

## LAPLACE TRANSFORM

### I. FIND THE LAPLACE TRANSFORM OF FOLLOWING FUNCTIONS:

1.  $f(t) = (t-1)^4, t > 4; f(t) = 0, 0 < t < 4$
2.  $f(t) = t, 0 < t < 1/2; f(t) = t-1, 1/2 < t < 1; f(t) = 0, t > 1$  [Ans:  $\frac{1}{s^2} - \frac{e^{-s}}{s^2} - \frac{e^{-s/2}}{s}$ ]
3.  $f(t) = 0, 0 < t < \pi; f(t) = \sin^2(t - \pi), t > \pi$  [Ans:  $\frac{e^{-\pi s}}{2s} - \frac{s \cdot e^{-\pi s}}{s^2 + 4}$ ]
4.  $\cos t \cdot \cos 2t \cdot \cos 3t$  [Ans:  $\frac{1}{4} \left( \frac{1}{s} + \frac{s}{s^2 + 2^2} + \frac{s}{s^2 + 4^2} + \frac{s}{s^2 + 6^2} \right)$ ]
5.  $(\sqrt{t} - 1)^2$  [Ans:  $\frac{1}{s^2} - \frac{\sqrt{\pi}}{s^{3/2}} + \frac{1}{s}$ ]
6.  $\frac{\cos \sqrt{t}}{\sqrt{t}}$  [Ans:  $\sqrt{\frac{\pi}{s}} \cdot e^{-1/4s}$ ]
7. If  $L[\sin \sqrt{t}] = \frac{\sqrt{\pi}}{2s\sqrt{s}} \cdot e^{-1/4s}$ , find  $L[\sin 2\sqrt{t}]$  [Ans:  $\frac{\sqrt{\pi}}{s\sqrt{s}} \cdot e^{-1/s}$ ]
8.  $\sinh(t/2) \cdot \sin(\sqrt{3}t/2)$  [Ans:  $\frac{\sqrt{3}}{2} \cdot \frac{s}{(s^4 + s^2 + 1)}$ ]
9.  $e^{4t} \sin^3 t$  [Ans:  $\frac{6}{(s^2 - 8s + 17)(s^2 - 8s + 25)}$ ]
10.  $\frac{\cos 2t \cdot \sin t}{e^t}$  [Ans:  $\frac{s^2 + 2s - 2}{(s^2 + 2s + 10)(s^2 + 2s + 2)}$ ]
11.  $e^{-4t} \sinh t \cdot \sin t$  [Ans:  $\frac{2(s+4)}{(s^2 + 6s + 10)(s^2 + 10s + 26)}$ ]
12.  $e^{2t} (1+t)^2$  [Ans:  $\frac{1}{(s-2)} + \frac{2}{(s-2)^2} + \frac{2}{(s-2)^3}$ ]
13. If  $L[f(t)] = \frac{s}{s^2 + s + 4}$ , find  $L[e^{-3t} f(2t)]$  [Ans:  $\frac{s+3}{s^2 + 8s + 10}$ ]
14.  $(1 + te^{-t})^3$  [Ans:  $\frac{1}{s} - \frac{3}{(s+1)^2} + \frac{6}{(s+2)^3} + \frac{6}{(s+3)^3}$ ]
15.  $t \sin^3 t$  [Ans:  $24 \cdot \frac{s(s+5)}{(s^2 + 1)^2 (s^2 + 9)^2}$ ]

16.  $t^5 \cosh t$

[Ans:  $60 \left( \frac{1}{(s-1)^6} + \frac{1}{(s+1)^6} \right)$ ]

17.  $t \sqrt{1 + \sin t}$

[Ans:  $4 \frac{(4s^2 + 4s - 1)}{(4s^2 + 1)^2}$ ]

18.  $t \left( \frac{\sin t}{e^t} \right)^2$

[Ans:  $\frac{1}{2} \left( -\frac{1}{(s+2)^2} + \frac{s^2 + 4s}{(s^2 + 4s + 8)} \right)$ ]

19. If  $L[f(t)] = \frac{s+3}{s^2+s+1}$ , find  $L[tf(2t)]$  [Ans:  $\frac{s^2+12s+8}{(s^2+2s+4)^2}$ ]

20.  $t e^{-2t} \sinh 4t$

[Ans:  $\frac{8(s+2)}{(s^2+4s-12)^2}$ ]

21.  $t \cos(\omega t - \alpha)$

[Ans:  $\frac{(s^2 - \omega^2) \cos \alpha - 2\omega s \sin \alpha}{(\omega^2 + s^2)^2}$ ]

22.  $(t \sinh 2t)^2$

[Ans:  $\frac{1}{2} \left( \frac{1}{(s-4)^3} + \frac{1}{(s+4)^3} \right)$ ]

23.  $(t + \sin 2t)^2$

[Ans:  $\frac{2}{s^3} + \frac{8s}{(s^2+4)^2} + \frac{1}{2s} - \frac{s}{2(s^2+4^2)}$ ]

24.  $\frac{1}{t} (1 - \cos t)$

[Ans:  $\frac{1}{2} \log \left( \frac{s^2+1}{s^2} \right)$ ]

25.  $\frac{1}{t} e^{-t} \sin t$

[Ans:  $\cot^{-1}(s+1)$ ]

26.  $\frac{\sin^2 2t}{t}$

[Ans:  $\frac{1}{4} \log \left( \frac{s^2+16}{s^2} \right)$ ]

27.  $\frac{1 - \cos t}{t^2}$

[Ans:  $\frac{\pi}{2} - \frac{s}{2} \log \left( \frac{s^2+1}{s^2} \right) - \tan^{-1} s$ ]

28. Find the Laplace transform of  $\frac{\sin at}{t}$ . Does Laplace transform of  $\frac{\cos at}{t}$  exist?  
[Ans:  $\cot^{-1} \frac{s}{a}$ , does not exist]

29.  $\frac{\cosh 2t \sin 2t}{t}$

[Ans:  $\pi + \tan^{-1} \left( \frac{s-2}{2} \right) + \tan^{-1} \left( \frac{s+2}{2} \right)$ ]

30.  $\frac{e^{-at} - \cos at}{t}$

[Ans:  $\log \left( \frac{\sqrt{s^2+a^2}}{s+a} \right)$ ]

31. Given that  $f(t) = t+1, 0 \leq t \leq 2$ , &  $f(t) = 3, t > 2$  find  $L[f(t)]$ ,  $L[f'(t)]$  &  $L[f''(t)]$   
[Ans:  $\frac{1}{s} + \frac{1}{s^2} (1 - e^{-2s})$ ,  $\frac{1}{s} (1 - e^{-2s})$ ,  $s^2 \left[ \frac{1}{s} + \frac{1}{s^2} (1 - e^{-2s}) \right] - s - 1$ ]

32. Find the Laplace transform of  $\frac{d}{dt} \left( \frac{\sin 3t}{t} \right)$  [Ans:  $s \cot^{-1}(s/3) - 3$ ]

33.  $\operatorname{erf} \sqrt{t}$

[Ans:  $\frac{1}{s\sqrt{s+1}}$ ]

34.  $\operatorname{erf} 2\sqrt{t}$

[Ans:  $\frac{2}{s\sqrt{s+4}}$ ]

35.  $e^{3t} t \operatorname{erf} \sqrt{t}$

[Ans:  $\frac{3s-7}{2(s-3)^2(s-2)^{3/2}}$ ]

36.  $\int_0^t \int_0^t \int_0^t t \sin t (dt)^3$

[Ans:  $\frac{2}{s^2(s^2+1)^2}$ ]

37.  $\int_0^t u e^{-3u} \cos^2 2u du$

[Ans:  $\frac{1}{2s(s+3)^2} + \frac{s^2+6s-7}{2s(s^2+6s+25)^2}$ ]

38.  $\int_0^t \frac{1-e^{-au}}{u} du$

[Ans:  $\frac{1}{s} \log\left(\frac{s-a}{s}\right)$ ]

39.  $t^{-1} \int_0^t e^{-u} \sin u du$

[Ans:  $\frac{1}{4} \log\left(\frac{s^2+2s+2}{s^2}\right) - \frac{1}{2} \cot^{-1}(s+1)$ ]

40.  $e^{-4t} \int_0^t u \sin 3u du$

[Ans:  $\frac{6}{(s^2+8s+25)^2}$ ]

41.  $\cosh t \int_0^t e^u \cosh u du$

[Ans:  $\frac{1}{2} \left[ \frac{s-2}{(s-1)^2(s-3)} + \frac{s}{(s+1)^2(s-1)} \right]$ ]

42.  $\int_0^t u e^{-3u} \sin^2 u du$

[Ans:  $\frac{1}{2s} \left[ \frac{1}{(s+3)^2} + \frac{s^2+6s+5}{(s^2+6s+13)^2} \right]$ ]

43.  $\frac{1}{t} (\cos at - \cos bt)$

[Ans:  $\frac{1}{2} \log\left(\frac{s^2+b^2}{s^2+a^2}\right)$ ]

44. Find  $L\{\cosh 2t \cdot \operatorname{erf} 3\sqrt{t}\}$  if  $L\{\operatorname{erf} \sqrt{t}\} = \frac{1}{s\sqrt{s+1}}$  [Ans:

$\frac{1}{2} \left[ \frac{3}{(s+2)\sqrt{s+7}} + \frac{3}{(s-2)\sqrt{s+11}} \right]$ ]

45. If  $L\left(2\sqrt{\frac{t}{\pi}}\right) = \frac{1}{s^{3/2}}$ , show that  $L\left(\frac{1}{\sqrt{\pi t}}\right) = \frac{1}{\sqrt{s}}$

46. A function  $f(t)$  obeys the equation  $f(t) + 2 \int_0^t f(t) dt = \cosh 2t$  find the Laplace

transform of  $f(t)$

[Ans:  $\frac{s^2}{(s^2-4)(s+2)}$ ]

## II. EVALUATE THE FOLLOWING INTEGRALS USING LAPLACE TRANSFORM:

1.  $\int_0^\infty e^{-2t} \sin^3 t dt$

[Ans: 6/65]

2. If  $\int_0^\infty e^{-2t} \sin(t+\alpha) \cos(t-\alpha) dt = 3/8$  then find  $\alpha$ .

[Ans:  $\pi/4$ ]

$$3. \int_0^{\infty} e^{-3t} t \sin t \, dt$$

[Ans: 3/50]

$$4. \text{ If } L[J_0(t)] = \frac{1}{\sqrt{s^2 + 1}}, \text{ prove that } \int_0^{\infty} e^{-3t} t J_0(4t) \, dt = 3/125$$

$$5. \int_0^{\infty} \frac{t^2 \sin 3t}{e^{2t}} \, dt$$

[Ans: 18/2197]

$$6. \int_0^{\infty} \frac{\cos at - \cos bt}{t} \, dt$$

[Ans:  $\log \frac{b}{a}$ ]

$$7. \int_0^{\infty} e^{-st} \frac{\sin^2(at/2)}{t} \, dt$$

[Ans:  $\frac{1}{2} \log \left( \frac{s^2 + a^2}{s^2} \right)$ ]

$$8. \text{ Prove that } \int_0^{\infty} e^{-st} \frac{\sin t \sinh t}{t} \, dt = \frac{1}{2} \tan^{-1} \left( \frac{2a}{1 + s^2 - a^2} \right)$$

$$9. \int_0^{\infty} \frac{e^{-t} - \cos t}{t e^{4t}} \, dt$$

[Ans:  $\log \frac{\sqrt{17}}{5}$ ]

$$10. \text{ Prove that } \int_0^{\infty} \frac{\sin 2t + \sin 3t}{t e^t} \, dt = \frac{3\pi}{4}$$

$$11. \int_0^{\infty} e^{-2t} \sinh t \frac{\sin t}{t} \, dt$$

[Ans:  $\frac{1}{2} \tan^{-1} \frac{1}{2}$ ]

$$12. \int_0^{\infty} e^{-t} \left( \int_0^t u^2 \sinh u \cosh u \, du \right) \, dt$$

[Ans:  $-\frac{2}{125}$ ]

$$13. \int_0^{\infty} e^{-4t} \left( \cosh t \int_0^t e^u \cosh u \, du \right) \, dt$$

[Ans: 31/225]

$$14. \text{ Prove that } \int_0^{\infty} e^{-st} \frac{\sin bt + \sin at}{t} \, dt = \pi - \tan^{-1} \left( \frac{s(a+b)}{ab - s^2} \right)$$

$$15. \int_0^{\infty} e^{-t} \sin^5 t \, dt$$

[Ans:  $\frac{3}{8}$ ]

$$16. \int_0^{\infty} \frac{\cos 4t - \cos 3t}{t} \, dt$$

[Ans:  $\log \frac{3}{4}$ ]

$$17. \int_0^{\infty} e^{-t} t^3 \sin t \, dt$$

[Ans: 0]

$$18. \int_{t=0}^{\infty} \int_{u=0}^t \frac{e^{-t} \sin u}{u} \, du \, dt$$

[Ans:  $\frac{\pi}{4s}$ ]