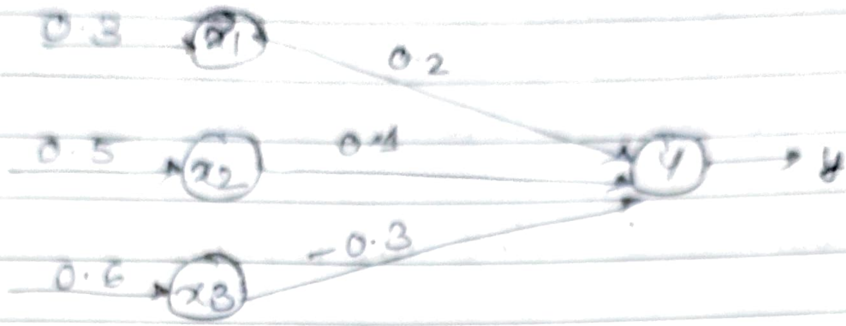
Module 1
Artificial neural network

1) For the network shown in figure calculate the net input to the output neuron.



Solution:-

$[x_1, x_2, x_3] = [0.3, 0.5, 0.6]$

$[w_1, w_2, w_3] = [0.2, 0.1, -0.3]$

The net input can be calculated as

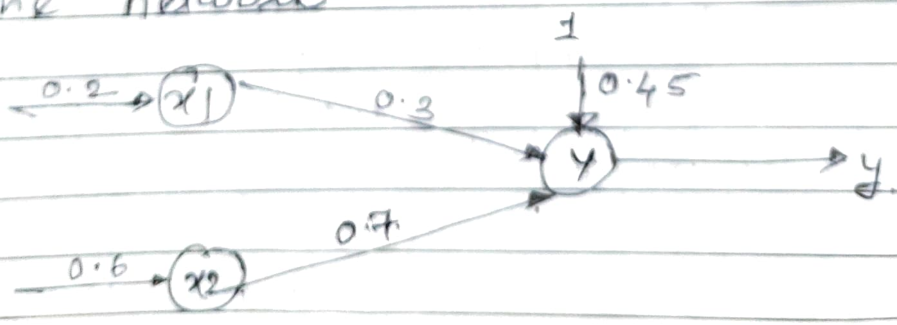
$y_{in} = x_1 w_1 + x_2 w_2 + x_3 w_3$

$= 0.3 \times 0.2 + 0.5 \times 0.1 + 0.6 \times -0.3$

$= 0.06 + 0.05 - 0.18$

$= -0.07$

2) Calculate the net input for the network shown in figure with bias included in the network



Solution:-

$Y_{in} = [x_1, x_2] = [0.2, 0.6]$

$[w_1, w_2] = [0.3, 0.7]$

bias $b = 0.45$, bias input $x_0 = 1$

$Y_{in} = b + x_1 w_1 + x_2 w_2$

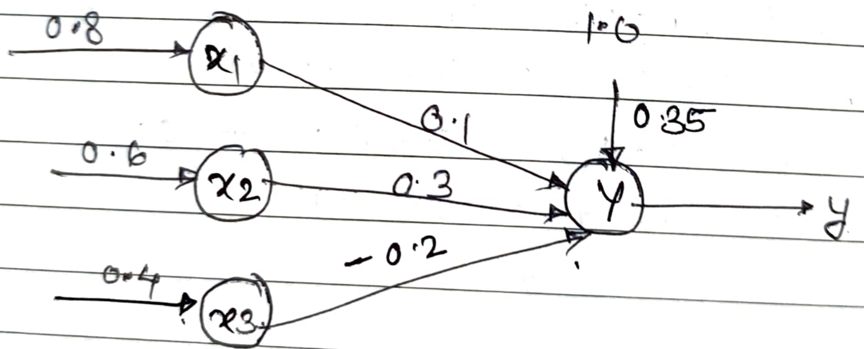
$= 0.45 + 0.2 \times 0.3 + 0.6 \times 0.7$

$= 0.45 + 0.06 + 0.42$

$= 0.93$

$\therefore$ $Y_{in} = 0.93$ is the net input.

3) Obtain the output of neuron Y for the network shown in figure using activation functions. as i) binary sigmoidal ii) bipolar sigmoidal.



Solution:-

The given network has three input neurons with bias and one output neuron. These form single layer network.

$[x_1, x_2, x_3] = [0.8, 0.6, 0.4]$

$[w_1, w_2, w_3] = [0.1, 0.3, -0.2]$

$b = 0.35$ (its input is always 1)

$$y_{in} = b + \sum_{i=1}^{n} x_i w_i$$

$$= b + x_1 w_1 + x_2 w_2 + x_3 w_3$$

$$= 0.35 + 0.8 \times 0.1 + 0.6 \times 0.3 + 0.4 \times -0.2$$

$$= 0.35 + 0.08 + 0.18 - 0.08$$

$$= 0.53$$

i) for binary sigmoidal activation function

$$y = f(y_{in}) = \frac{1}{1+e^{-y_{in}}} = \frac{1}{1+e^{-0.53}} = 0.625$$

ii) For bipolar sigmoidal activation function

$$y = f(y_{in}) = \frac{2}{1+e^{-y_{in}}} - 1 = \frac{2}{1+e^{-0.53}} - 1 = 0.259$$

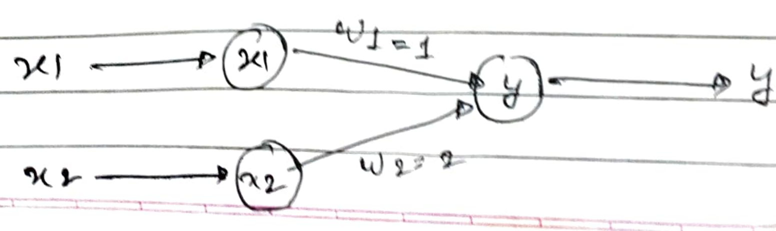
4) Implement AND function using McCulloch-Pitts neuron (take binary data)

Solution:-

Truth table for AND

x_1	x_2	y
1	1	1
1	0	0
0	1	0
0	0	0

Assume the weights be $w_i = 1$.



For an AND function, the output is high if bothe the inputs are high.

$\therefore$ $x_1 = 1$ $x_2 = 1$

For this case,

net input $y_{in} = x_1 w_1 + x_2 w_2$

$= 1 \times 1 + 1 \times 1$

$= 2$

Based on this input set, the threshold is calculated as 2 (θ). So if the ~~thresho~~ ~~netput~~ net input value is greater than or equal to 2, the neuron fires, else it doesnot fire.

This can also be calcuted as.

$$\theta \geq nw - p$$

Here $n=2$, $w=1$ (excitatory input) and $p=0$ (no inhibitory weights).

Substituting these values in above equation

$$\theta \geq 2 \times 1 - 0 \implies \theta \geq 2$$

Thus the output of neuron Y can be written as

$$y = f(y_{in}) = \begin{cases} 1 & \text{if } y_{in} \geq 2 \\ 0 & \text{if } y_{in} < 2 \end{cases}$$

where "2" represents the threshold value.

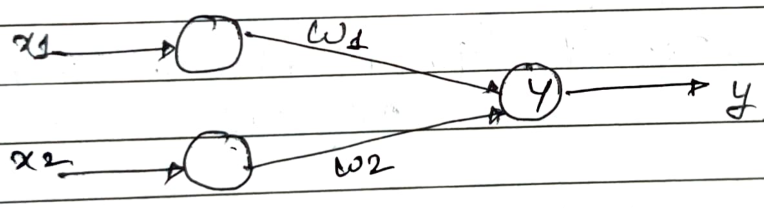
5) Implement ANDNOT function using Mc-Culloch-Pitts neuron. (use binary data)

Solution :-

In case of ANDNOT function, the response is true if the first input is true and the second input is false. For all other input variations, the output is false. The truthtable for ANDNOT is

x_1	x_2	y
0	0	0
0	1	0
1	0	1
1	1	0

The given function gives an output only when $x_1 = 1$ and $x_2 = 0$. The weights have to be decided only after the analysis.



Case 1 :- Assume that both weights ~~are~~ w_1, w_2 are exitatory. i.e

$$w_1 = w_2 = 1$$

for input

$(1,1)$ - $y_{in} = 1 \times 1 + 1 \times 1 = 2$

$(1,0)$ - $y_{in} = 1 \times 1 + 0 \times 1 = 1$

$(0,1)$ - $y_{in} = 0 \times 1 + 1 \times 1 = 1$

$(0,0)$ - $y_{in} = 0 \times 1 + 0 \times 1 = 0$

It is not possible to fire the neuron for input $(1,0)$ only. Hence the weights are not souitable.

Case 2:

Assume one weight as excitatory and the other as inhibirtory i.e

$w_1 = 1, \quad w_2 = -1$

for the inputs calculate net input.

(1, 1)	$y_{in} =$	$1 \times 1 + 1 \times -1$	$= 0$
(1, 0)	$y_{in} =$	$1 \times 1 + 0 \times -1$	$= 1$
(0, 1)	$y_{in} =$	$0 \times 1 + 1 \times -1$	$= -1$
(0, 0)	$y_{in} =$	$0 \times 1 + 0 \times -1$	$= 0$

from the calculated net inputs, now it is possible to fire the neuron for input (1, 0) only by fixing a threshold of 1.

i.e $\theta \geq 1$ for Y unit. Thus

$w_1 = 1, \quad w_2 = -1, \quad \theta \geq 1.$

The value of θ is calculated using the following:

$\theta \geq nw - p$

$\theta \geq 2 \times 1 - 1$ (for "p" inhibitory only magnitude is considered)

Thus the output of neuron Y can be written as

$$y = f(y_{in}) = \begin{cases} 1 & \text{if } y_{in} \geq 1 \\ 0 & \text{if } y_{in} < 1 \end{cases}$$

6) Implement XOR function using Mc-Culloch-Pitts neuron

Solution :-

The truth table for XOR function is

x_1	x_2	y
0	0	0
0	1	1
1	0	1
1	1	0

In this case the output is "ON" for only odd number of 1's. for the rest it is 'OFF'. XOR function cannot be represented by simple and single logic function; it is represented as

$$y = x_1 \bar{x}_2 + \bar{x}_1 x_2$$

$$y = z_1 + z_2$$

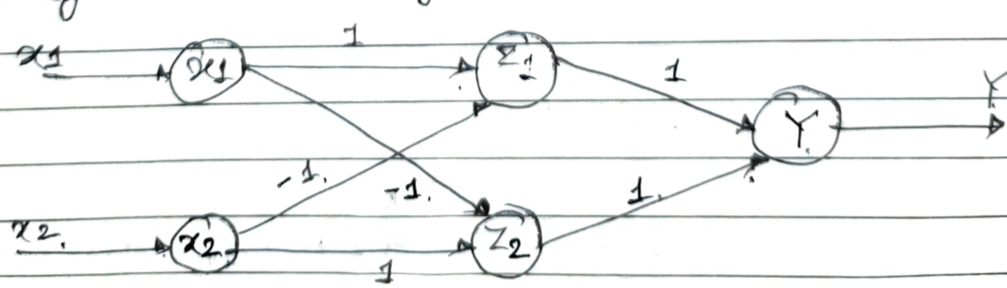
Where

$Z_1 = x_1 \bar{x}_2$ (function 1)

$Z_2 = \bar{x}_1 x_2$ (function 2)

$y = Z_1 (OR) Z_2$ (function 3)

A single-layer net is not sufficient to represent the function. An intermediate layer is necessary.



- First function ($Z_1 = x_1 \bar{x}_2$)....

The truth table for function Z_1 is shown in figure.

x_1	x_2	Z_1
0	0	0
0	1	0
1	0	1
1	1.	0.

The net representation is given as.

Case 1: Assume both weights are excitatory i.e

$w_{11} = w_{21} = 1$.

calculate the net inputs for inputs

$(0,0) \quad Z_1 in = 0 \times 1 + 0 \times 1 = 0$

$(0,1) \quad Z_1 in = 0 \times 1 + 1 \times 1 = 1$

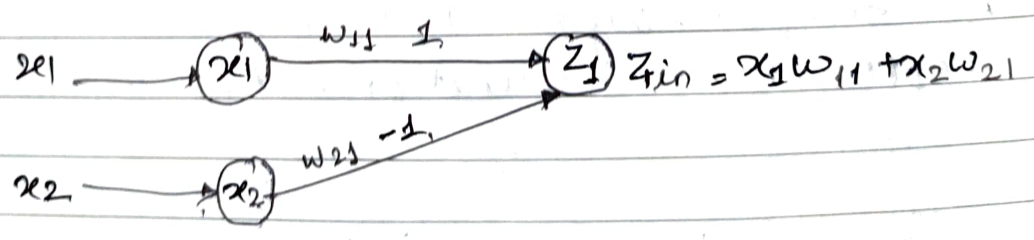
$(1,0) \quad Z_1 in = 1 \times 1 + 0 \times 1 = 1$

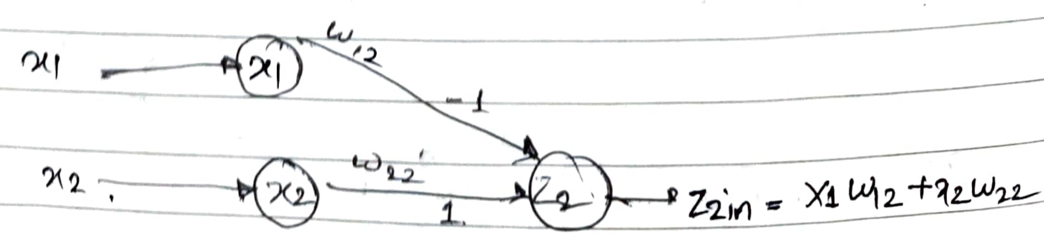
$(1,1) \quad Z_1 in = 1 \times 1 + 1 \times 1 = 2$.

Hence, it is not possible to obtain function Z_1 using these weights.

Case 2: Assume one weight as excitatory and the other as inhibitory i.e.

$w_{11} = 1 \qquad w_{21} = -1$.





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Calculate the net inputs

$(0,0)$ $Z_{1in} = 0\times1 + 0\times-1 = 0$

$(0,1)$ $Z_{1in} = 0\times1 + 1\times-1 = -1$

$(1,0)$ $Z_{1in} = 1\times1 + 0\times-1 = 1$

$(1,1)$ $Z_{1in} = 1\times1 + 1\times-1 = 0$

On the basis of the calculated net input, it is possible to get the required output

$\therefore$ $w_{11} = +1$ $w_{21} = -1$, $\theta \geq 1$ for Z_1 neuron

Second function := $(z_2 = \bar{x_1} x_2)$.

The truth table for function Z_2 is shown below

x_1	x_2	Z_2
0	0	0
0	1	1
1	0	0
1	1	0

Case 1: Assume both weights are excitatory

i.e $w_{12} = w_{22} = 1$

Calculate the net input

$(0,0) = Z_{2in} = 0\times1 + 0\times1 = 0$

$(0,1) = Z_{2in} = 0\times1 + 1\times1 = 1$

$(1,0) - Z_{2in} = 1\times0 + 1\times1 = 1$

$(1,1) = Z_{2in} = 1\times1 + 1\times1 = 2$

Hence it is not possible to obtain function Z_2 using these weights

Case 2: Assume one weight as excitatory and other as inhibitory i.e

$w_{12} = -1$ $w_{22} = 1$.

Now calculate the net inputs.

(0,0) $z_{2in} = 0 \times -1 + 0 \times 1 = 0$

(0,1) $z_{2in} = 0 \times -1 + 1 \times 1 = 1$

(1,0) $z_{2in} = 1 \times -1 + 0 \times 1 = -1$

(1,1) $z_{2in} = 1 \times -1 + 1 \times 1 = 0$

Thus based on this calculated net input it is possible to get the required output

$w_{12} = -1$, $w_{22} = 2$, $\theta \geq 1$,

Third function ($y = z_1$ OR z_2)

x_1	x_2	z_1	z_2	y
0	0	0	0	0
0	1	0	1	1
1	0	1	0	1
1	1	0	0	0

Here the net input is calculated as

$y_{in} = z_1 v_1 + z_2 v_2$

Case 1: Assume both weights as excitatory ie

$v_1 = v_2 = 1$.

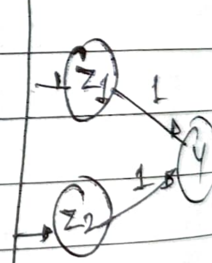
Now calculate net input

(0,0) $y_{in} = 0 \times 1 + 0 \times 1 = 0$

(0,1) $y_{in} = 0 \times 1 + 1 \times 1 = 1$

(1,0) $y_{in} = 1 \times 1 + 0 \times 1 = 1$

(1,1) $y_{in} = 1 \times 1 + 1 \times 1 = 0$



$\therefore \theta = 1$.

$\therefore$ The analysis is made for XOR function using M-P neuron. Thus for XOR function the weights are

$w_{11} = w_{22} = 1$

$w_{12} = w_{21} = -1$

$v_1 = v_2 = 1$

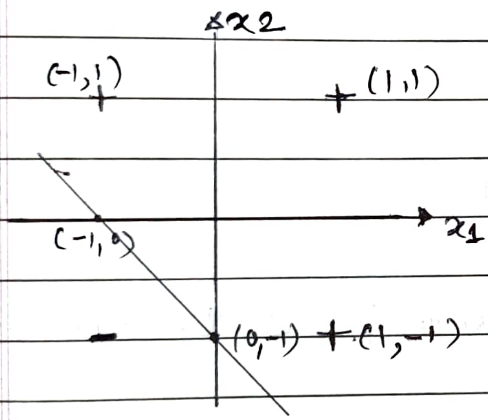
7) Using the linear seperability concept, obtain the response for OR function. (take bipolar inputs and bipolar outputs).

Solution:-

Truth table for "OR" using bipolar input-output

x_1	x_2	y
1	-1	1
-1	1	1
-1	-1	-1
1	1	1.

If output is '1' it is denoted as "+1" else "-1".



Assuming the coordinates ~~as~~ $(-1,0)$ and $(0,-1)$ as (x_1, y_1), (x_2, y_2). The ~~sp~~ slope m can be calculated as.

$$m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{-1-0}{0-(-1)} = \frac{-1}{1} = -1$$

We now calculate c

$$y = mx + c$$
$$c = y - mx$$
$$c = y_1 - mx_1$$
$$= 0 - (-1)(-1)$$
$$= -1$$

Using this value.

$$y = mx + c$$
$$= -1x - 1$$
$$= -x - 1.$$

Here the quadrants are not x and y, but x_1, and x_2, so,

$$x_2 = -x_1 - 1.$$

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This can be written as

$$x_2 = -\frac{w_1}{w_2} x_1 - \frac{b}{w_2} \quad (2)$$

Comparing equation 1 and 2, we get

$$\frac{w_1}{w_2} = \frac{1}{1} \qquad \frac{b}{w_2} = \frac{1}{1}$$

$\therefore w_1 = 1, \; w_2 = 1, \; b = 1$

Calculating the net input and output of OR using these weights and bias

x_1	x_2	b	$y_{in} = b + x_1 w_1 + x_2 w_2$	y
1	1	1	3	1
1	-1	1	1	1
-1	1	1	1	1
-1	-1	1	-1	-1

Thus the output of neuron y can be written as

$$y = f(y_{in}) = \begin{cases} 1 & \text{if } y_{in} \geq 1 \\ 0 & \text{if } y_{in} < 1 \end{cases}$$

where $\theta = 1$.