# Programming Paradigms

Lecture 11. Lists and arithmetic in Prolog

#### Outline

- Clarification on terms
- Syntax for lists
- Implementing recursive predicates on lists
- Arithmetic in Prolog

connected(1, 2).

```
connected(2, 3).

path(From, From, stop).
path(From, To, ) :- connected(From, Next).
```

```
connected(1, 2).
connected(2, 3).
```

```
path(From, From, stop).
path(From, To, go(From, To)) :- connected(From, Next).
```

connected(1, 2).

connected (1, 2).

```
connected(2, 3).

path(From, From, stop).
path(From, To, go(From, To)) :- connected(From, Next).
path(From, To, go(From, Next, Path))
   :- connected(From, Next), path(Next, To, Path).
```

connected (1, 2).

```
path(From, From, stop).
path(From, To, go(From, To)) :- connected(From, Next).
path(From, To, go(From, Next, Path))
   :- connected(From, Next), path(Next, To, Path).
?- path(1, 1, Path)
```

```
connected (1, 2).
connected(2, 3).
path(From, From, stop).
path(From, To, go(From, To)) :- connected(From, Next).
path(From, To, go(From, Next, Path))
  :- connected(From, Next), path(Next, To, Path).
?- path(1, 1, Path)
Path = stop
```

```
connected (1, 2).
connected(2, 3).
path(From, From, stop).
path(From, To, go(From, To)) :- connected(From, Next).
path(From, To, go(From, Next, Path))
  :- connected(From, Next), path(Next, To, Path).
?- path(1, 2, Path)
Path = go(1, 2)
```

```
connected (1, 2).
connected(2, 3).
path(From, From, stop).
path(From, To, go(From, To)) :- connected(From, Next).
path(From, To, go(From, Next, Path))
  :- connected(From, Next), path(Next, To, Path).
?- path(1, 3, Path)
Path = go(1, 2, go(2, 3))
```

# List syntax

[1, 2, 3]

#### List syntax

```
[1, 2, 3]
```

```
[hello, X, flight(boston, To), 10]
```

#### List syntax

```
[1, 2, 3]
[hello, X, flight(boston, To), 10]
[[], hello, X, [1, 2]]
```

List syntax: the | operator

?- [Head | Tail] = [1, 2, 3]

# List syntax: the | operator

```
?- [Head|Tail] = [1, 2, 3]
Head = 1,
Tail = [2, 3]
```

## List syntax: the | operator

```
?- [Head|Tail] = [1, 2, 3]
Head = 1,
Tail = [2, 3]
```

?- [First, Second | Tail] = [1, 2, 3, 4, 5]

## List syntax: the operator

```
?-[Head|Tail] = [1, 2, 3]
Head = 1,
Tail = [2, 3]
?- [First, Second | Tail] = [1, 2, 3, 4, 5]
First = 1,
Second = 2,
Tail = [3, 4, 5]
```

#### Anonymous variable

```
?- [X1,X2,X3,X4|Tail] = [1, 2, 3, 4, 5]
X1 = 1,
X2 = 2,
X3 = 3,
X4 = 4,
Tail = [5]
```

#### Anonymous variable

```
?- [_,X2,_,X4|_] = [1, 2, 3, 4, 5]
X2 = 2,
X4 = 4
```

member(X, [X]).

 $member(X, [X|_]).$ 

```
member(X, [X|_]).
member(X, [_|T]) :- member(X, T).
```

```
member (X, [X|_]).
member (X, [_|T]) :- member (X, T).
?- member (c, [a, b, c, d]).
true
```

```
member (X, [X|_]).
member (X, [_|T]) :- member (X, T).

?- member (c, [a, b, c, d]).
true

?- member (X, [a, b, c, d]).
```

```
member (X, [X|_]).
member (X, [_|T]) :- member (X, T).

?- member (c, [a, b, c, d]).
true

?- member (X, [a, b, c, d]).
X = a
```

```
member (X, [X|_]).
member(X, [\_|T]) :- member(X, T).
?- member(<mark>c</mark>, [a, b, c, d]).
true
?- member(X, [a, b, c, d]).
X = a
X = b
```

```
member (X, [X|_]).
member(X, [\_|T]) :- member(X, T).
?- member(<mark>c</mark>, [a, b, c, d]).
true
?- member(X, [a, b, c, d]).
X = a
X = b
X = C
```

X = d

```
member (X, [X|_]).
member(X, [\_|T]) :- member(X, T).
?- member(c, [a, b, c, d]).
true
?- member(X, [a, b, c, d]).
X = a
X = b
X = C
```

```
?- 8 is 6+2. yes
```

```
?- 8 is 6+2.
yes
?- 12 is 6*2.
yes
?- -2 is 6-8.
yes
?- 3 is 6/2.
yes
```

```
?- 8 is 6+2.
yes
?- 12 is 6*2.
yes
?- -2 is 6-8.
yes
?- 3 is 6/2.
yes
?- 1 is mod(7,2).
yes
```

```
?- X is 6+2. X = 8
```

```
?- X is 6+2. X = 8
```

?- 8 is 6+Y.

```
?- X is 6+2.
X = 8
?- 8 is 6+Y.
```

Arguments are not sufficiently instantiated In:
[1] 8 is 6+\_1634

### Arithmetic in Prolog

```
?- X is 6+2.
X = 8

six_plus(X, Y) :- X is 6+Y.
```

### Arithmetic in Prolog

```
?- X is 6+2.
X = 8

six_plus(X, Y) :- X is 6+Y.
?- six_plus(X, 2).
X = 8
```

length([], 0).

```
length([], 0).
length([_|T], N) :-
```

```
length([], 0).
length([_|T], N) :- length(T, M),
```

```
length([], 0).
length([_|T], N) :- length(T, M), N is M+1.
```

```
length([], 0).
length([_|T], N) :- length(T, M), N is M+1.
?- length([a, b, c], X)
X = 3
```

```
binary([0]).
binary([1]).
binary([0|T]) :- binary(T).
binary([1|T]) :- binary(T).
```

```
binary([0]).
binary([1]).
binary([0|T]) :- binary(T).
binary([1|T]) :- binary(T).
?- binary(X), length(X, 3).
```

```
binary([0]).
binary([1]).
binary([0|T]) :- binary(T).
binary([1|T]) :- binary(T).
?- binary(X), length(X, 3).
X = [0, 0, 0]
```

```
binary([0]).
binary([1]).
binary([0|T]) :- binary(T).
binary([1|T]) :- binary(T).
?- binary(X), length(X, 3).
X = [0, 0, 0]
X = [0, 0, 1]
```

```
binary([0]).
binary([1]).
binary([0|T]) :- binary(T).
binary([1|T]) :- binary(T).
?- binary(X), length(X, 3).
X = [0, 0, 0]
X = [0, 0, 1]
(loops indefinitely)
```

?- trace, binary(X), length(X, 3).

```
?- trace, binary(X), length(X, 3).
Exit:binary([0])
```

```
?- trace, binary(X), length(X, 3).
Exit:binary([0])
Fail:length([0], 3)
```

```
?- trace, binary(X), length(X, 3).
Exit:binary([0])
Fail:length([0], 3)
Exit:binary([1])
```

```
?- trace, binary(X), length(X, 3).
Exit:binary([0])
Fail:length([0], 3)
Exit:binary([1])
Fail:length([1], 3)
```

```
?- trace, binary(X), length(X, 3).
Exit:binary([0])
Fail:length([0], 3)
Exit:binary([1])
Fail:length([1], 3)
Exit:binary([0, 0])
```

```
?- trace, binary(X), length(X, 3).
Exit:binary([0])
Fail:length([0], 3)
Exit:binary([1])
Fail:length([1], 3)
Exit:binary([0, 0])
Fail:length([0, 0], 3)
```

```
?- trace, binary(X), length(X, 3).
Exit:binary([0])
Fail:length([0], 3)
Exit:binary([1])
Fail:length([1], 3)
Exit:binary([0, 0])
Fail:length([0, 0], 3)
Exit:binary([0, 1])
```

```
?- trace, binary(X), length(X, 3).
Exit:binary([0])
Fail:length([0], 3)
Exit:binary([1])
Fail:length([1], 3)
Exit:binary([0, 0])
Fail:length([0, 0], 3)
Exit:binary([0, 1])
Fail:length([0, 1], 3)
```

```
?- trace, binary(X), length(X, 3).
Exit:binary([0])
Fail:length([0], 3)
Exit:binary([1])
Fail:length([1], 3)
Exit:binary([0, 0])
Fail:length([0, 0], 3)
Exit:binary([0, 1])
Fail:length([0, 1], 3)
Exit:binary([0, 0, 0])
```

```
?- trace, binary(X), length(X, 3).
Exit:binary([0])
Fail:length([0], 3)
Exit:binary([1])
Fail:length([1], 3)
Exit:binary([0, 0])
Fail:length([0, 0], 3)
Exit:binary([0, 1])
Fail:length([0, 1], 3)
Exit:binary([0, 0, 0])
Exit:length([0, 0, 0], 3)
```

```
?- trace, binary(X), length(X, 3).
 Exit:binary([0])
 Fail:length([0], 3)
 Exit:binary([1])
 Fail:length([1], 3)
 Exit:binary([0, 0])
 Fail:length([0, 0], 3)
 Exit:binary([0, 1])
 Fail:length([0, 1], 3)
 Exit:binary([0, 0, 0])
 Exit:length([0, 0, 0], 3)
X = [0, 0, 0]
```

```
?- trace, binary(X), length(X, 3).
 Exit:binary([0])
                                Exit:binary([0, 0, 1])
 Fail:length([0], 3)
 Exit:binary([1])
 Fail:length([1], 3)
 Exit:binary([0, 0])
 Fail:length([0, 0], 3)
 Exit:binary([0, 1])
 Fail:length([0, 1], 3)
 Exit:binary([0, 0, 0])
 Exit:length([0, 0, 0], 3)
X = [0, 0, 0]
```

```
?- trace, binary(X), length(X, 3).
 Exit:binary([0])
                                Exit:binary([0, 0, 1])
 Fail:length([0], 3)
                                Exit:length([0, 0, 1], 3)
 Exit:binary([1])
 Fail:length([1], 3)
 Exit:binary([0, 0])
 Fail:length([0, 0], 3)
 Exit:binary([0, 1])
 Fail:length([0, 1], 3)
 Exit:binary([0, 0, 0])
 Exit:length([0, 0, 0], 3)
X = [0, 0, 0]
```

```
?- trace, binary(X), length(X, 3).
 Exit:binary([0])
                                Exit:binary([0, 0, 1])
 Fail:length([0], 3)
                                Exit:length([0, 0, 1], 3)
 Exit:binary([1])
                               X = [0, 0, 1]
 Fail:length([1], 3)
 Exit:binary([0, 0])
 Fail:length([0, 0], 3)
 Exit:binary([0, 1])
 Fail:length([0, 1], 3)
 Exit:binary([0, 0, 0])
 Exit:length([0, 0, 0], 3)
X = [0, 0, 0]
```

```
?- trace, binary(X), length(X, 3).
 Exit:binary([0])
                                Exit:binary([0, 0, 1])
 Fail:length([0], 3)
                                Exit:length([0, 0, 1], 3)
 Exit:binary([1])
                               X = [0, 0, 1]
 Fail:length([1], 3)
                                Exit:binary([0, 0, 0, 0])
 Exit:binary([0, 0])
                                Fail:length([0, 0, 0, 0], 3)
 Fail:length([0, 0], 3)
 Exit:binary([0, 1])
 Fail:length([0, 1], 3)
 Exit:binary([0, 0, 0])
 Exit:length([0, 0, 0], 3)
X = [0, 0, 0]
```

```
?- trace, binary(X), length(X, 3).
 Exit:binary([0])
                                Exit:binary([0, 0, 1])
 Fail:length([0], 3)
                                Exit:length([0, 0, 1], 3)
 Exit:binary([1])
                               X = [0, 0, 1]
 Fail:length([1], 3)
                                Exit:binary([0, 0, 0, 0])
 Exit:binary([0, 0])
                                Fail:length([0, 0, 0, 0], 3)
 Fail:length([0, 0], 3)
                                Exit:binary([0, 0, 0, 1])
 Exit:binary([0, 1])
                                Fail:length([0, 0, 0, 1], 3)
 Fail:length([0, 1], 3)
 Exit:binary([0, 0, 0])
 Exit:length([0, 0, 0], 3)
X = [0, 0, 0]
```

```
?- trace, binary(X), length(X, 3).
 Exit:binary([0])
                                Exit:binary([0, 0, 1])
 Fail:length([0], 3)
                                Exit:length([0, 0, 1], 3)
 Exit:binary([1])
                               X = [0, 0, 1]
 Fail:length([1], 3)
                                Exit:binary([0, 0, 0, 0])
 Exit:binary([0, 0])
                                Fail:length([0, 0, 0, 0], 3)
 Fail:length([0, 0], 3)
                                Exit:binary([0, 0, 0, 1])
 Exit:binary([0, 1])
                                Fail:length([0, 0, 0, 1], 3)
 Fail:length([0, 1], 3)
                                Exit:binary([0, 0, 0, 0, 0])
 Exit:binary([0, 0, 0])
                                Fail:length([0, 0, 0, 0, 0], 3)
 Exit:length([0, 0, 0], 3)
X = [0, 0, 0]
```

```
binary([0]).
binary([1]).
binary([0|T]) :- binary(T).
binary([1|T]) :- binary(T).
?- length(X, 3), binary(X).
```

```
binary([0]).
binary([1]).
binary([0|T]) :- binary(T).
binary([1|T]) :- binary(T).
?- length(X, 3), binary(X).
X = [0, 0, 0]
```

```
binary([0]).
binary([1]).
binary([0|T]) :- binary(T).
binary([1|T]) :- binary(T).
?- length(X, 3), binary(X).
X = [0, 0, 0]
X = [0, 0, 1]
```

```
binary([0]).
binary([1]).
binary([0|T]) :- binary(T).
binary([1|T]) :- binary(T).
?- length(X, 3), binary(X).
X = [0, 0, 0]
X = [0, 0, 1]
X = [1, 1, 1]
```

```
binary([0]).
binary([1]).
binary([0|T]) :- binary(T).
binary([1|T]) :- binary(T).
?- length(X, 3), binary(X).
X = [0, 0, 0]
X = [0, 0, 1]
X = [1, 1, 1]
false
```

## Appending lists

append([], Y, Y).

```
append([], Y, Y).
append([H|X], Y, [H|Z]) :-
```

```
append([], Y, Y).
append([H|X], Y, [H|Z]) :- append(X, Y, Z).
```

```
append([], Y, Y).
append([H|X], Y, [H|Z]) :- append(X, Y, Z).
?- append([1,2], [3,4], R)
```

```
append([], Y, Y).
append([H|X], Y, [H|Z]) :- append(X, Y, Z).
?- append([1,2], [3,4], R)
R = [1,2,3,4]
```

```
append([], Y, Y).
append([H|X], Y, [H|Z]) :- append(X, Y, Z).
?- append([1,2], [3,4], R)
R = [1,2,3,4]
?- append(X, Y, [1, 2, 3])
```

```
append ([], Y, Y).
append ([H|X], Y, [H|Z]) :- append (X, Y, Z).
?- append ([1,2], [3,4], R)
R = [1,2,3,4]
?- append (X, Y, [1, 2, 3])
X = [], Y = [1, 2, 3]
```

```
append([], Y, Y).
append([H|X], Y, [H|Z]) :- append(X, Y, Z).
?- append([1,2], [3,4], R)
R = [1,2,3,4]
?- append(X, Y, [1, 2, 3])
X = [], Y = [1, 2, 3]
X = [1], Y = [1, 2]
```

```
append([], Y, Y).
append([H|X], Y, [H|Z]) :- append(X, Y, Z).
?- append([1,2], [3,4], R)
R = [1,2,3,4]
?- append(X, Y, [1, 2, 3])
X = [], Y = [1, 2, 3]
X = [1], Y = [1, 2]
X = [1, 2], Y = [1]
```

```
append([], Y, Y).
append([H|X], Y, [H|Z]) :- append(X, Y, Z).
?- append([1,2], [3,4], R)
R = [1,2,3,4]
?- append(X, Y, [1, 2, 3])
X = [], Y = [1, 2, 3]
X = [1], Y = [1, 2]
X = [1, 2], Y = [1]
X = [1, 2, 3], Y = []
```

reverse([], []).

```
reverse([], []).
reverse([H|T], Z) :-
```

```
reverse([], []).
reverse([H|T], Z) :- reverse(T, X),
```

```
reverse([], []).
reverse([H|T], Z) :- reverse(T, X), append(X, [H], Z).
```

```
reverse([], []).
reverse([H|T], Z) :- reverse(T, X), append(X, [H], Z).
```

reverse([a, b, c], Z)

reverse([b, c], X1), append(X1, [a], Z)

```
reverse([], []).
reverse([H|T], Z) :- reverse(T, X), append(X, [H], Z).
1. reverse([a, b, c], Z)
```

```
reverse([], []).
reverse([H|T], Z) :- reverse(T, X), append(X, [H], Z).

1. reverse([a, b, c], Z)
2. reverse([b, c], X1), append(X1, [a], Z)
```

reverse([c], X2), append(X2, [b], X1), append(X1, [a], Z)

```
reverse([], []).
reverse([H|T], Z) :- reverse(T, X), append(X, [H], Z).
```

- 1. reverse([a, b, c], Z)
  2. reverse([b, c], X1), append(X1, [a], Z)
- 2. reverse([b, c], X1), append(X1, [a], Z)
- 3. reverse([<mark>c</mark>], X2), append(X2, [<mark>b</mark>], X1), append(X1, [<mark>a</mark>], Z)
- 4. reverse([], X3), append(X3, [c], X2), append(X2, [b], X1), append(X1, [a], Z

```
reverse([], []).
reverse([H|T], Z) :- reverse(T, X), append(X, [H], Z).

1. reverse([a, b, c], Z)
2. reverse([b, c], X1), append(X1, [a], Z)
3. reverse([c], X2), append(X2, [b], X1), append(X1, [a], Z)
4. reverse([], X3), append(X3, [c], X2), append(X2, [b], X1), append(X1, [a], Z)
5. X3 = []
    append([], [c], X2), append(X2, [b], X1), append(X1, [a], Z)
```

append([c], [b], X1), append(X1, [a], Z)

reverse([], []).

6. X2 = [c]

```
reverse([H|T], Z) :- reverse(T, X), append(X, [H], Z).

1. reverse([a, b, c], Z)
2. reverse([b, c], X1), append(X1, [a], Z)
3. reverse([c], X2), append(X2, [b], X1), append(X1, [a], Z)
4. reverse([], X3), append(X3, [c], X2), append(X2, [b], X1), append(X1, [a], Z)
5. X3 = []
```

append([], [c], X2), append(X2, [b], X1), append(X1, [a], Z)

append([c], [b], X1), append(X1, [a], Z)

reverse([], []).

6. X2 = [c]

7. X1 = [c, b]

append([c, b], [a], Z)

```
reverse([H|T], Z) :- reverse(T, X), append(X, [H], Z).

1. reverse([a, b, c], Z)
2. reverse([b, c], X1), append(X1, [a], Z)
3. reverse([c], X2), append(X2, [b], X1), append(X1, [a], Z)
4. reverse([], X3), append(X3, [c], X2), append(X2, [b], X1), append(X1, [a], Z)
5. X3 = []
    append([], [c], X2), append(X2, [b], X1), append(X1, [a], Z)
```

reverse([], []).

7. X1 = [c, b]

8. Z = [c, b, a]

append([c, b], [a], Z)

```
reverse([H|T], Z) :- reverse(T, X), append(X, [H], Z).

1. reverse([a, b, c], Z)
2. reverse([b, c], X1), append(X1, [a], Z)
3. reverse([c], X2), append(X2, [b], X1), append(X1, [a], Z)
4. reverse([], X3), append(X3, [c], X2), append(X2, [b], X1), append(X1, [a], Z)
5. X3 = []
    append([], [c], X2), append(X2, [b], X1), append(X1, [a], Z)
6. X2 = [c]
    append([c], [b], X1), append(X1, [a], Z)
```

```
reverse([], Acc, Acc).
reverse([Head|Tail], Acc, Result)
:- reverse(Tail, [Head|Acc], Result).
```

```
reverse([], Acc, Acc).
reverse([Head|Tail], Acc, Result)
   :- reverse(Tail, [Head|Acc], Result).

1. reverse([a, b, c], [], Z)
```

2. reverse([b, c], [a], Z)

```
reverse([], Acc, Acc).
reverse([Head|Tail], Acc, Result)
:- reverse(Tail, [Head|Acc], Result).

1. reverse([a, b, c], [], Z)
```

```
reverse ([], Acc, Acc).
reverse ([Head|Tail], Acc, Result)
:- reverse (Tail, [Head|Acc], Result).

1. reverse([a, b, c], [], Z)
2. reverse([b, c], [a], Z)
3. reverse([c], [b, a], Z)
```

```
reverse([], Acc, Acc).
reverse([Head|Tail], Acc, Result)
:- reverse(Tail, [Head|Acc], Result).

1. reverse([a, b, c], [], Z)
2. reverse([b, c], [a], Z)
3. reverse([c], [b, a], Z)
4. reverse([], [c, b, a], Z)
```

```
reverse([], Acc, Acc).
reverse([Head|Tail], Acc, Result)
    :- reverse(Tail, [Head|Acc], Result).
  reverse([a, b, c], [], Z)
  reverse([<mark>b</mark>, c], [a], Z)
3. reverse([c], [b, a], Z)
4. reverse([], [c, b, a], Z)
5. Z = [c, b, a]
```

## Homework (self-study)

- Read about Declarative Debugging of Prolog Programs
   https://www.metalevel.at/prolog/debugging
- 2. Read **Chapters 6 and 10** of Learn Prolog Now! <a href="http://www.let.rug.nl/bos/lpn/lpnpage.php?pagetype=html&pageid=lpn-htmlch1">http://www.let.rug.nl/bos/lpn/lpnpage.php?pagetype=html&pageid=lpn-htmlch1</a>
- Work through the exercises from both chapters, using SWISH <a href="https://swish.swi-prolog.org">https://swish.swi-prolog.org</a>

# What was the most unclear part of the lecture for you?

See Moodle

## References

1. Learn Prolog Now!