Fidelities

Saturday, November 27, 2021 6:17 PM

Fidelity

Quantum control is hard as there are continuous sets of possible positions on the bloch sphere, so lots of room for errors

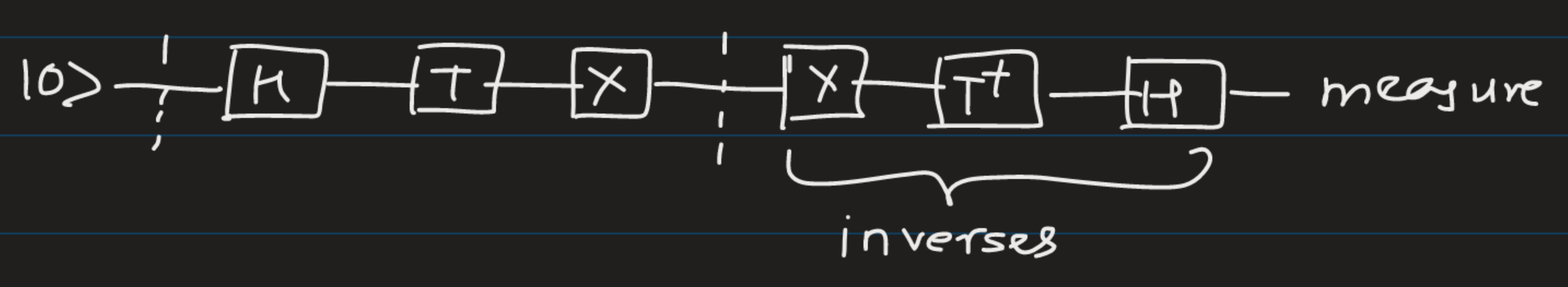
We quantify quantum gates using fidelity

Perfect Gates have fidelity of 100% $f = 1$

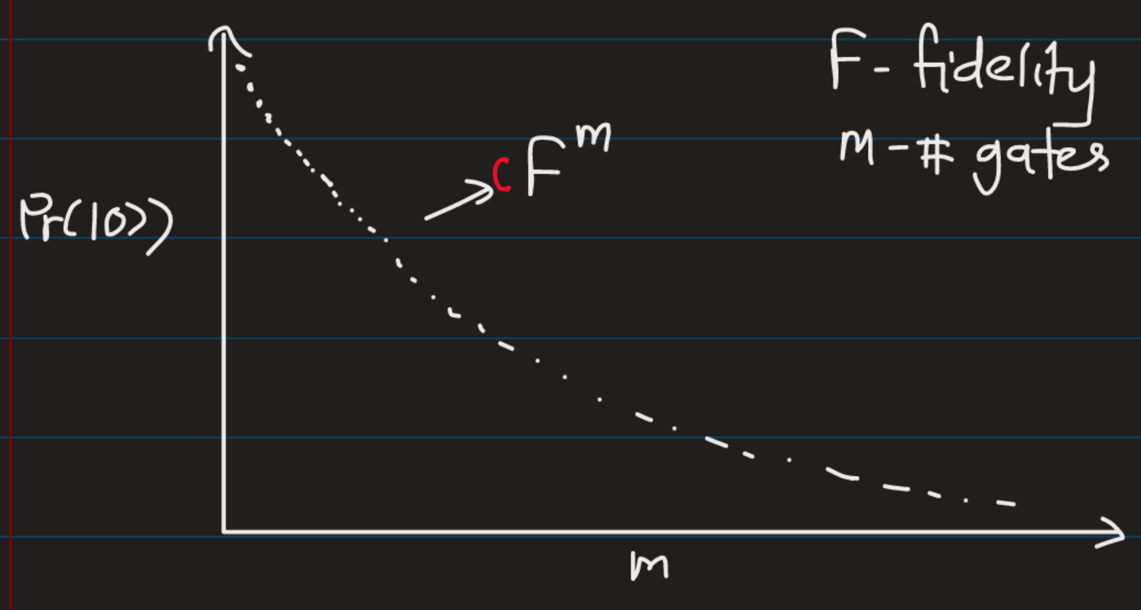
$$\boxed{\therefore f = 1 - \text{error rate}}$$

How to measure gate fidelity : Randomized benchmarking

- Initialize to some state lets say $|0\rangle$
- Gates = $\{H, T, X\}$
- Apply random gates from set to qubit
- Apply inverse of set of these gates
- Measure



- After measurement, if we get $|0\rangle$, the gates have $f = 1$ else $Pr(|0\rangle)$ decreases exponentially with time



F - fidelity
m - # gates

Ex $f = 0.995$ (99.5%)
$m = 200$

Overall fidelity : $(0.995)^{200} = 0.367$ (36.7%) not good

For a good quantum circuit, overall fidelity > 66%.

For $f = 0.999$ $m = 200$
$F_{overall} = 0.999^{200} = 81.9\%$

state fidelity - from qiskit.quantum_info import state_fidelity

f = state_fidelity $(\underbrace{\text{state1}, \text{state2}}_{\text{quantum states}}$, validate = True$)$

validate if state1 & state2 are valid quantum states

$$F(\rho_1, \rho_2) = \left(Tr\left(\sqrt{\sqrt{\rho_1}\rho_2\sqrt{\rho_1}}\right)\right)^2$$

mixed state, for pure states $F(\rho_1, \rho_2) = |\langle\psi_1|\psi_2\rangle|^2$

$F(\rho_1, \rho_2) = F(\rho_2, \rho_1)$

state fidelity is the measure of closeness of two quantum state

gate fidelity - from qiskit.quantum_info import average_gate_fidelity

F_{avg} = average_gate_fidelity(channel, target = None, required_cp = True, required_tp = True)

channel: quantum channel/ operator
target: target unitary operator (I if None)

$$F_{avg}(\underset{\text{channel}}{\mathcal{E}}, \overset{\text{target}}{U}) = \int d\psi \langle\psi|U^{\dagger}\mathcal{E}(|\psi\rangle\langle\psi|)U|\psi\rangle$$

$$= \frac{F_{pro}(\mathcal{E}, U) + 1}{d + 1}$$

F_{pro}: process fidelity
d: dimensions of $\mathcal{E}$ (channel)

process fidelity - process_fidelity (arguments same as avg gate fidelity)

$$F_{pro}(\mathcal{E}, F) = F(\rho_{\mathcal{E}}, \rho_F)$$

$$F_{pro}(\mathcal{E}, U) = \frac{Tr(S_U^{\dagger} S_{\mathcal{E}})}{d^2}$$

$S_U, S_{\mathcal{E}}$ → superoperators