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Batch:	Roll No.: 16010121045
Experiment / assignment / tutorial No. 4	
Grade: AA/ AB / BB / BC / CC / CD / DD	
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Q1) $\dfrac{5s^2+8s-1}{(s+3)(s^2+1)}$ Find laplace inverse.

A) $\phi(s) = \dfrac{5s^2+8s-1}{(s+3)(s^2+1)}$

$$= \frac{3s}{(s^2+1)} - \frac{1}{s^2+1} + \frac{2}{s+3} \quad \text{(By Partial fraction)}$$

$$\therefore L^{-1}[\phi(s)] = L^{-1}\left[\frac{3s}{s^2+1} - \frac{1}{s^2+1} + \frac{2}{s+3}\right]$$

$$= 3L^{-1}\left[\frac{s}{s^2+1}\right] - L^{-1}\left[\frac{1}{s^2+1}\right] + 2L^{-1}\left[\frac{1}{s+3}\right]$$

(By linearity prop)

$$= 3\cos t - \sin t + 2e^{-3t}$$

Q2) $\dfrac{1}{s}\ln\left(1+\dfrac{1}{s^2}\right)$. Find using convulation theorem.

convulation theorem states:

If $L[f_1(t)] = \phi_1(s)$ & $L[f_2(t)] = \phi_2(s)$

$$L^{-1}[\phi_1(s) - \phi_2(s)] = \int_0^t f_1(u)\cdot f_2(t-u)\,du$$

$\therefore$ Let $\phi_1(s) = \dfrac{1}{s}$ & $\phi_2(s) = \ln\left(1+\dfrac{1}{s^2}\right)$

$$L^{-1}[\phi_1(s)] \Rightarrow L^{-1}\left[\frac{1}{s}\right] \Rightarrow \underbrace{1}_{f_1(u)} \quad \text{(By formula)}$$

$$\therefore L^{-1}\left[\ln\left(1+\frac{1}{s^2}\right)\right]$$

$$L^{-1}\left[\ln\left(\frac{s^2+1}{s^2}\right)\right] \Rightarrow L^{-1}\left[\phi_2(s)\right]$$

$$\phi_2'(s) = \frac{s^2}{s^2+1} \qquad \Rightarrow L^{-1}\left[\ln(s^2+1)\right] - L^{-1}\left[\ln(s^2)\right]$$

$$L^{-1}\left[\phi_2(s)\right] = L^{-1}\left[\phi_2'(s)\right] \cdot (-1)^1 \cdot t^1$$

$$= -t \cdot L^{-1}\left[\frac{s^2}{s^2+1}\right]$$

$$= -t \cdot \cos t$$

$$\therefore L^{-1}\left[\phi_1(s) - \phi_2(s)\right] = \int_0^t 1$$

$$\Rightarrow L^{-1}\left[\ln(s^2+1)\right] - 2L^{-1}\left[\ln s\right]$$

$$\cancel{\Rightarrow -\frac{2}{u}(1-\cos u)}$$

$$\Rightarrow \frac{2\cos u}{u} - \frac{2}{u} \Rightarrow -\frac{2}{u}(1-\cos u)$$

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$$\therefore L^{-1}[\phi_1(s) - \phi_2(s)] = \int_0^t 1 \cdot \frac{-2}{u}(\cos u - 1)\, du$$

By convulation theorem.

Q3) $f(t) = \begin{cases} \frac{kt}{T} & \text{for } 0 < t < T \\ f(t+T) & \text{otherwise} \end{cases}$ (Periodic funⁿ)

$$f(t) = \frac{kt}{T}$$

$$f(t+T) = \frac{k(t+T)}{T}$$

$$= \frac{kt}{T} + k$$

$$f(t) = \frac{kt}{T} \; ; \; 0 < t < T$$

$$L[f(t)] = \frac{1}{1-e^{-st}} \int_0^T e^{-st} \underbrace{\frac{kt}{T}}_{f(t)}\, dt$$

$\therefore$ The Period is T

as $f(t) = f(t+T)$

$$= \frac{1}{1-e^{-sT}} \int_0^T e^{-st} \frac{kt}{T}\, dt$$

$$= \frac{K}{T(1-e^{-st})} \int_0^T e^{-sT} t \, dt$$

By party.

$$= \frac{K}{T(1-e^{-st})} \left[t \left(\frac{e^{-st}}{-s} \right) - \int_0^T \frac{dt}{dt} \int e^{-3t} dt \, dt \right]$$

$$= \frac{K}{T(1-e^{-st})} \left[\frac{T e^{-sT}}{-s} - \left[\frac{e^{-sT}}{s^2} - \frac{1}{s^2} \right] \right]$$

$$\Rightarrow K \left[\frac{1}{Ts^2} - \frac{e^{-sT}}{s(1-e^{-sT})} \right]$$