

Consider the following set of input vectors

$$x_1 = \begin{bmatrix} 1 \\ -2 \\ 0 \\ -1 \end{bmatrix}$$

$$x_2 = \begin{bmatrix} 5 \\ 0 \\ 1.5 \\ -0.5 \\ -1 \end{bmatrix}$$

$$x_3 = \begin{bmatrix} -1 \\ 1 \\ 0.5 \\ -1 \end{bmatrix}$$

$d_1 = -1$ ,  $d_2 = -1$ ,  $d_3 = 1$  are the desired responses for  $x_1, x_2, x_3$  respectively.

Initial weight vector  $w_1 = \begin{bmatrix} 1 \\ -1 \\ 0 \\ 0.5 \end{bmatrix}$

randomly chosen learning constant  $c = 0.1$   
 $\lambda = 1$  for  $\text{fnet}$ .

Use delta learning rule to calculate final weights. Use bipolar continuous activation function.

Solution: =

$$\text{net}_i = w_i^T x_i$$

$$o_i = f(\text{net}_i) = \frac{2}{1 + \exp^{-\lambda \text{net}}} - 1$$

$$f'(\text{net}_i) = \frac{1}{2} (1 - o_i^2)$$

step 1:

first training pair,  $x_1, d_1$ .

$$x_1 = \begin{bmatrix} 1 \\ -2 \\ 0 \\ -1 \end{bmatrix}$$

$$d_1 = -1$$

$$\begin{aligned} \text{net}_1 &= w_1^t \cdot x_1 \\ &= [1 \quad -1 \quad 0 \quad 0.5] \begin{bmatrix} 1 \\ -2 \\ 0 \\ -1 \end{bmatrix} = 2.5 \end{aligned}$$

$$o^1 = f(\text{net}^1) = \frac{2}{1 + \exp^{-1 \times 2.5}} - 1 = 0.848$$

$$f'(\text{net}^1) = \frac{1}{2} [1 - (o^1)^2] =$$

$$= \frac{1}{2} [1 - 0.848^2]$$

$$= 0.140$$

$$w^2 = c \cdot (d_1 - o^1) f'(\text{net}^1) x_1 + w^1$$

$$= (0.1) \times (-1 - 0.848) \times 0.140 \times \begin{bmatrix} 1 \\ -2 \\ 0 \\ -1 \end{bmatrix} + \begin{bmatrix} 1 \\ -1 \\ 0 \\ 0.5 \end{bmatrix}$$

$$= \begin{bmatrix} 0.974 \\ -0.948 \\ 0 \\ 0.526 \end{bmatrix}$$

Step 2:

Input is vector  $x_2$ , weight vector is  $w^2$ .

$$x_2 - \text{net}^2 = w^2 \cdot x_2$$

$$= [0.974 \quad -0.948 \quad 0 \quad 0.526] \begin{bmatrix} 0 \\ 1.5 \\ -0.5 \\ -1 \end{bmatrix}$$

$$= -1.948$$

$$o^2 = f(\text{net}^2) = \frac{2}{1 + \exp^{-\lambda \text{net}^2}} - 1 = -0.75$$

$$\begin{aligned} f'(\text{net}^2) &= \frac{1}{2} [1 - (o^2)^2] \\ &= \frac{1}{2} [1 - (-0.75)^2] \\ &= 0.218 \end{aligned}$$

$$w^3 = C \cdot (d_2 - o^2) \cdot f'(\text{net}^2) \times 2 + w^2$$

$$(0.1) \times (-1 + 0.75) \times 0.218 \times \begin{bmatrix} 0 \\ 1.5 \\ -0.5 \\ -1 \end{bmatrix} + \begin{bmatrix} 0.974 \\ -0.948 \\ 0 \\ 0.526 \end{bmatrix}$$

$$= \begin{bmatrix} 0.974 \\ -0.956 \\ 0.002 \\ 0.531 \end{bmatrix}$$

Step 3:

Input is  $x_3$ , weight vector is  $w^3$

$$\text{net}^3 = w^3 \cdot x_3$$

$$= [0.974, -0.956, 0.002, 0.531] \begin{bmatrix} -1 \\ 1 \\ 0.5 \\ -1 \end{bmatrix}$$

$$= -2.46$$

$$o^3 = f(\text{net}^3) = \frac{2}{1 + \exp^{-\lambda \text{net}^3}} - 1 = -0.842$$

$$f'(net^3) = \frac{1}{2} [1 - (0^3)^2]$$

$$= \frac{1}{2} [1 - (-0.842)^2]$$

$$= 0.145$$

$$f'(net^3) =$$

$$w^4 = c (d_3 - 0^3) \cdot f'(net^3) \cdot x_3 + w^3$$

$$= (0.1) \times (1 - (-0.842)) \times 0.145 \times \begin{bmatrix} -1 \\ 1 \\ 0.5 \\ -1 \end{bmatrix} + \begin{bmatrix} 0.974 \\ -0.956 \\ 0.002 \\ 0.531 \end{bmatrix}$$

$$= \begin{bmatrix} 0.947 \\ -0.929 \\ 0.016 \\ 0.505 \end{bmatrix}$$