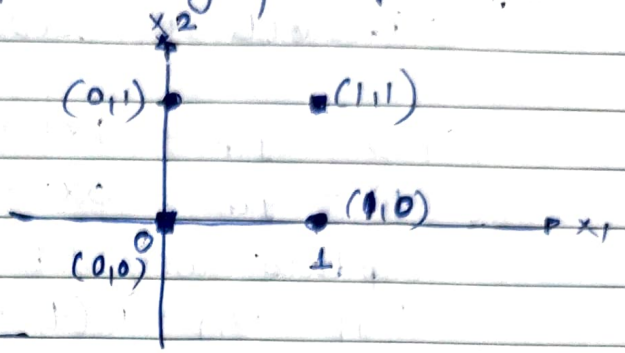
XOR solved example using perceptron.

x_1	x_2	y
0	0	0
0	1	1
1	0	1
1	1	0



Data is not linearly seperable.

$y = x_1 \bar{x_2} + \bar{x_1} x_2$

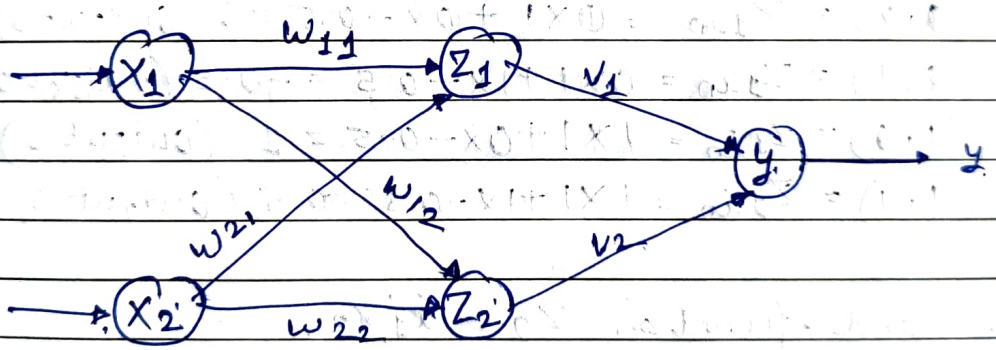
$y = Z_1 + Z_2$

where

$Z_1 = x_1 \bar{x_2}$ (function 1)

$Z_2 = \bar{x_1} x_2$ (function 2)

$y = Z_1$ OR Z_2 (function 3)



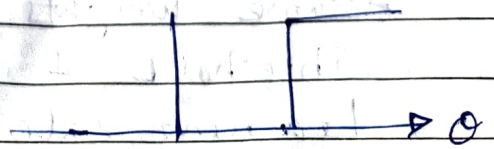
first function $Z_1 = x_1 \bar{x_2}$

(t = taget)

x_1	x_2	Z_1
0	0	0
0	1	0
1	0	1
1	1	0

Activation function

$$f(y_{in}) = \begin{cases} 1 & \text{if } y_{in} \geq \theta \\ 0 & \text{if } y_{in} < \theta \end{cases}$$



unipolar discrete

Assume the initial inputs $w_{11} = w_{21} = 1$
Threshold = 1, learning rate = 1.5

(0,0) - $Z_{1in} = 0\times1 + 0\times1 = 0$ (output = 0) = t
(0,1) - $Z_{1in} = 0\times1 + 1\times1 = 1$ (output 1) ≠ t

update weights

$w_{ij} = w_{ij} + \eta(t-o)x_i$

$w_{11} = 1 + 1.5(0-1)\times0$
$= 0$

$w_{21} = 1 + 1.5(0-1)\times1$
$= -0.5$

~~$\Delta w_{21} = 1.5(0-1)\times1$~~
$=$

$w_{11} = 1 \quad w_{21} = -0.5$

(0,0) - $Z_{1in} = 0\times1 + 0\times-0.5 = 0$ (output 0) = t
(0,1) - $Z_{1in} = 0\times1 + 1\times-0.5 = -0.5$ (output 0) = t
(1,0) - $Z_{1in} = 1\times1 + 0\times-0.5 = 1$ (output 1) = t
(1,1) - $Z_{1in} = 1\times1 + 1\times-0.5 = 0.5$ (output 0) = t

Second function $Z_2 = \bar{x}_1 x_2$

(target t)

x_1	x_2	Z_2
0	0	0
0	1	1
1	0	0
1	1	0

activation function

$$f(y_{in}) = \begin{cases} 1 & \text{if } y_{in} \geq \theta \\ 0 & \text{if } y_{in} < \theta \end{cases}$$

Assume weights are $w_{21} = w_{22} = 1$
Threshold = 1
Learning rate = 1.5

(0,0) → $Z_{2in} = 0 \times 1 + 0 \times 1 = 0$ (output 0) = t

(0,1) → $Z_{2in} = 0 \times 1 + 1 \times 1 = 1$ (output 1) = t

(1,0) = $Z_{2in} = 1 \times 1 + 0 \times 1 = 1$ (output 1) ≠ t.

update weights.

$w_{ij} = w_{ij} + \eta (t - o) \cdot x_i$

$w_{12} = 1 + 1.5 (0 - 1) * 1$

$= -0.5$

$w_{22} = 1 + 1.5 (0 - 1) * 0$

$= 1.$

$\therefore w_{12} = -0.5$, $w_{22} = 1$

(0,0) = $Z_{2in} = 0 \times -0.5 + 0 \times 1 = 0$ (output 0) = t

(0,1) = $Z_{2in} = 0 \times -0.5 + 1 \times 1 = 1$ (output 1) = t

(1,0) = $Z_{2in} = 1 \times -0.5 + 0 \times 1 = -0.5$ (output 0) = t

(1,1) = $Z_{2in} = 1 \times -0.5 + 1 \times 1 = 0.5$ (output 0) = t

Third function $y = Z_1$ OR Z_2

$y_{in} = Z_1 v_1 + Z_2 v_2$

Assume $v_1 = v_2 = 1$, threshold = 1, learning rate = 1.5

(0,0) = $y_{in} = 0 \times 1 + 0 \times 1 = 0$ (output = 0) = t

(0,1) = $y_{in} = 0 \times 1 + 1 \times 1 = 1$ (output = 1) = t

(1,0) = $y_{in} = 1 \times 1 + 0 \times 1 = 1$ (output 1) = t

(1,0) = $y_{in} = 0 \times 1 + 0 \times 1 = 0$ (output 0) = t

X_1	X_2	Z_1	Z_2	y
0	0	0	0	0
0	1	0	1	1
1	0	1	0	1
1	1	0	0	0

$w_{11} = 1$, $w_{21} = -0.5$

$w_{12} = -0.5$, $w_{22} = 1$

$v_1 = v_2 = 1$