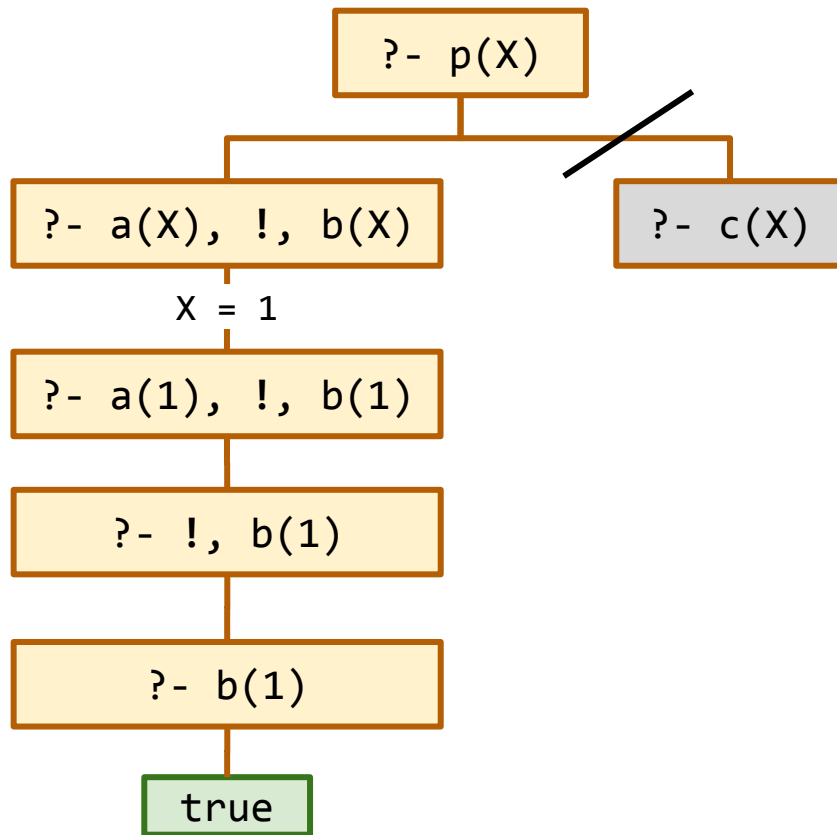


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# The Cut: general example

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a(X) :- b(X).  
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b(X) :- d(X), !, e(X).  
b(2).

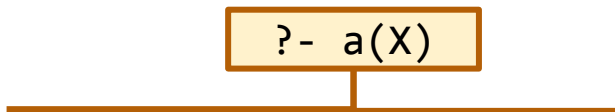
d(3). d(4).  
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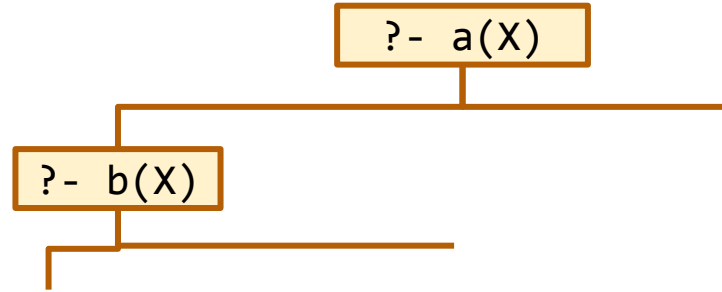


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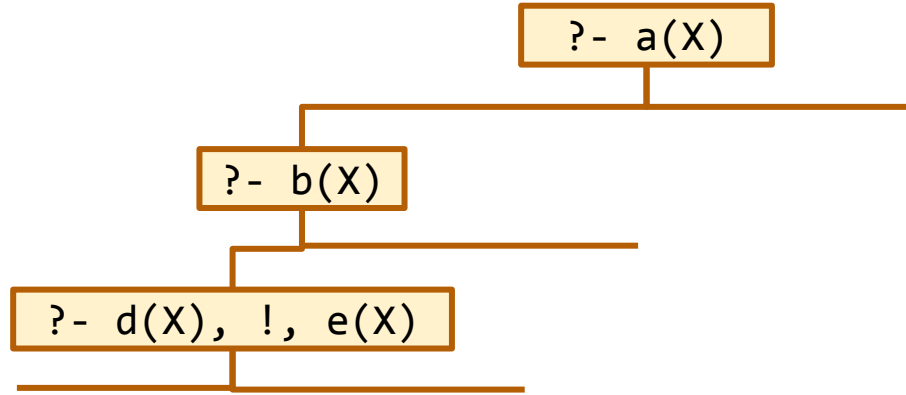


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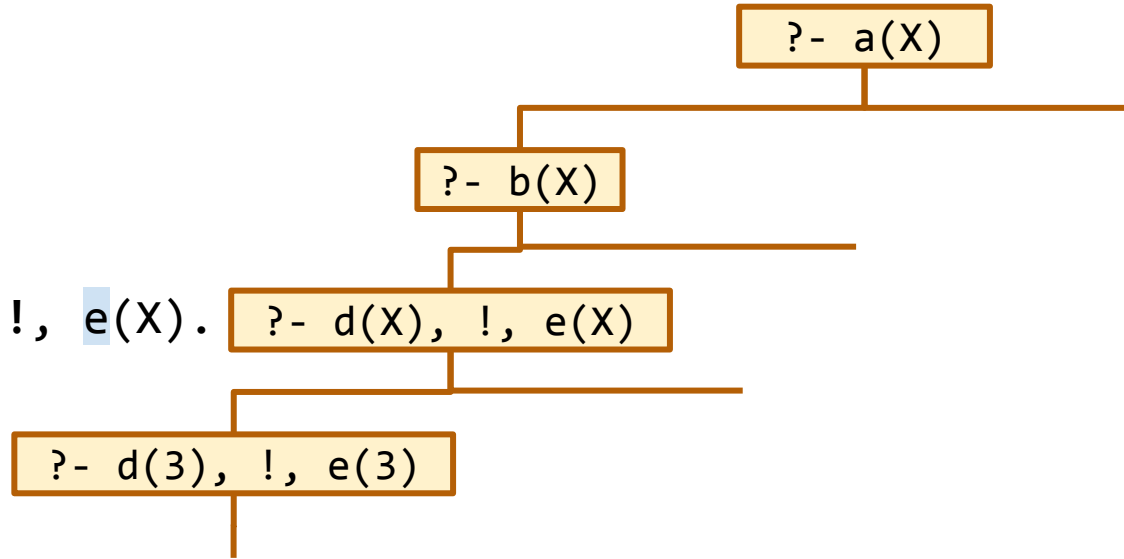


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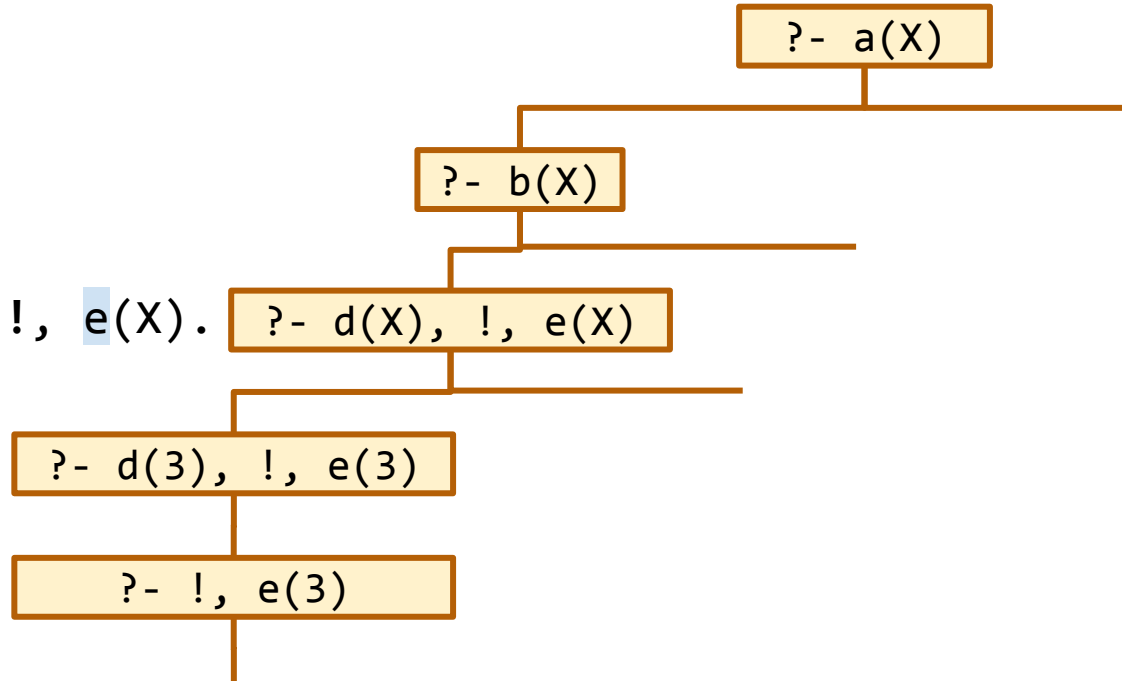


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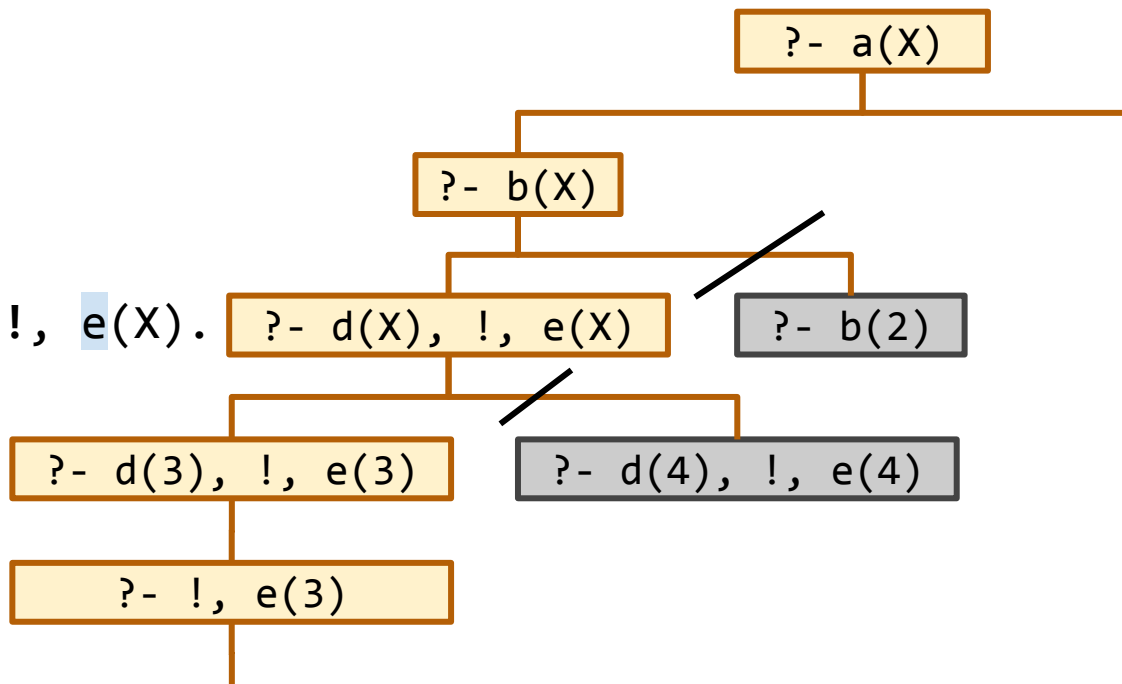


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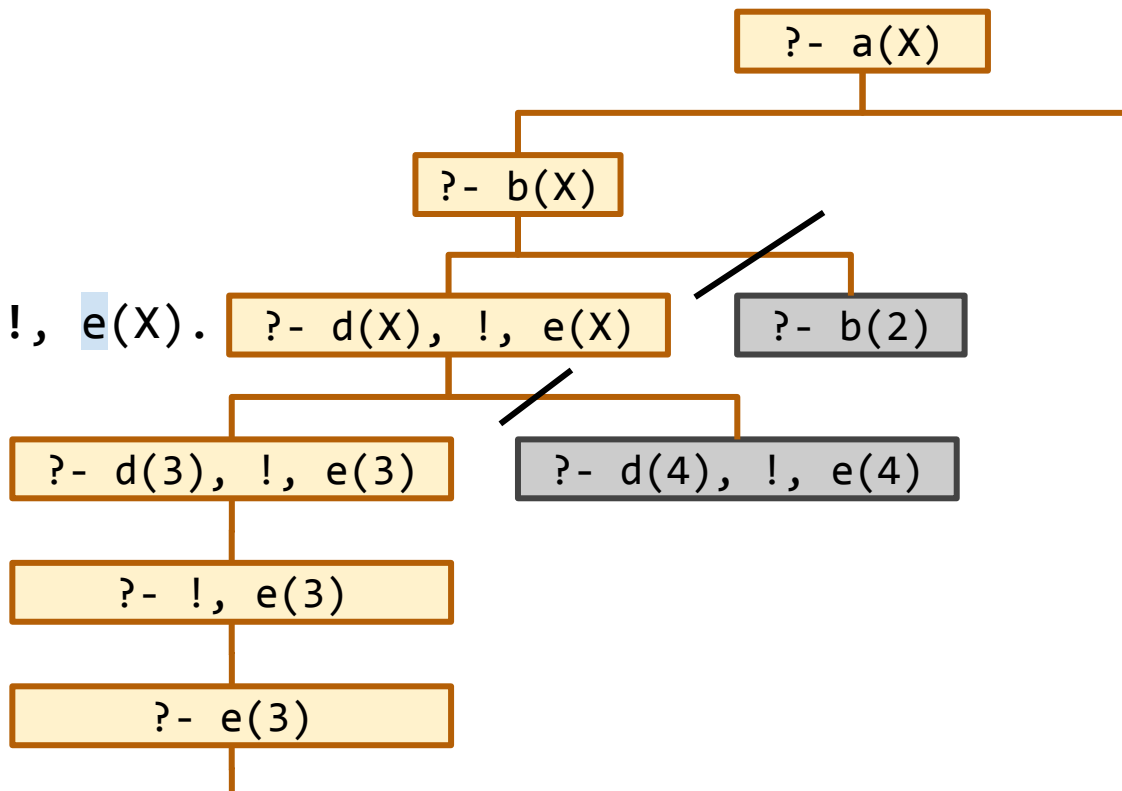


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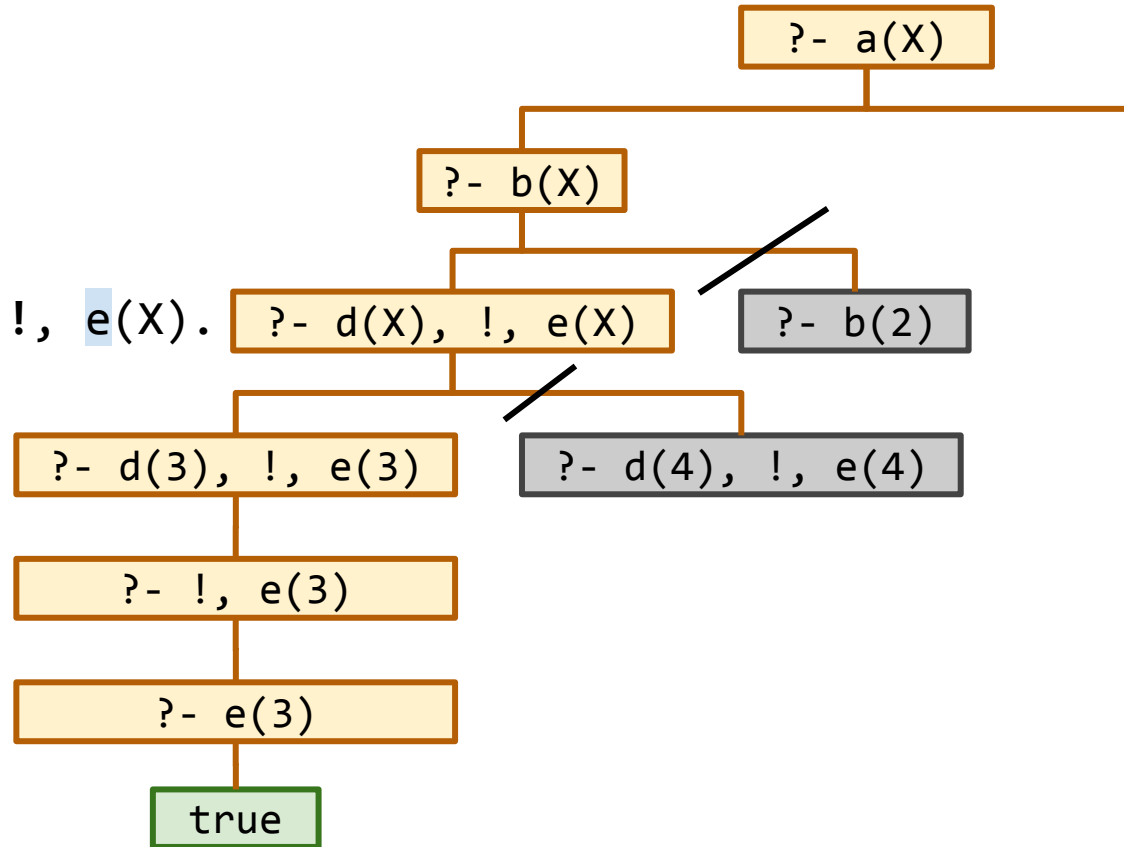


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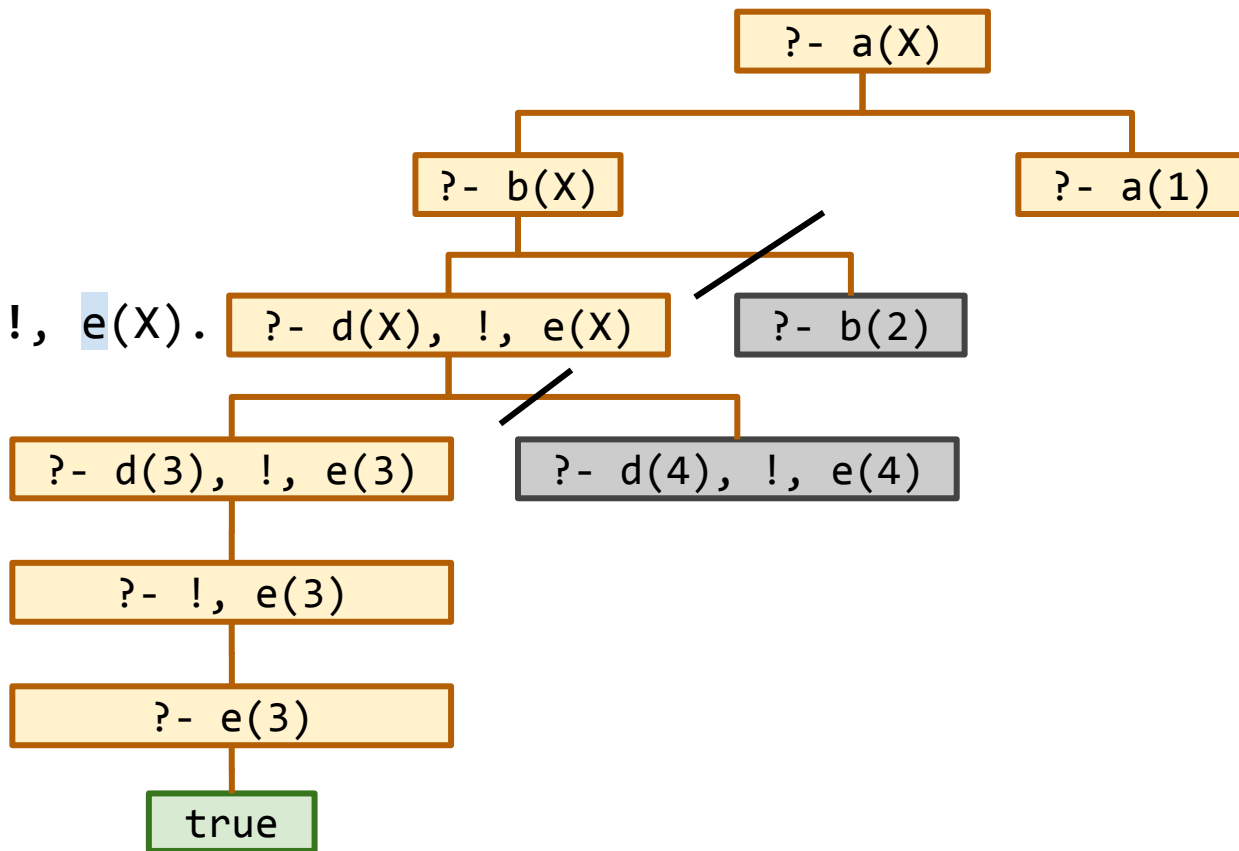


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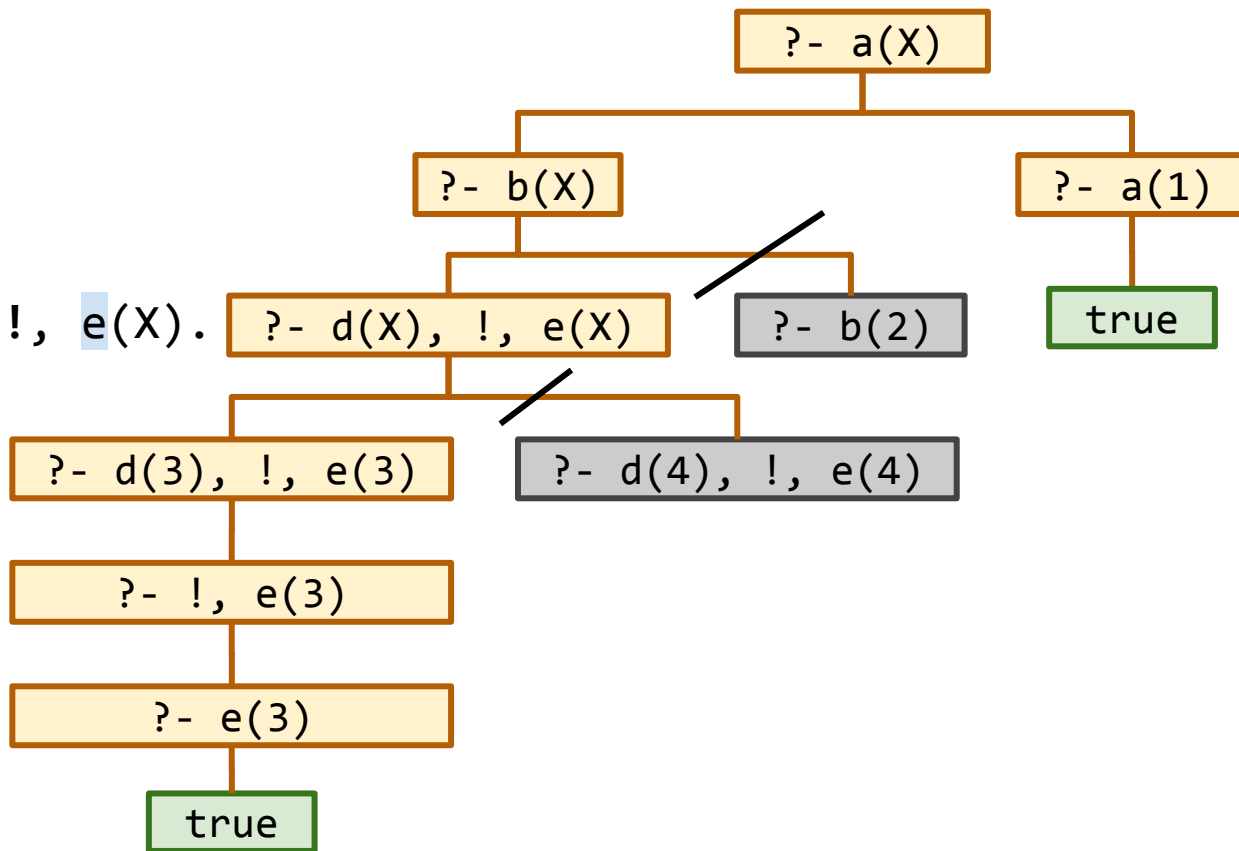


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## Using Prolog's cut for efficiency: max

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max(X, Y, X) :- X > Y.  
max(X, Y, Y) :- X <= Y.
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What happens when we execute this query?  
?- max(3, 2, X)

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## No Cut vs Green Cut vs Red Cut

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max(X, Y, X) :- X > Y.  
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Green cut. Improves efficiency.  
Safe to remove (does not change  
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max(X, Y, Y).
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Red cut. Improves efficiency.  
**Unsafe** to remove  
(will change the meaning).

## Using Prolog's cut to fix overlapping branches

```
interleave([], Y, Y).  
interleave(X, [], X).  
interleave([HX|TX], [HY|TY], [HX, HY|T])  
    :- interleave(TX, TY, T).
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← Overlapping solutions

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## Negation using cut-fail combination

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**false**

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?- notMember(4, [1, 2, 3]).
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notMember(_, []).  
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notMember(X, [_|T]) :- notMember(X, T).
```

```
?- notMember(2, [1, 2, 3]).
```

**false**

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```

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## Negation as failure

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neg(Goal) :- Goal, !, fail.  
neg(_Goal).
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## Negation as failure vs Logical negation

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```
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neg(Goal) :- Goal, !, fail.  
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false
```

```
?- notMember(4, [1, X, 3])  
false
```

**What was the most  
unclear part of the  
lecture for you?**

See Moodle

# References

1. Learn Prolog Now!