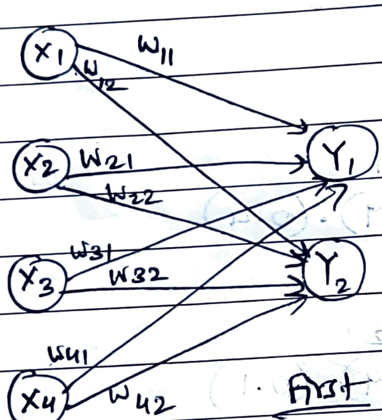
Mod – 3 SC

Q. Train hetero-associative network, with Hebb's Rule, to store input row vector s to output row vector t (s : t).

	s_1	s_2	s_3	s_4	t_1	t_2
1)	1	0	1	0	1	0
2)	1	0	0	1	1	0
3)	1	1	0	0	0	1
4)	0	0	1	1	0	1

Ans.



All $w = 0$ at start

First Pattern :

$\therefore w_{11\,new} = w_{11}\,old + x_1 y_1 = 0 + (1 \times 1) = 1$

$w_{12\,new} = w_{12}\,old + x_1 y_2 = 0 + (1 \times 0) = 0$

$w_{21\,new} = w_{21}\,old + x_2 y_1 = 0 + (0) = 0$

$w_{22\,new} = w_{22}\,old + x_2 y_2 = 0 + 0 = 0$

$w_{31\,new} = w_{31}\,old + x_3 y_1 = 0 + (1 \times 1) = 1$

$w_{32\,new} = w_{32}\,old + x_3 y_2 = 0$

$w_{41\,new} = w_{41}\,old + x_4 y_1 = 0$

$w_{42\,new} = w_{42}\,old + x_4 y_2 = 0$

Second Pattern :

$w_{11\,new} = 1 + (1 \times 1) = 2$

$w_{12\,new} = 0 + (1 \times 0) = 0$

$w_{21\,new} = 0$, $w_{22\,new} = 0$

$w_{31\,new} = 1$, $w_{32\,new} = 0$

$w_{41\,new} = 0 + (1 \times 1) = 1$

$w_{42\,new} = 0$

Third Pattern ↴

W_{11} new = $2+(1\times0) = 2$, W_{12} new = $0+(1\times1) = 1$
W_{21} new = $0+(1\times0) = 0$, W_{22} new = $0+(1\times1) = 1$
W_{31} new = $1+(0\times0) = 1$, W_{32} new = 0
W_{41} new = $1+(0\times0) = 1$, W_{42} new = 0

Fourth Pattern ↴

W_{11} new = $2+(0\times0) = 2$, W_{12} new = $1+(0\times1) = 1$
W_{21} new = $0+(0\times0) = 0$, W_{22} new = $1+(0\times1) = 1$
W_{31} new = $1+(1\times0) = 1$, W_{32} new = $0+(1\times1) = 1$
W_{41} new = $1+(1\times0) = 1$, W_{42} new = $0+(1\times1) = 1$

Testing with input. $[0,0,1,1]$ [output should be $[0,1]$]

$\therefore y_1 = w_{11}x_1 + w_{21}x_2 + w_{31}x_3 + w_{41}x_4$

$y_1 = (2\times0) + (0\times0) + (1\times1) + (1\times1) = 2$

~~Using binary activation function~~,

y_1 ~~=~~ 1 Similarly,

$y_2 = w_{12}x_1 + w_{22}x_2 + w_{32}x_3 + w_{42}x_4$

$y_2 = (1\times0) + (1\times0) + (1\times1) + (1\times1) = 2$

$\therefore f(y) = [f(y_1), f(y_2)] \leftarrow$ (binary activation)

$= [1, 1]$

Q. Train the hetero-associative memory network using ~~with~~ outer products rule to store input row vectors. $s = (s_1, s_2, s_3, s_4)$ to output row vector $t = (t_1, t_2)$

	s_1	s_2	s_3	s_4	t_1	t_2
1)	1	0	1	0	1	0
2)	1	0	0	1	1	0
3)	1	1	0	0	0	1
4)	0	0	1	1	0	1

(continuing) →

Ans. for pair 1:

~~$W = s^t \times t$~~ $\rightarrow$ $W = s^T \cdot t$

$$\therefore W = \begin{bmatrix} 1 \\ 0 \\ 1 \\ 0 \end{bmatrix} \begin{bmatrix} 1 & 0 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 0 \\ 1 & 0 \\ 0 & 0 \end{bmatrix}$$

for pair 2:

$W = s^T t$

$$\therefore W = \begin{bmatrix} 1 \\ 0 \\ 0 \\ 1 \end{bmatrix} \begin{bmatrix} 1 & 0 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 0 \\ 0 & 0 \\ 1 & 0 \end{bmatrix}$$

for pair 3:

$$W = s^T t = \begin{bmatrix} 1 \\ 1 \\ 0 \\ 0 \end{bmatrix} \begin{bmatrix} 0 & 1 \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ 0 & 1 \\ 0 & 0 \\ 0 & 0 \end{bmatrix}$$

for pair 4:

$$W = s^T t = \begin{bmatrix} 0 \\ 0 \\ 1 \\ 1 \end{bmatrix} \begin{bmatrix} 0 & 1 \end{bmatrix} = \begin{bmatrix} 0 & 0 \\ 0 & 0 \\ 0 & 1 \\ 0 & 1 \end{bmatrix}$$

$\therefore W_{final} = \sum W$ for all pairs

$$= \begin{bmatrix} 2 & 1 \\ 0 & 1 \\ 1 & 1 \\ 1 & 1 \end{bmatrix}$$

Q: Train hetero-associative, and then test with training data. (using outer product's rule)

Ans.

	s_1	s_2	s_3	s_4	t_1	t_2
1)	1	0	0	0	0	1
2)	1	1	0	0	0	1
3)	0	0	0	1	1	0
4)	0	0	1	1	1	0

[~~Dig~~ Diagram like one on page 3]

[f used for binary activation]

Ans. For pair 1: (Training)

$$W = s^T t = \begin{bmatrix} 1 \\ 0 \\ 0 \\ 0 \end{bmatrix} \begin{bmatrix} 0 & 1 \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ 0 & 0 \\ 0 & 0 \\ 0 & 0 \end{bmatrix} //$$

For pair 2: $W = s^T t = \begin{bmatrix} 1 \\ 1 \\ 0 \\ 0 \end{bmatrix} \begin{bmatrix} 0 & 1 \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ 0 & 1 \\ 0 & 0 \\ 0 & 0 \end{bmatrix} //$

For pair 3: $W = s^T t = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 1 \end{bmatrix} \begin{bmatrix} 1 & 0 \end{bmatrix} = \begin{bmatrix} 0 & 0 \\ 0 & 0 \\ 0 & 0 \\ 1 & 0 \end{bmatrix} //$

For pair 4: $W = s^T t = \begin{bmatrix} 0 \\ 0 \\ 1 \\ 1 \end{bmatrix} \begin{bmatrix} 1 & 0 \end{bmatrix} = \begin{bmatrix} 0 & 0 \\ 0 & 0 \\ 1 & 0 \\ 1 & 0 \end{bmatrix} //$

$\therefore W = \sum W$ for all pairs $= \begin{bmatrix} 0 & 2 \\ 0 & 1 \\ 1 & 0 \\ 2 & 0 \end{bmatrix} //$

(Testing) for pair 1:

$t = \begin{bmatrix} 1 & 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} 0 & 2 \\ 0 & 1 \\ 1 & 0 \\ 2 & 0 \end{bmatrix} = \begin{bmatrix} 0 & 2 \end{bmatrix} //$ $\therefore f(t) = \begin{bmatrix} 0 & 1 \end{bmatrix} //$

for pair 2: $t = \begin{bmatrix}1 & 1 & 0 & 0\end{bmatrix}\begin{bmatrix}0 & 2\\0 & 1\\1 & 0\\2 & 0\end{bmatrix} = \begin{bmatrix}0 & 3\end{bmatrix}$ //

$f(t) = \begin{bmatrix}0 & 1\end{bmatrix}$ //

for pair 3: $t = \begin{bmatrix}0 & 0 & 0 & 1\end{bmatrix}\begin{bmatrix}0 & 2\\0 & 1\\1 & 0\\2 & 0\end{bmatrix} = \begin{bmatrix}2 & 0\end{bmatrix}$ //

$f(t) = \begin{bmatrix}1 & 0\end{bmatrix}$ //

for pair 4: $t = \begin{bmatrix}0 & 0 & 1 & 1\end{bmatrix}\begin{bmatrix}0 & 2\\0 & 1\\1 & 0\\2 & 0\end{bmatrix} = \begin{bmatrix}3 & 0\end{bmatrix}$ //

$f(t) = \begin{bmatrix}1 & 0\end{bmatrix}$ //

To test for 'similar' data:

changing $\begin{bmatrix}1 & 1 & 0 & 0\end{bmatrix}$ to $\begin{bmatrix}0 & 1 & 0 & 0\end{bmatrix}$ (similar)

$\therefore \begin{bmatrix}0 & 1 & 0 & 0\end{bmatrix}\begin{bmatrix}0 & 2\\0 & 1\\1 & 0\\2 & 0\end{bmatrix} = \begin{bmatrix}0 & 1\end{bmatrix}$ //

$\therefore f(t) = \begin{bmatrix}0 & 1\end{bmatrix}$ //

for 'dissimilar' data:

Use any input not present

$\therefore \begin{bmatrix}1 & 1 & 1 & 1\end{bmatrix}\begin{bmatrix}0 & 2\\0 & 1\\1 & 0\\2 & 0\end{bmatrix} = \begin{bmatrix}3 & 3\end{bmatrix}$

$\therefore f(t) = \begin{bmatrix}1 & 1\end{bmatrix}$ //

<u>NOTE</u>: In bipolar, question may ask to test using 'missing' data, just replace one of the values with zero. (only in ~~binary~~ bipolar, not in binary)

(Bidirectional Associative Memory)

Q. Construct and test a BAM network to ~~compare~~ associate letters E and F with simple input output vectors.

Target vector for E : [-1, 1]
Target vector for F : [1, 1]
Display matrix size : 5 x 3
Take * as 1 and • as -1

```
* * *        * * *
* • •        * * *
* * *        * • •
* • •        * • •
* * *        * • •
 "E"          "F"
target: [-1, 1]   [1, 1]
```

Ans.

Input letters	Input (1x15)	Targets	Weights
E	[1 1 1 1 -1 -1 1 1 1 1 -1 -1 1 1 1]	[-1, 1]	w_1
F	[1 1 1 1 1 1 1 -1 -1 1 -1 -1 1 -1 -1]	[1, 1]	w_2

First ~~Left to right~~ Training:

(Using bipolar activation function for both forward and backward since all 1 and -1)

$$W = \sum S^T(p)\,t(p)$$

$$\therefore w_1 \overset{(\text{for 'E'})}{=} \underset{(15\times1)}{\begin{bmatrix} 1\\1\\1\\1\\-1\\-1\\1\\1\\1\\1\\-1\\-1\\1\\1\\1 \end{bmatrix}} \underset{(1\times2)}{\begin{bmatrix} -1 & 1 \end{bmatrix}} = \underset{(15\times2)}{\begin{bmatrix} -1 & 1\\ -1 & 1\\ -1 & 1\\ -1 & 1\\ 1 & -1\\ 1 & -1\\ -1 & 1\\ -1 & 1\\ -1 & 1\\ -1 & 1\\ 1 & -1\\ 1 & -1\\ -1 & 1\\ -1 & 1\\ -1 & 1 \end{bmatrix}}$$

$$\text{Similarly, } w_2 \overset{(\text{for 'F'})}{=} \underset{(15\times2)}{\begin{bmatrix} 1 & 1\\ 1 & 1\\ 1 & 1\\ 1 & 1\\ 1 & 1\\ 1 & 1\\ 1 & 1\\ -1 & -1\\ -1 & -1\\ 1 & 1\\ -1 & -1\\ -1 & -1\\ 1 & 1\\ -1 & -1\\ -1 & -1 \end{bmatrix}} \qquad \therefore W = \begin{bmatrix} 0 & 2\\ 0 & 2\\ 0 & 2\\ 0 & 2\\ 2 & 0\\ 2 & 0\\ 0 & 2\\ -2 & 0\\ -2 & 0\\ 0 & 2\\ 0 & -2\\ 0 & -2\\ 0 & 2\\ -2 & 0\\ -2 & 0 \end{bmatrix} = w_1 + w_2$$

Now testing forward direction :

$\therefore$ first 'E' x W (gives $(1\times15)\times(15\times2) = (1\times2)$)

$\therefore Y_{in_1} = [-12, 18]$ $\quad \therefore Y_1 = f(Y_{in_1}) = [-1, 1]$ → (target achieved!)

Next 'F' x W (gives $(1\times15)\times(15\times2) = (1\times2)$ matrix)

$\therefore Y_{in_2} = [12, 18]$ $\quad \therefore Y_2 = f(Y_{in_2}) = [1, 1]$ → (target achieved!)

Now testing for backwords direction:

$\therefore$ First 'E' = $Y_1 \times W^T$ (gives $(1\times2)\times(2\times15) = (1\times15)$ matrix)

$\therefore E_{in} = [2\ 2\ 2\ 2\ -2\ -2\ 2\ 2\ 2\ 2\ -2\ -2\ 2\ 2\ 2]$

$\therefore f(E_{in}) = [1\ 1\ 1\ -1\ -1\ 1\ 1\ 1\ -1\ -1\ 1\ 1\ 1\ 1]$ → (target achieved!)

Next 'F' = $Y_2 \times W^T$ (gives $(1\times2)\times(2\times15) = (1\times15)$ matrix)

$\therefore F_{in} = [2\ 2\ 2\ 2\ 2\ 2\ 2\ -2\ -2\ 2\ -2\ -2\ 2\ -2\ -2]$

$\therefore f(F_{in}) = [1\ 1\ 1\ 1\ 1\ 1\ 1\ -1\ -1\ 1\ -1\ -1\ 1\ -1\ -1]$ → (target achieved!)

[Hopfield Network]

Q. Construct ~~Test~~ an auto associative discrete hopfield network with input vector $[1\ 1\ 1\ -1]$. Test the hopfield network with missing entries in 1st and 2nd components of the stored vector.

Ans. Since bipolar inputs, $W = \sum s^T(p)\, t(p)$ [s = input, t = output]

Here s and t are same, so

$$W = \begin{bmatrix} 1 \\ 1 \\ 1 \\ -1 \end{bmatrix} \begin{bmatrix} 1 & 1 & 1 & -1 \end{bmatrix} = \begin{bmatrix} 1 & 1 & 1 & -1 \\ 1 & 1 & 1 & -1 \\ 1 & 1 & 1 & -1 \\ -1 & -1 & -1 & 1 \end{bmatrix}$$

$\because W_{ii} = 0$ (In hopfield) → [weight matrix with no self connections]

$$W = \begin{bmatrix} 0 & 1 & 1 & -1 \\ 1 & 0 & 1 & -1 \\ 1 & 1 & 0 & -1 \\ -1 & -1 & -1 & 0 \end{bmatrix}$$

Let random order be $Y_1\, Y_4\, Y_3\, Y_2$

Now, for input vector, we know 1st and 2nd values missing from stored vector $\therefore$ Inputs vector $X = [0\ 0\ 1\ -1]$

$Y = [0\ 0\ 1\ -1]$

$$Y_{in_i} = X_i + \sum_{j=1}^{4} y_j w_{ji}$$

$$\therefore Y_{in_1} = X_1 + \begin{bmatrix} 0 & 0 & 1 & -1 \end{bmatrix} \begin{bmatrix} 0 \\ 1 \\ 1 \\ -1 \end{bmatrix} = 2$$

$\therefore f(Y_{in_1}) = 1$ (Applying activation)

(continued) →

$\therefore y = [1 \ 0 \ 1 \ -1]$

Next is y_4 so $y_{in4} = x_4 + \sum_{j=1}^{4} y_j w_{j4}$

$$= -1 + [1 \ 0 \ 1 \ -1] \begin{bmatrix} -1 \\ -1 \\ -1 \\ 0 \end{bmatrix} = -3$$

$\therefore f(y_{in4}) = -1$

$\therefore y = [1 \ 0 \ 1 \ -1]$

Next $\rightarrow$ $y_{in3} = x_3 + \sum_{j=1}^{4} y_j w_{j3}$

$$= 1 + [1 \ 0 \ 1 \ -1] \begin{bmatrix} 1 \\ 1 \\ 0 \\ -1 \end{bmatrix} = 3$$

$\therefore f(y_{in3}) = 1$

$\therefore y = [1 \ 0 \ 1 \ -1]$

Lastly $\rightarrow$ $y_{in2} = x_2 + \sum_{j=1}^{4} y_j w_{j2}$

$$= 0 + [1 \ 0 \ 1 \ -1] \begin{bmatrix} 1 \\ 0 \\ 1 \\ -1 \end{bmatrix} = 3$$

$\therefore f(y_{in2}) = 1$

$\therefore$ Finally $y = [1 \ 1 \ 1 \ -1]$

Q. Construct a auto-associative network to store the vectors x_1, x_2, x_3.
$x_1 = [1\ 1\ 1\ 1\ 1]$, $x_2 = [1\ -1\ -1\ 1\ -1]$, $x_3 = [-1\ 1\ -1\ -1\ -1]$
Find weight matrix with no self-connection. Calculate the energy of the stored patterns using discrete hopfield network test patterns, if the test patterns are given as below:

Test: $x_1 = [1\ 1\ 1\ -1\ -1]$
$x_2 = [1\ -1\ -1\ -1\ -1]$
$x_3 = [1\ 1\ -1\ -1\ -1]$

Compare the test pattern's energy with the stored pattern's energy.

Ans. $W = \sum_{p=1}^{3} s^T(p)t(p)$

$$W = \begin{bmatrix}1\\1\\1\\1\\1\end{bmatrix}\begin{bmatrix}1 & 1 & 1 & 1 & 1\end{bmatrix} + \begin{bmatrix}1\\-1\\-1\\1\\-1\end{bmatrix}\begin{bmatrix}1 & -1 & -1 & 1 & -1\end{bmatrix} + \begin{bmatrix}-1\\1\\-1\\-1\\-1\end{bmatrix}\begin{bmatrix}-1 & 1 & -1 & -1 & -1\end{bmatrix}$$

$$\therefore W = \begin{bmatrix}3 & -1 & 1 & 3 & 1\\ -1 & 3 & 1 & -1 & 1\\ 1 & 1 & 3 & 1 & 3\\ 3 & -1 & 1 & 3 & 1\\ 1 & 1 & 3 & 1 & 3\end{bmatrix}$$

Since $W_{ii} = 0 \rightarrow$ [no self connections]

$$W = \begin{bmatrix}0 & -1 & 1 & 3 & 1\\ -1 & 0 & 1 & -1 & 1\\ 1 & 1 & 0 & 1 & 3\\ 3 & -1 & 1 & 0 & 1\\ 1 & 1 & 3 & 1 & 0\end{bmatrix}$$

For energy:

☆ $\Rightarrow E_i = 0.5\,[x_i W^T x_i^T]$ (only for Hopfield network)

$$\therefore E_1 = 0.5\begin{bmatrix}1 & 1 & 1 & 1 & 1\end{bmatrix}\begin{bmatrix}0 & -1 & 1 & 3 & 1\\ -1 & 0 & 1 & -1 & 1\\ 1 & 1 & 0 & 1 & 3\\ 3 & -1 & 1 & 0 & 1\\ 1 & 1 & 3 & 1 & 0\end{bmatrix}\begin{bmatrix}1\\1\\1\\1\\1\end{bmatrix}$$

$\therefore E_1 = +10$

$\left.\begin{array}{l}E_2 = -6\\ E_3 = -10\end{array}\right\}$ similarly to E_1

Now testing,

First test pattern = [1 1 1 -1 1] = x_1

$y = [1\ 1\ 1\ -1\ 1]$

Choose unit 4 for updation first, (random)

$$\therefore y_{in4} = x_4 + \sum_{j=1}^{5} y_j x_{j4}$$

$$= -1 + [1\ 1\ 1\ -1\ 1]\begin{bmatrix} 3 \\ -1 \\ 1 \\ 0 \\ 1 \end{bmatrix} = 3$$

$\therefore f(y_{in4}) = 1$

$\therefore y = [1\ 1\ 1\ 1\ 1]$ → [Since this is equal to x_1 (of stored vector) therefore it has converged]

Second test pattern = $[1\ -1\ -1\ -1\ -1] = x_2$

$y = [1\ -1\ -1\ -1\ -1]$

Choosing unit 4 for updation first (random)

$$\therefore y_{in4} = x_4 + \sum_{j=1}^{5} y_j w_{j4}$$

$$= -1 + [1\ -1\ -1\ -1\ -1]\begin{bmatrix} 3 \\ -1 \\ 1 \\ 0 \\ 1 \end{bmatrix} = 1$$

$\therefore f(y_{in4}) = 1$

$\therefore y = [1\ -1\ -1\ 1\ -1]$ → [Since this is equal to x_2 of stored vector therefore it has converged]

→ Similarly do for third test pattern (use unit 1 for updation).

→ Then, calculate energy and compare

Module 5 – Fuzzy Logic

→ Fuzzy Proposition:

Q. $X = \{a, b, c, d\}$

~~A = {(a, 0}~~ $A = \{(a, 0), (b, 0.8), (c, 0.6), (d, 1)\}$

$B = \{(1, 0.2), (2, 1), (3, 0.8), (4, 0)\}$

$C = \{(1, 0), (2, 0.4), (3, 1), (4, 0.8)\}$

$Y = \{1, 2, 3, 4\}$

(a) If x is A then y is B

$\therefore R = (A \times B) \cup (\sim A \times Y)$

~~A×~~ $A \times B$ uses min. membership value,

$$\therefore A \times B = \begin{array}{c} \\ a \\ b \\ c \\ d \end{array} \begin{array}{c} \begin{array}{cccc} 1 & 2 & 3 & 4 \end{array} \\ \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0.2 & 0.8 & 0.8 & 0 \\ 0.2 & 0.6 & 0.6 & 0 \\ 0.2 & 1 & 0.8 & 0 \end{bmatrix} \end{array}$$

Next, $\sim A \times Y$ also uses min. membership value

~~$\sim A \times Y$ =~~

$$\cancel{\begin{array}{c|cccc} & 1 & 2 & 3 & 4 \\ \hline a & 0.2 & 1 & 0.8 & 0 \\ b & 0.2 & 0.2 & 0.2 & 0 \\ c & 0.2 & 0.4 & 0.4 & 0 \\ d & 0 & 0 & 0 & 0 \end{array}}$$

$$\sim A \times Y = \begin{array}{c} \\ a \\ b \\ c \\ d \end{array} \begin{array}{c} \begin{array}{cccc} 1 & 2 & 3 & 4 \end{array} \\ \begin{bmatrix} 1 & 1 & 1 & 1 \\ 0.2 & 0.2 & 0.2 & 0.2 \\ 0.4 & 0.4 & 0.4 & 0.4 \\ 0 & 0 & 0 & 0 \end{bmatrix} \end{array}$$

Now, for union, using max. membership value,

$$\therefore (A \times B) \cup (\sim A \times Y) = \begin{array}{c} \\ a \\ b \\ c \\ d \end{array} \begin{array}{c} \begin{array}{cccc} 1 & 2 & 3 & 4 \end{array} \\ \begin{bmatrix} 1 & 1 & 1 & 1 \\ 0.2 & 0.8 & 0.8 & 0.2 \\ 0.4 & 0.6 & 0.6 & 0.4 \\ 0.2 & 1 & 0.8 & 0 \end{bmatrix} \end{array}$$

Q. Let $A = \{a_1, a_2\}$, $B = \{b_1, b_2, b_3\}$, $C = \{c_1, c_2\}$. Let R be relation from A to B, defined by matrix:

$$\begin{array}{c|ccc} & b_1 & b_2 & b_3 \\ \hline a_1 & 0.4 & 0.5 & 0 \\ a_2 & 0.2 & 0.8 & 0.2 \end{array}$$

Let S be relation from B to C, defined by matrix:

$$\begin{array}{c|cc} & c_1 & c_2 \\ \hline b_1 & 0.2 & 0.7 \\ b_2 & 0.3 & 0.8 \\ b_3 & 1 & 0 \end{array}$$

Find : (i) Max-min composition of R and S.
(ii) Max-product composition of R and S.

Ans. Let $T = R \circ S$

(i) Since max-min composition,

$$T(a_1, c_1) = \max(\min(0.4, 0.2), \min(0.5, 0.3), \min(0, 1))$$
$$= \max(0.2, 0.3, 0) = 0.3$$

Similarly, $T(a_1, c_2) = \max(\min(0.4, 0.7), \min(0.5, 0.8), \min(0, 0))$
$$= \max(0.4, 0.5, 0) = 0.5$$

$$T(a_2, c_1) = \max(\min(0.2, 0.2), \min(0.8, 0.3), \min(0.2, 1))$$
$$= \max(0.2, 0.3, 0.2) = 0.3$$

$$T(a_2, c_2) = \max(\min(0.2, 0.7), \min(0.8, 0.8), \min(0.2, 0))$$
$$= \max(0.2, 0.8, 0) = 0.8$$

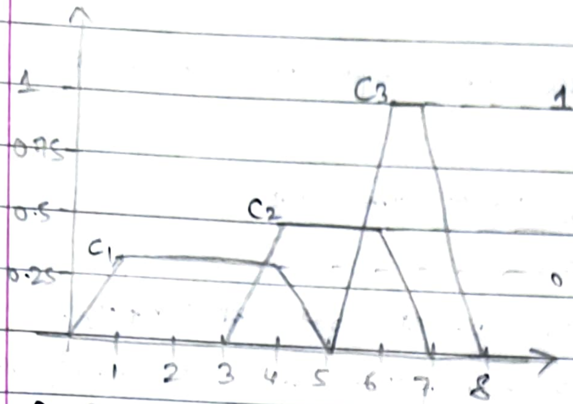
$$\therefore T = R \circ S = \begin{array}{c|cc} & c_1 & c_2 \\ \hline a_1 & 0.3 & 0.5 \\ a_2 & 0.3 & 0.8 \end{array}$$

(ii) Since max-product composition,

$$T(a_1, c_1) = \max(0.4 \times 0.2, 0.5 \times 0.3, 0 \times 1) = \max(0.08, 0.15, 0) = 0.15$$
$$T(a_1, c_2) = \max(0.4 \times 0.7, 0.5 \times 0.8, 0 \times 0) = \max(0.28, 0.4, 0) = 0.4$$
$$T(a_2, c_1) = \max(0.2 \times 0.2, 0.8 \times 0.3, 0.2 \times 1) = \max(0.04, 0.24, 0.2) = 0.24$$
$$T(a_2, c_2) = \max(0.2 \times 0.7, 0.8 \times 0.8, 0.2 \times 0) = \max(0.14, 0.64, 0) = 0.64$$

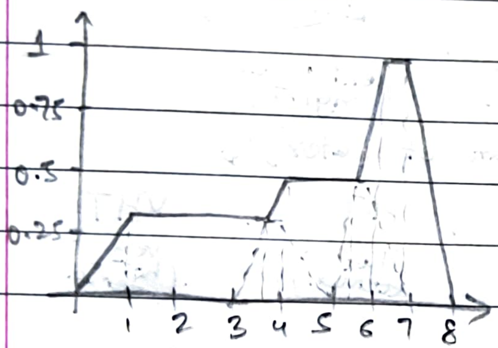
$$\therefore T = R \circ S = \begin{array}{c|cc} & c_1 & c_2 \\ \hline a_1 & 0.15 & 0.4 \\ a_2 & 0.24 & 0.64 \end{array}$$

Q.



Perform defuzzification using mean of max (MoM), centroid

Ans. Aggregation of graphs:



(i) Mean of Max (MoM) $= \frac{6+7}{2} = 6.5$

(ii) Centroid method $= x^* = \frac{(0.3\times1)+(0.3\times3.5)+(0.5\times4)+(0.5\times5.5)+(1\times6)+(1\times7)}{0.3+0.3+0.5+0.5+1+1}$

$\therefore x^* = {}^{18.6}/_{3.6}$

(iii) Centre of sum method:

Areas of individual fuzzy sets : $A(C_1) = 1.2$, $A(C_2) = 1.5$, $A(C_3) = 2$

Centres of individual fuzzy sets : $c(C_1) = 2.5$, $c(C_2) = 5$, $c(C_3) = 6.5$

$\therefore x^* = \frac{(2.5\times1.2)+(1.5\times5)+(2\times6.5)}{1.2+1.5+2}$ $\quad \therefore x^* = 5$

(iv) Weighted average method:

Membership values of fuzzy sets (at centre): 0.3, 0.5, 1

Centres of fuzzy sets : 2.5, 5, 6.5

$\therefore x^* = \frac{(0.3\times2.5)+(0.5\times5)+(1\times6.5)}{0.3+0.5+1}$ $\quad \therefore x^* = 5.41$

Q. Design a fuzzy controller to regulate the temperature of a domestic shower. Assume that:
(a) the temperature is adjusted by single mixer tap,
(b) the flow of water is constant,
(c) control variable is the ratio of hot to cold water input.
The design should clearly mention descriptors used for fuzzy sets, and the control variables, and set of rules to generate control action and defuzzification.

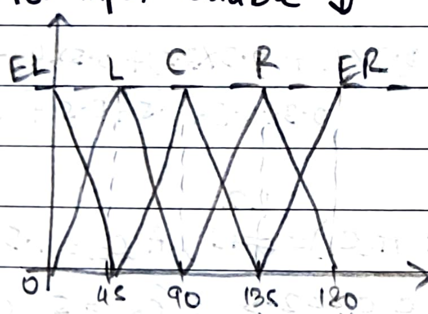
Ans. <u>Step 1</u>: Mention descriptors.
Descriptors for input variable (position of mixer tap) ↓

EL (extreme left), L (left), C (centre), R (right), ER (extreme right)

Descriptors for output variable (temperature of water) ↓

VCT (very cold temp.), CT (cold temp.), WT (warm temp.), HT (hot temp.), VHT (very hot temp.)

<u>Step 2</u>: Membership functions
(1) For input variable ↓



$\mu_{EL}(x) = \frac{45-x}{45}$ in $0 \le x \le 45$

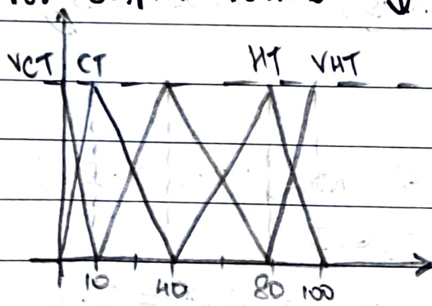
$\mu_L(x) = \frac{x}{45}$ in $0 \le x \le 45$; $\frac{90-x}{45}$ in $45 < x \le 90$

$\mu_C(x) = \frac{x-45}{45}$ in $45 \le x \le 90$; $\frac{135-x}{45}$ in $90 < x \le 135$

$\mu_R(x) = \frac{x-90}{45}$ in $90 \le x \le 135$; $\frac{180-x}{45}$ in $135 < x \le 180$

$\mu_{ER}(x) = \frac{x-135}{45}$ in $135 \le x \le 180$

(2) For output variable ↓



$\mu_{VCT}(y) = \frac{10-y}{10}$ in $0 \le y \le 10$

$\mu_{CT}(y) = \frac{y}{10}$ in $0 \le y \le 10$; $\frac{40-y}{30}$ in $10 < y \le 40$

$\mu_{WT}(y) = \frac{y-10}{30}$ in $10 \le y \le 40$; $\frac{80-y}{40}$ in $40 < y \le 80$

$\mu_{HT}(y) = \frac{y-40}{40}$ in $40 \le y \le 80$; $\frac{100-y}{20}$ in $80 < y \le 100$

$\mu_{VHT}(y) = \frac{y-80}{20}$ in $80 \le y \le 100$

Step 3: Rulebase

Input		Output
EL	———	VCT
L	———	CT
C	———	WT
R	———	HT
ER	———	VHT

Step 4: Rule evaluation

Tap Position $75°$ ↴

∴ For $75°$, both μ_L and μ_c applicable.

$\mu_L(x) = \frac{90-x}{45} = \frac{15}{45} = 1/3$

$\mu_c(x) = \frac{x-45}{45} = \frac{75-45}{45} \therefore \frac{30}{45} = 2/3$

Step 5: Defuzzification

We can use any method, so using max method

$\max\left(\frac{1}{3}, \frac{2}{3}\right) = \frac{2}{3} \rightarrow \therefore \mu_c(x)$ is chosen

Since $\mu_c(x)$ chosen, it corresponds to WT

∴ Temperature of water is 'warm' (room temp.)

∴ We now know $\mu_{WT}(y) = 2/3$

$\therefore \mu_{WT}(y) = \frac{y-10}{30}$ in $10 \le y \le 40$ $\quad \frac{80-y}{40}$ in $40 < y \le 80$

$\frac{y-10}{30} = \frac{2}{3} \quad \therefore y = 30°C$

$\frac{80-y}{40} = \frac{2}{3} \quad \therefore y = 80 - \frac{80}{3}$

$y = 53.3°C \approx 53°C$

$\therefore y = \frac{30+53}{2} = \frac{83}{2} = 41.5°C$

Q. Design a fuzzy controller to determine wash time of a domestic washing machine. Assume that input is dirt and grease on clothes. Use three descriptors for input variables and five descriptors for output variables. Derive set of rules for controller action and defuzzification. The design should be supported by figures wherever possible. Show that, if the clothes are soiled to a large degree, then the wash time will be more and vice versa.

Ans. **Step 1 :** Mention descriptors

Input variables:

Descriptors for dirt: SD (small dirt), MD (medium dirt), LD (large dirt)

Descriptors for grease: SG (small grease), MG (medium grease), LG (large grease)

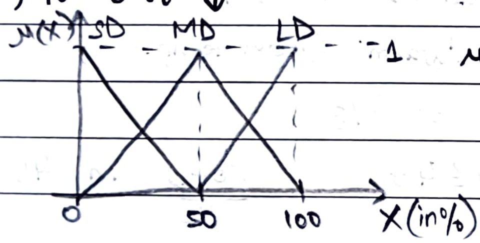
Output variable:

Descriptors for time: VS (very small), S (small), M (medium), L (large), VL (very large)

Step 2 : Membership functions

For input variables :

(1) For dirt ↓

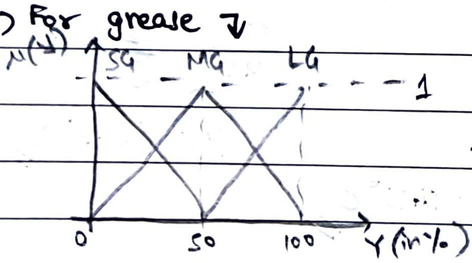


$\mu_{SD}(x) = \frac{50-x}{50}$ in $0 \le x \le 50$

$\mu_{MD}(x) = \frac{x}{50}$ in $0 \le x \le 50$; $\frac{100-x}{50}$ in $50 < x \le 100$

$\mu_{LD}(x) = \frac{x-50}{50}$ in $50 \le x \le 100$

(2) For grease ↓



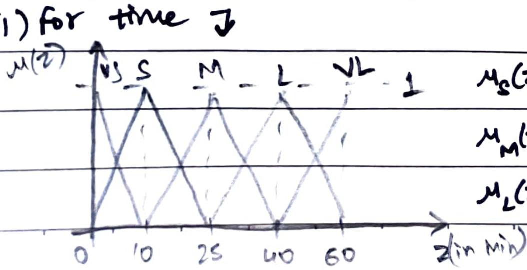
$\mu_{SG}(y) = \frac{50-y}{50}$ in $0 \le y \le 50$

$\mu_{MG}(y) = \frac{y}{50}$ in $0 \le y \le 50$, $\frac{100-y}{50}$ in $50 < y \le 100$

$\mu_{LG}(y) = \frac{y-50}{50}$ in $50 \le y \le 100$

For output variables :

(1) For time ↓



$\mu_{VS}(z) = \frac{10-z}{10}$ in $0 \le z \le 10$

$\mu_{S}(z) = \frac{z}{10}$ in $0 \le z \le 10$; $\frac{25-z}{15}$ in $10 < z \le 25$

$\mu_{M}(z) = \frac{z-10}{15}$ in $10 \le z \le 25$; $\frac{40-z}{15}$ in $25 < z \le 40$

$\mu_{L}(z) = \frac{z-25}{15}$ in $25 \le z \le 40$; $\frac{60-z}{20}$ in $40 < z \le 60$

$\mu_{VL}(z) = \frac{z-40}{20}$ in $40 \le z \le 60$

Step 3: Rulebase

X \ Y	SG	MG	LG
SD	VS	M	L
MD	S	M	L
LD	M	L	VL

Step 4: Rule evaluation

Assuming dirt is 60% and grease is 70%

$\therefore \mu_{MD}(x) = \frac{100-60}{50} = 0.8$ $\qquad \mu_{LD}(x) = \frac{60-50}{50} = 0.2$

$\mu_{MG}(y) = \frac{100-70}{50} = 0.6$ $\qquad \mu_{LG}(y) = \frac{70-50}{50} = 0.4$

strength of rules ↓

(1) Medium dirt and medium grease = min (0.8, 0.6) = 0.6

(2) Medium dirt and large grease = min (0.8, 0.4) = 0.4

(3) ~~Medium~~ Large dirt and medium grease = min (0.2, 0.6) = 0.2

(4) Large dirt and large grease = min (0.2, 0.4) = 0.2

X \ Y	SG	MG	LG
SD			
MD		0.6	0.4
LD		0.2	0.2

Since 0.6 is max. strength, it is chosen. We can see that the position is occupied by 'M' descriptor in output variable.

Step 5: Defuzzification

$\therefore \mu_M(z) = \frac{z-10}{15}$ in $10 \leq z \leq 25$; $\frac{40-z}{15}$ in $25 < z \leq 40$

$\frac{z-10}{15} = 0.6 \quad \therefore z-10 = 9$

$z = 19$

$\frac{40-z}{15} = 0.6 \quad \therefore 40-z = 9$

$\therefore z = 31$

$\therefore z = \frac{19+31}{2} = \frac{50}{2} = 25$ minutes