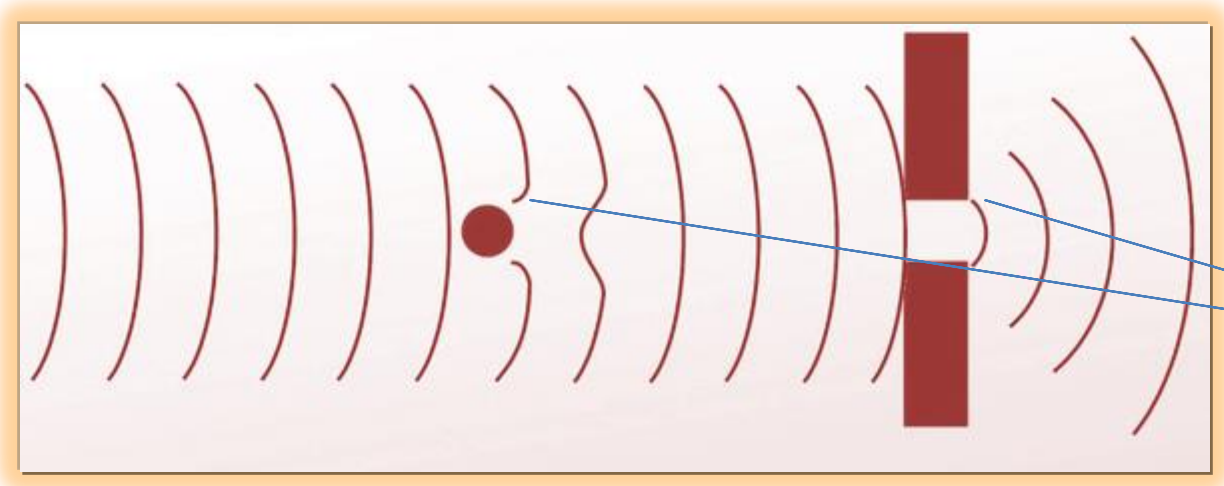
UPH004 – Applied Physics

Diffraction

(Semester: July– December 2022)

What is diffraction?

Diffraction is phenomenon which is defined as the bending of waves around corner by an obstacle or through an aperture into the region of geometrical shadow of the obstacle/aperture.



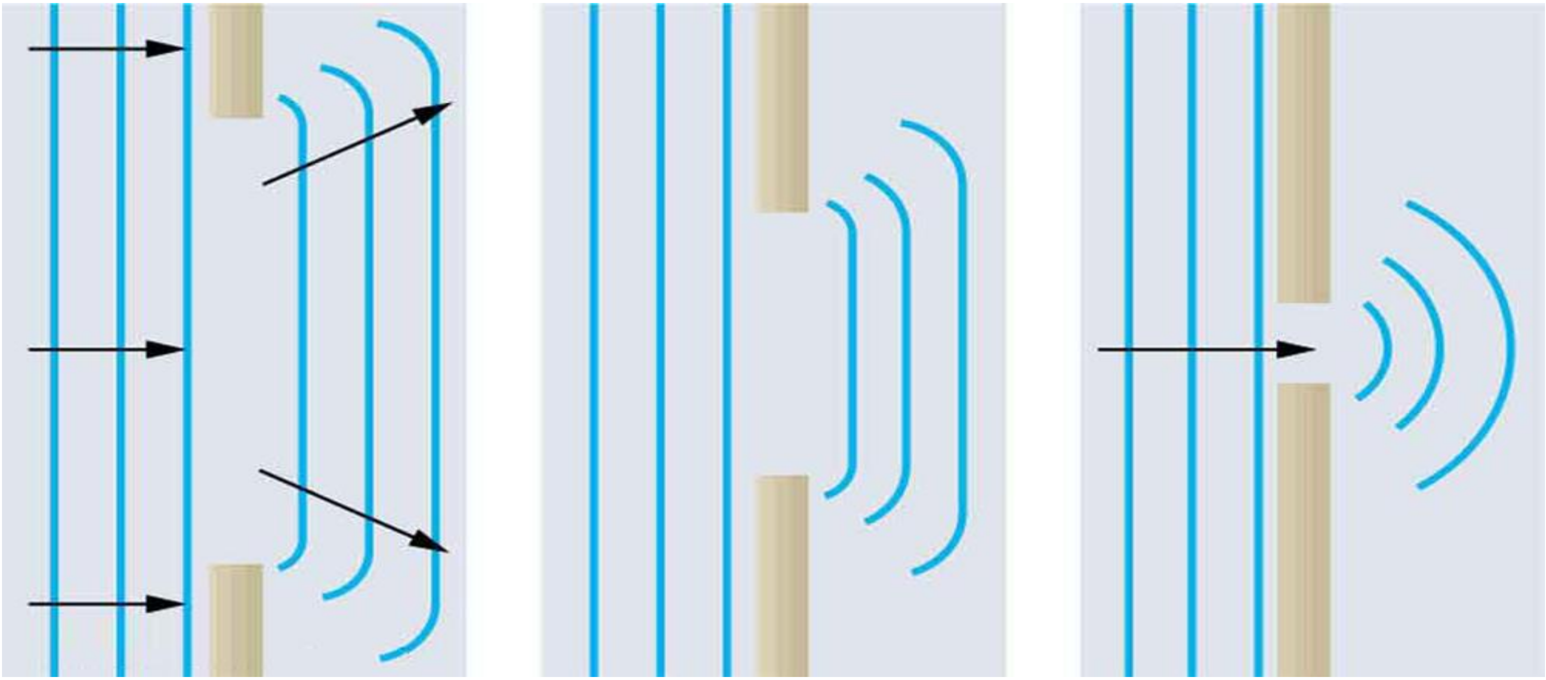
Bending of wave around the corner

How does it occur?

❖ The obstacle, aperture, slit or object become a secondary source of the propagating wave.

❖ Then the wave emanating from secondary source interfere to give rise to diffraction.

Condition for diffraction

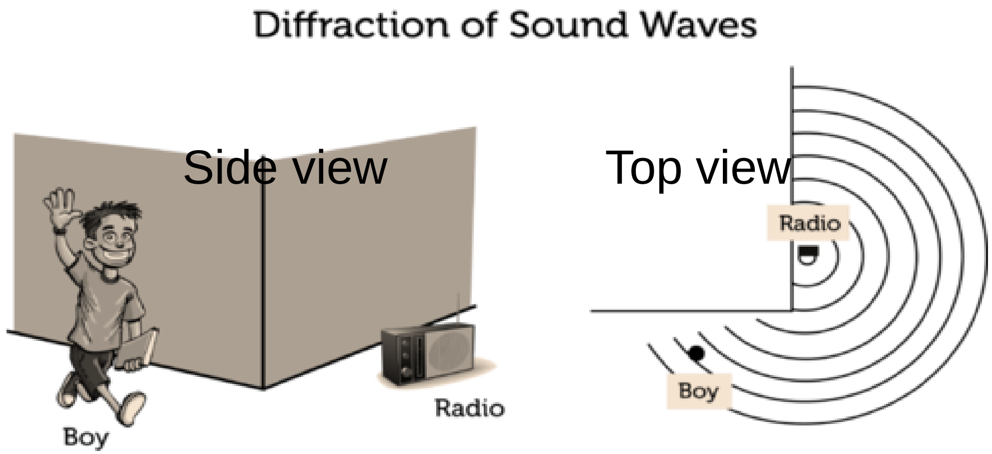


Condition for diffraction

aperture size or slit width $a \approx \lambda$

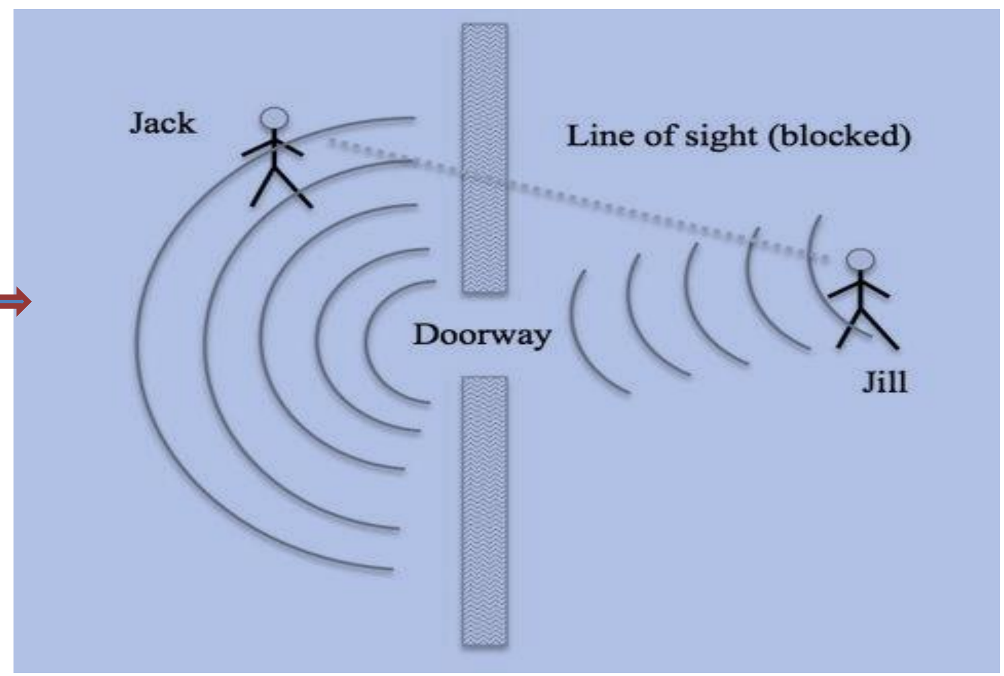
Can we use the fact that minima occurs for $b\sin\theta = m\lambda$ in order to explain the condition $a \approx \lambda$?

Our experience with diffraction in daily life



Q1: Can we observe diffraction due to door or window for Ultrasonic wave?

Another example →

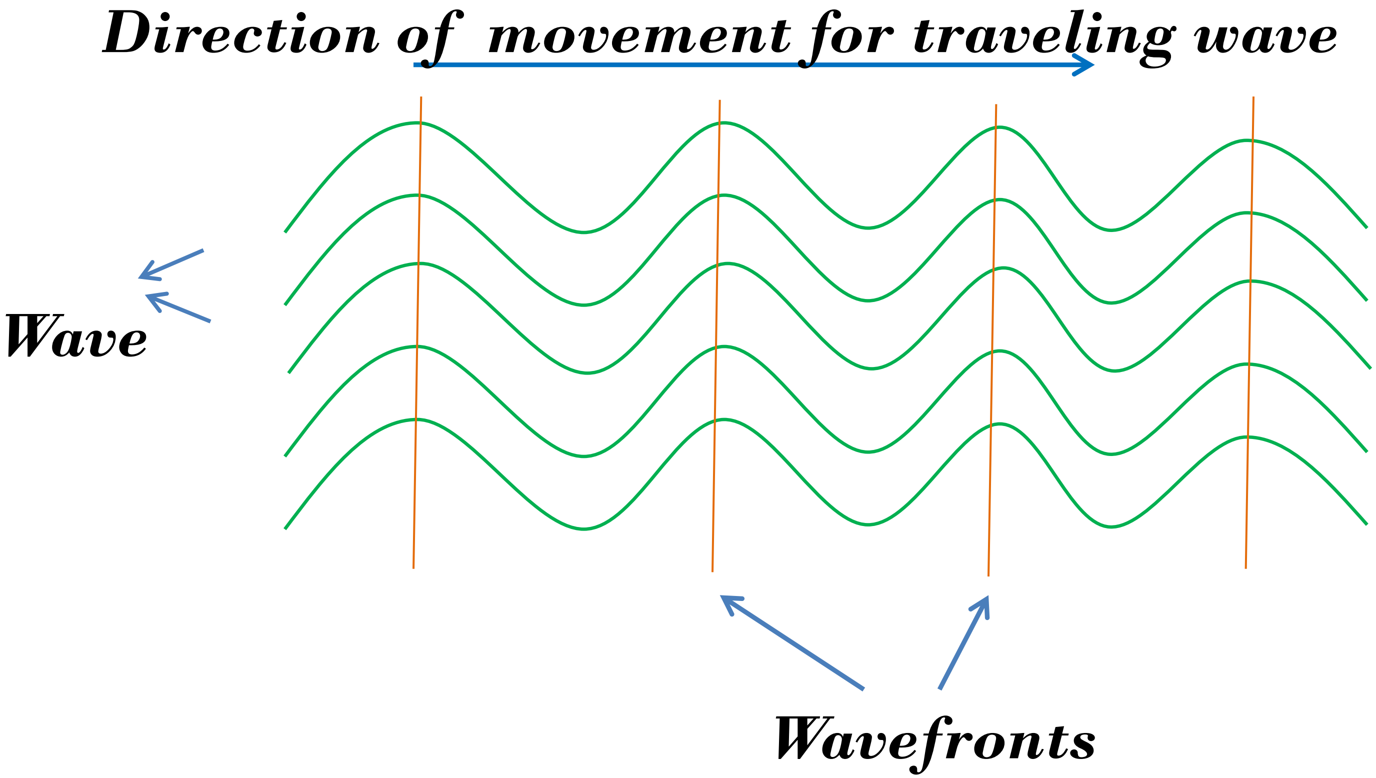


Diffraction: water wave

Q2: Why cannot we experience diffraction of light wave in our daily life?

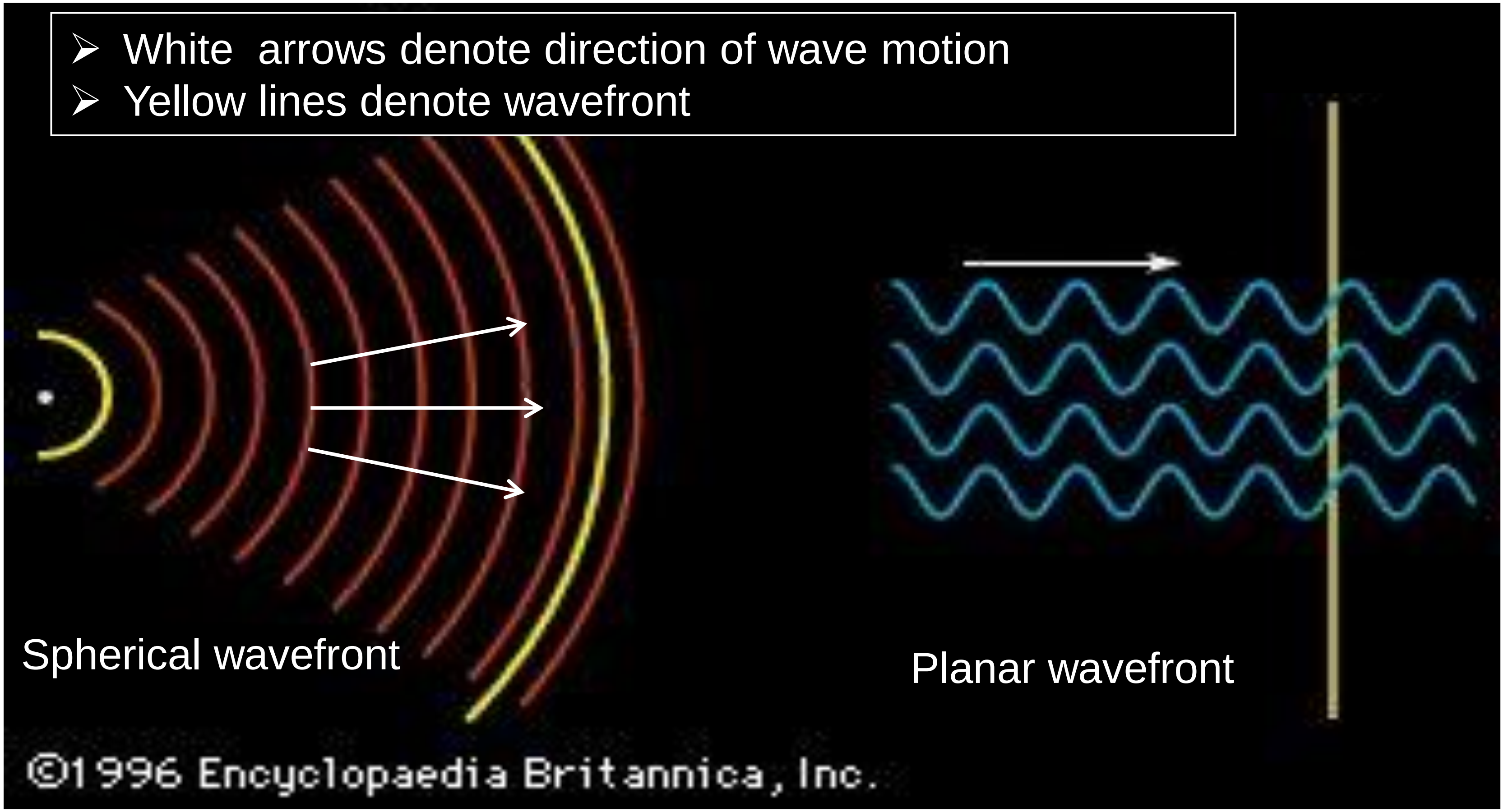
Wavefront

The wavefront is the set or collection of all points on a wave which are in phase with each other (phase difference is strictly zero).



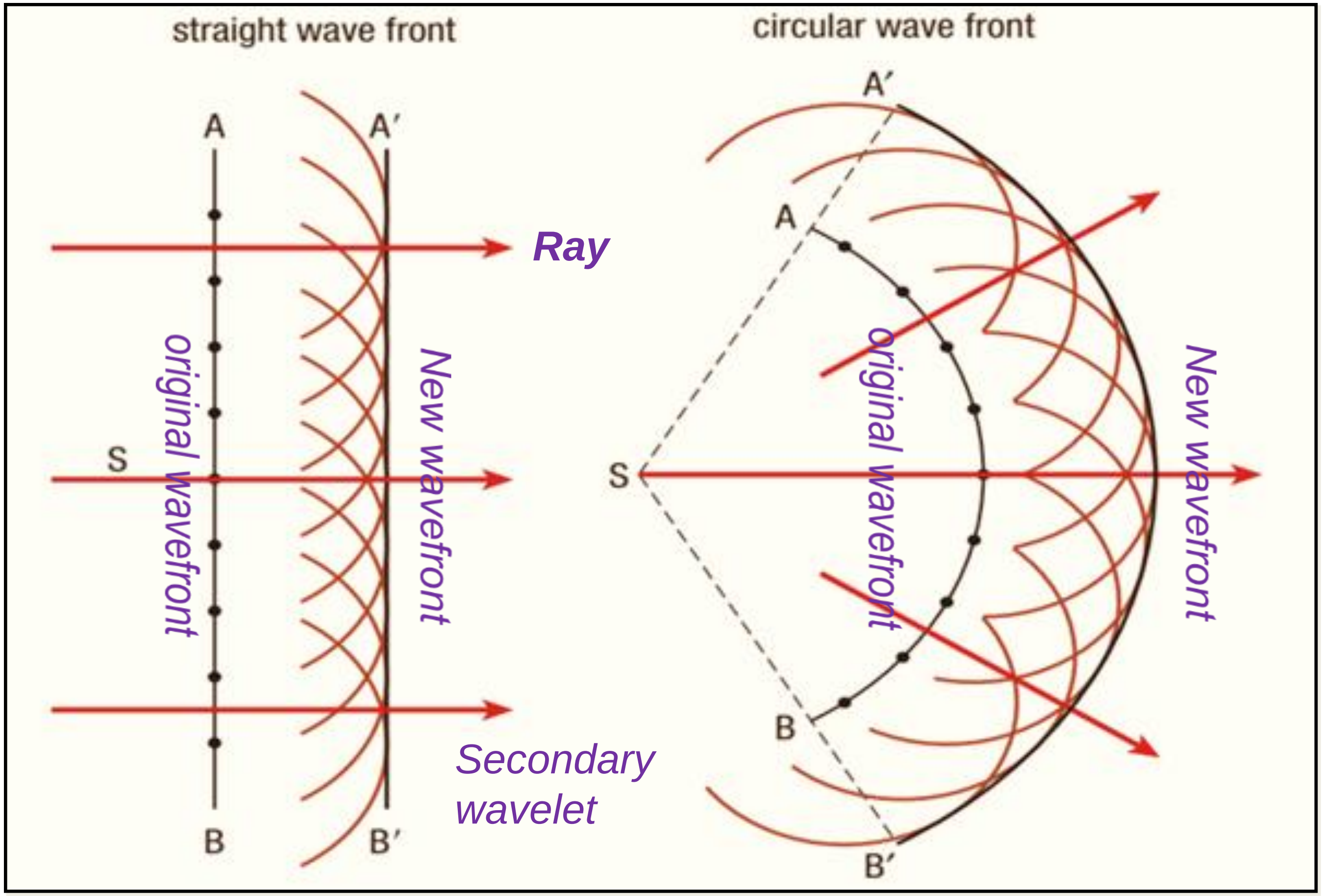
Type of wavefronts

1. *Planar*
2. *Circular*
3. *Cylindrical*
4. *Spherical*



Huygens's principle:

Each and every point of a wavefront of light may be regarded as new sources of wavelets that expand in every direction at a rate equal to the velocity propagation.



Difference between interference and diffraction:

- In interference, minima may be perfectly dark while this may not be the case for diffraction.
- In interference, all maxima can be of same intensity but they have varying intensity in diffraction.
- Fringe width could be equal in some cases in interference while they are never equal in diffraction.
- Two separate wavefronts originating from two different coherent sources interact in the interference, while secondary wavelets originating from same wavefronts interact with each other to give rise to diffraction.

Diffraction is a special case of Interference

Diffraction is simply the $N \rightarrow \infty$ limit of interference, where N is the number of secondary source defined on a single wavefront. It may seem thus that there is no need to introduce a new term for it. But on the other hand, a specific type of patterns occur for specific reason. That is why a separate term "diffraction" is often used.

Fresnel class and Fraunhofer class of diffraction

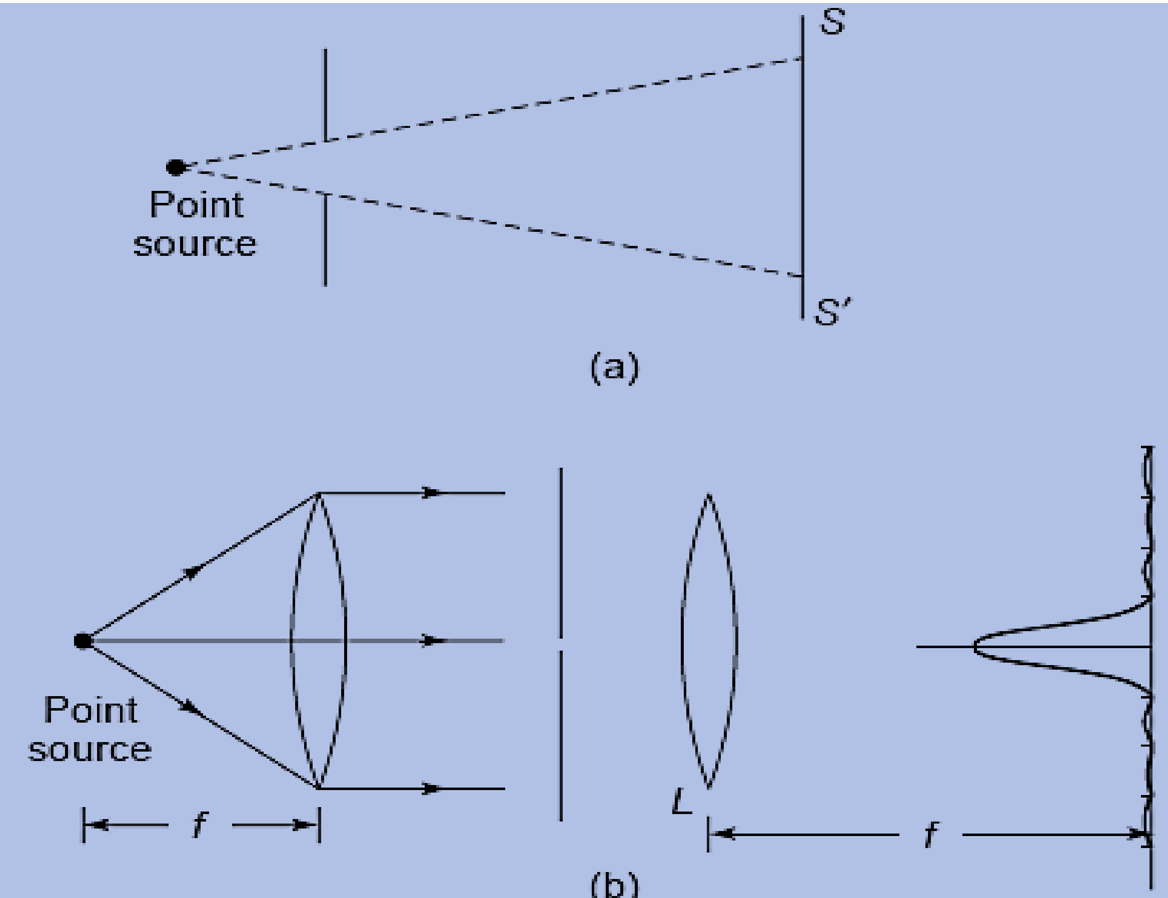
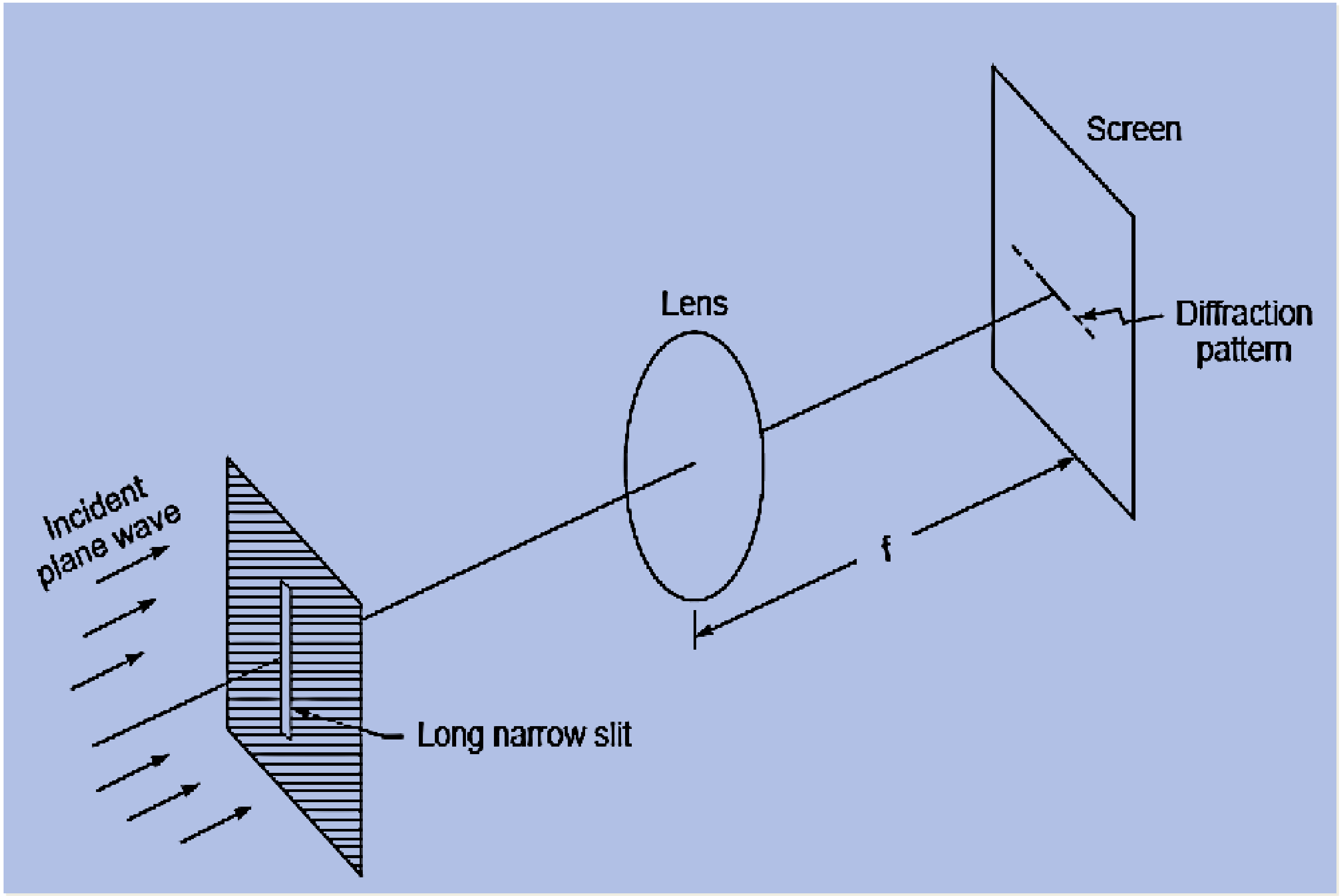


Fig. 18.2 (a) When either the source or the screen (or both) is at a finite distance from the aperture, the diffraction pattern corresponds to the Fresnel class. (b) In the Fraunhofer class both the source and the screen are at infinity.

Diffraction at single slit:



Diffraction due to slits

Assumptions: (i) the source of light is a coherent source of light (ii) single slit is a one dimensional object (like a line segment without much width). Each point of line segment is acting as a source of light.

Our aim: to calculate intensity due to single slit and multiple slits placed at equal distance.

Strategy: consider equally spaced points along the width of slits. Let these points to be acting as secondary source of light. Light emanating from these point sources interfere. We want to obtain the resultant electric field due to all the points and hence the Intensity.

Light emanating from these points turn out to have these form

$a \cos(\omega t)$

$a \cos(\omega t - \varphi)$

$a \cos(\omega t - 2\varphi)$

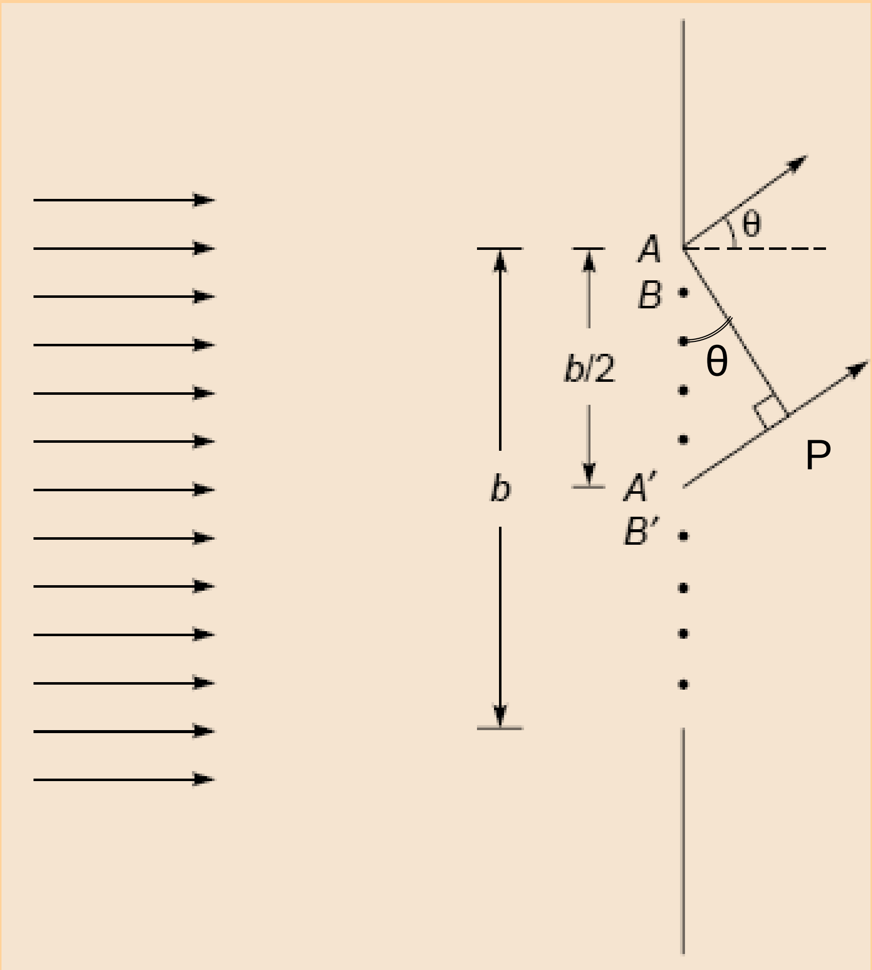
....

....

$a \cos(\omega t - (n-1)\varphi)$

where a is the amplitude of electric field, ω is the frequency, φ is the phase difference due to path difference from two neighb oring points.

A qualitative way to understand the condition for diffraction minima



Divide the slit in two parts. Suppose we are interested in minima. Then, for each point A in one part, there exists a point A' in other part so that path difference at angle θ is $\lambda/2$

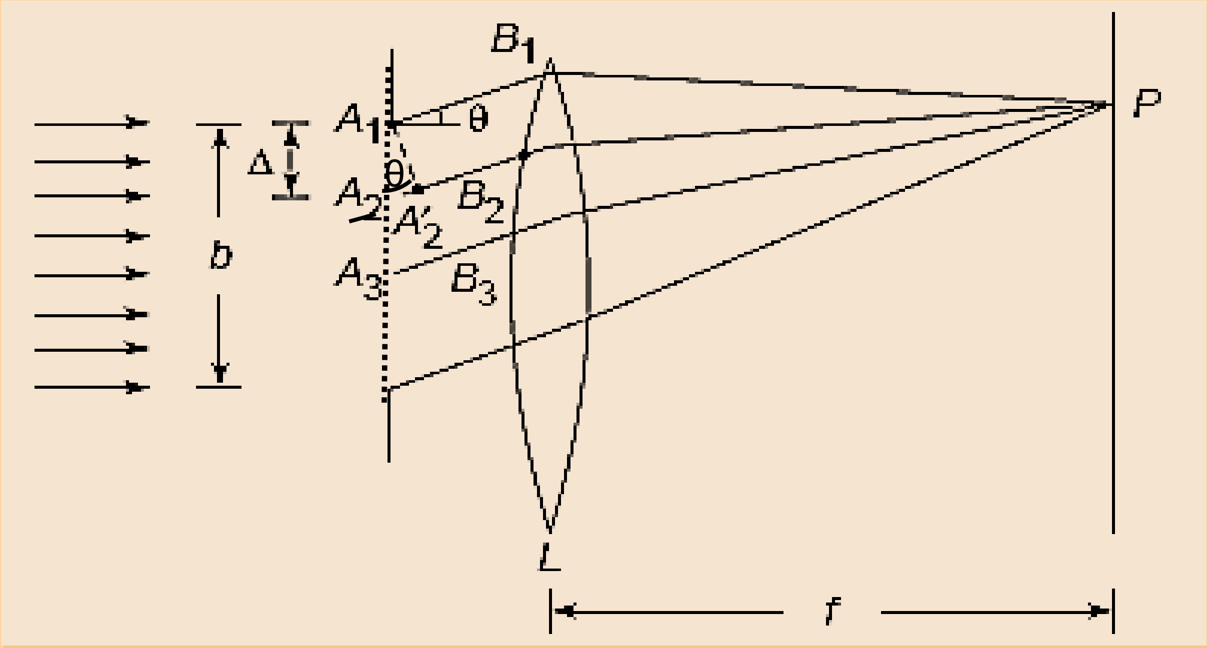
$$A'P = (b/2)sin(\theta) = \lambda/2$$

Thus, $bsin(\theta) = \lambda$

which is the condition for first minima

Note: this is only to give an qualitative idea

Calculation of resultant intensity in diffraction (single slit)

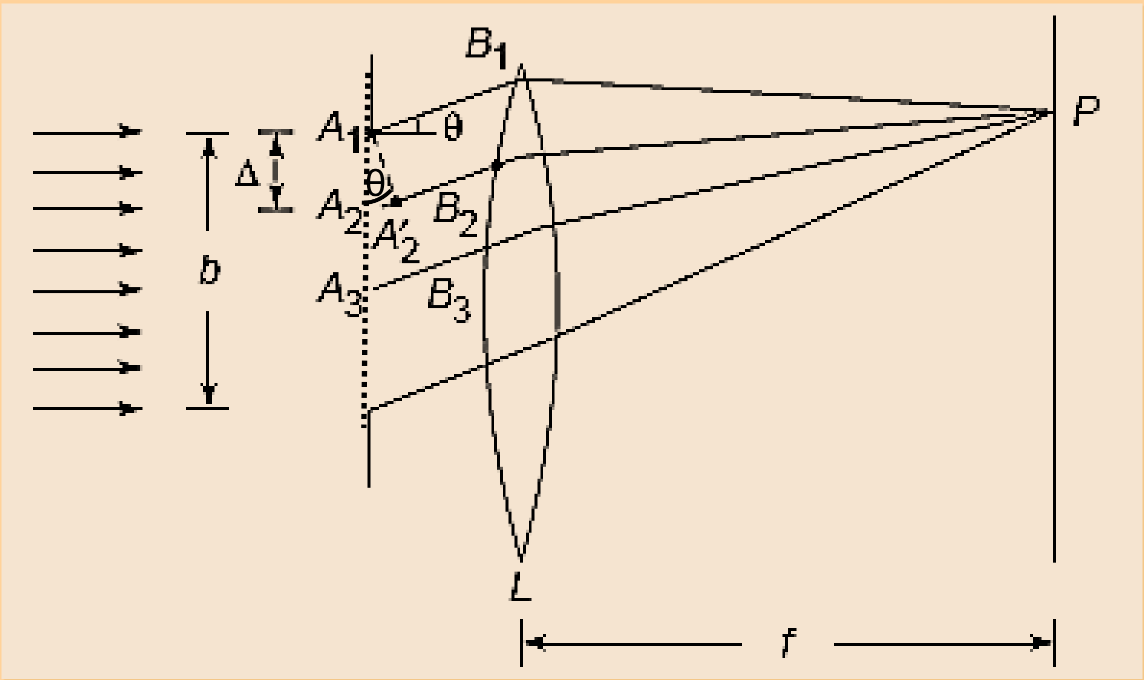


- Consider n equidistant points (A_1, A_2, A_3 …..) along the slit.
- According to Huygens principle, secondary wavelets emerge from these n points.
- These wavelets interfere, which results into diffraction.
- Let the distance between two consecutive points be

$A_1A_2 = A_2A_3 = A_3A_4 = \ldots\ldots\ldots\ldots = \Delta$.

- Then, slit width $b = (n – 1)\Delta$.

Phase difference ϕ between two consecutive points



- Consider rays A_1B_1, A_2B_2, A_3B_3, etc making an angle θ with the line perpendicular to the screen.
- $A_1A'_2$ is perpendicular to A_2B_2

- Additional path traversed by the ray A_2B_2 *in* comparison to A_1B_1 will be $A_2A'_2$, where $A_2A'_2 = \Delta\sin(\theta)$.
- Phase difference between rays A_1B_1 and A_2B_2 is $\varphi = (2\pi/\lambda)\Delta\sin(\theta)$
- If the field at point P due to point source A_1 is "$a\cos(\omega t)$", then the field due to the A_2 will be "$a\cos(\omega t - \varphi)$".

Calculation of resultant intensity in diffraction (single slit) ?

- Similarly, field due to the disturbance emanating from A_3 is "$a\cos(\omega t - 2\varphi)$" and so on. For A_n it will be "$a\cos(\omega t - (n\text{-}1)\varphi)$".

- Resultant field at point P will be,

$\boldsymbol{E} = a\cos(\omega t) + a\cos(\omega t - \varphi) + a\cos(\omega t - 2\varphi) + \ldots\ldots + a\cos(\omega t - (n\text{-}1)\varphi)$ where $\varphi = (2\pi/\lambda).\Delta.\sin(\theta)$

- But mathematically,

$$a\cos(\omega t) + a\cos(\omega t - \varphi) + a\cos(\omega t - 2\varphi) + \ldots\ldots\ldots\ldots + a\cos(\omega t - (n\text{-}1)\varphi)$$

$$= a\frac{\sin(n\varphi/2)}{\sin(\varphi/2)}\cos\left(\omega t - \frac{1}{2}(n-1)\varphi\right)$$

Note: the whole exercise is to obtain the amplitude of resultant electric field.

Calculation of resultant intensity in diffraction (single slit) ?

Thus, $E = a\dfrac{\sin(n\varphi/2)}{\sin(\varphi/2)}\cos\left(\omega t - \dfrac{1}{2}(n-1)\varphi\right) = E_\theta \cos\left(\omega t - \dfrac{1}{2}(n-1)\varphi\right)$

where the amplitude E_θ of the resultant field is given by

$E_\theta = a\dfrac{\sin(n\varphi/2)}{\sin(\varphi/2)}$ with $\varphi = (2\pi/\lambda).\Delta.\sin(\theta)$

When number of points *n* very large

In the limit of $\boldsymbol{n \to \infty}$ and $\boldsymbol{\Delta \to 0}$, we can see that $\boldsymbol{n\Delta \to b}$,

Therefore, $\dfrac{n\varphi}{2} = \dfrac{\pi}{\lambda} n \Delta \sin(\theta) \rightarrow \dfrac{\pi}{\lambda} b \sin(\theta)$

Also, if $\boldsymbol{n \to \infty}$ then $\boldsymbol{\varphi \to 0;\ \sin(\varphi/2) = \varphi/2}$

Recall $\varphi = \dfrac{2\pi}{\lambda}\dfrac{b\sin(\theta)}{n}$

Note: the whole exercise is to obtain the amplitude of resultant electric field

Calculation of resultant intensity in diffraction (single slit)

Therefore, $$E_\theta \approx a\frac{\sin(n\varphi/2)}{\varphi/2} = na\frac{\sin(\pi b\sin(\theta)/\lambda)}{\pi b\sin(\theta)/\lambda}$$

$$= A\frac{\sin\beta}{\beta}$$

$$where\,,\ A = na\,;\ \beta = \frac{\pi b\sin(\theta)}{\lambda} = n\frac{\varphi}{2}$$

Thus,

$$E = A\frac{\sin(\beta)}{\beta}\cos\left(\omega t - \frac{1}{2}(n-1)\varphi\right)$$

$$E = A\frac{\sin(\beta)}{\beta}\cos(\omega t - \beta)\,,\ because\ n \rightarrow \infty$$

and $n\phi/2 \approx (n-1)\phi/2$

Intensity distribution for this will be given by :

$$I = I_0\frac{\sin^2\beta}{\beta^2}$$

where I_0 ($I_0 = A^2$) gives intensity at θ=0.

Minima

For minima, intensity should be zero, which is possible when $\sin\beta = 0$ *or* $\beta = m\pi$, $m \neq 0$

Thus, $\frac{\pi b \sin(\theta)}{\lambda} = m\pi$ because, $\beta = \frac{\pi b \sin(\theta)}{\lambda}$

which implies $b \sin\theta = m\lambda$, with $m = \pm 1, \pm 2, \pm 3, \ldots\ldots$ **(for minima)**

- First minimum occurs for $\theta = \pm \sin^{-1}(\lambda/b)$,
  Second minimum occurs for $\theta = \pm \sin^{-1}(2\lambda/b)$
  and so on.

- Maximum possible value of $\sin(\theta)$ is 1, therefore maximum possible value of m is integer close to b/λ.

Central maximum

- If $m = 0$, or $\beta = 0$. $\sin\beta/\beta$ will have an indeterminate form.

- In the limit $\beta \to 0$ or $\theta = 0$, $(\sin\beta)/\beta = 1$ and $I = I_0$, which corresponds to the maximum of value of intensity. This point corresponds to center of central maxima.

Other maxima

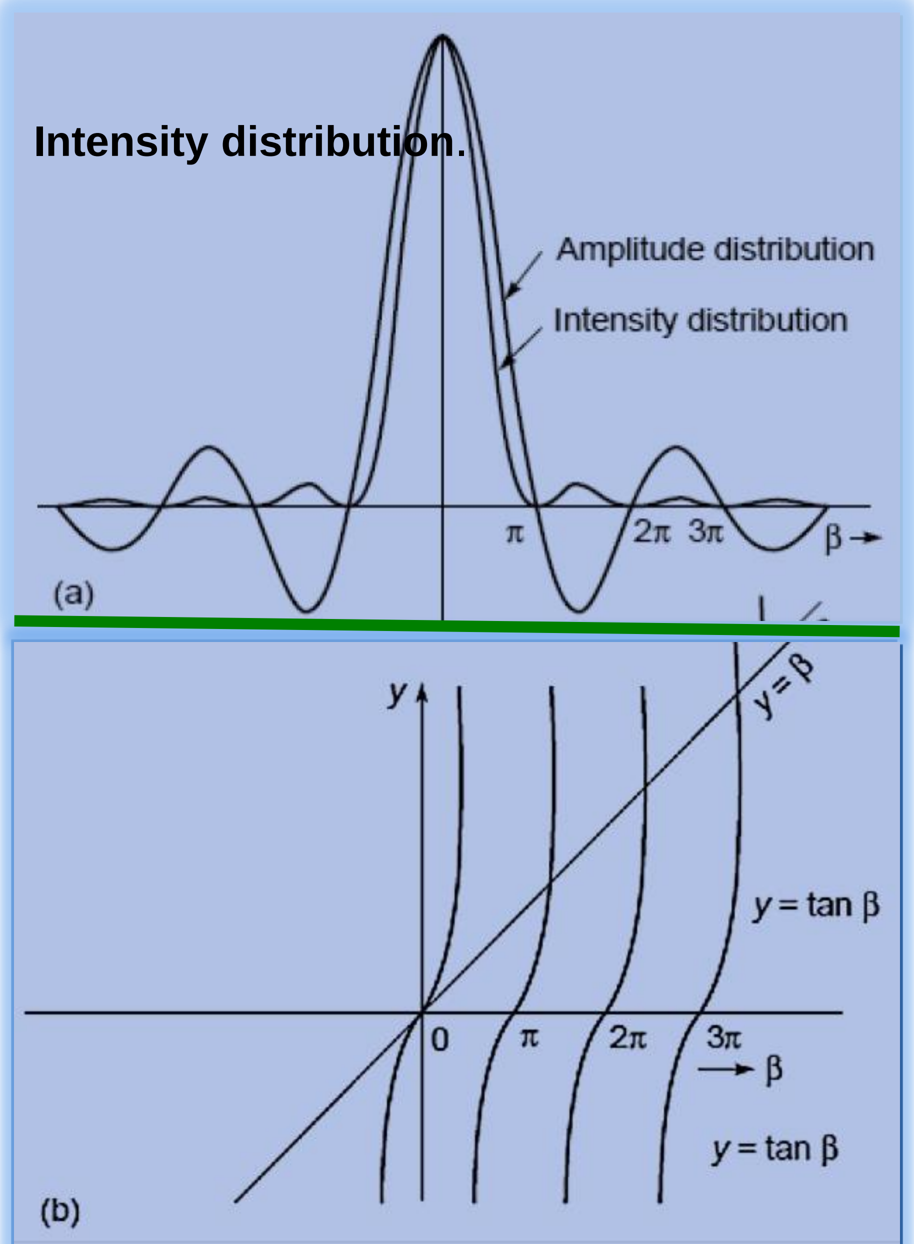
Differentiate intensity in order to find maximum or minimum

Intensity due to single slit $$I = I_0 \frac{\sin^2\beta}{\beta^2}$$

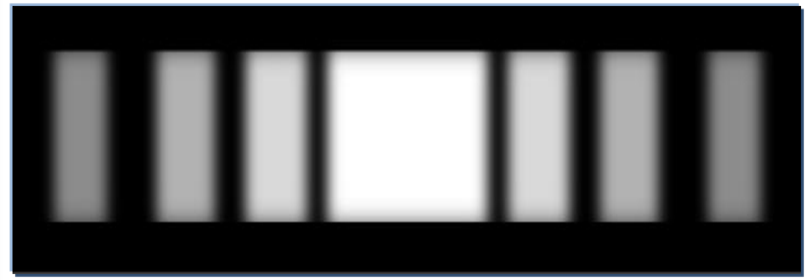
Differentiating with Respect to β $$\frac{dI}{d\beta} = I_0 \left(\frac{2\sin\beta\cos\beta}{\beta^2} - \frac{2\sin^2\beta}{\beta^3} \right) = 0$$

$$\Rightarrow \sin\beta(\beta - \tan\beta) = 0$$

Condition ***tan(β) = β*** gives maxima. ***β=0*** gives central maxima. Rest of roots are found by intersection of curves ***y = β*** and *y* = ***tan(β)***. ***(Note sin(β) = 0 gives minima condition)***



Intensity pattern on the screen will look like this



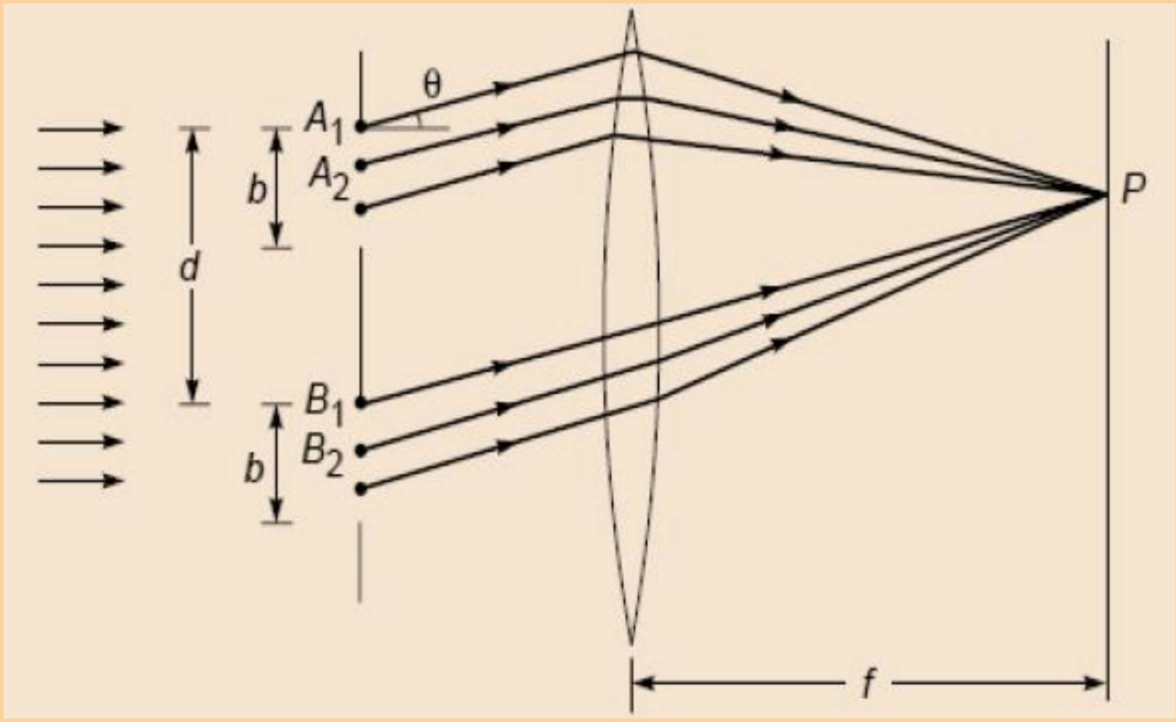
Roots of equation tan(β) = β. Few roots are: β=1.43π, 2.86π and so on.

Calculation of intensity patterns due to double slits

❖ Two slits each of width *b* and separated by distance *d*.

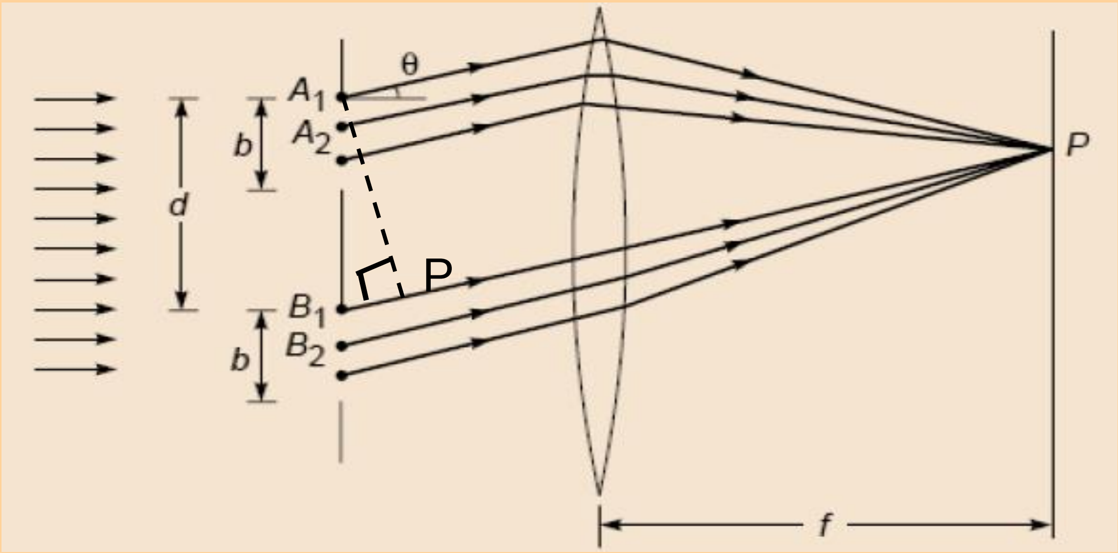
❖ Consider *n* equidistant points (A_1, A_2, A_3 …..) along the slit 1 and (B_1, B_2, B_3 …..) along slit 2.

❖ Distance between A_1 and B_1 or between A_2 and B_2 and so on is *d*.



Resultant intensity distribution will be a combination of the single-slit diffraction pattern and the interference pattern produced by two point sources separated by distance "*d*".

Calculation of intensity patterns due to double slits



❖ A_1P is perpendicular to ray emanating from B_1

❖ Path difference between rays emanating from A_1 *and* B_1 *is* B_1P

❖ Phase difference between rays from A_1 and B_1, $\phi_1 = (2\pi \times B_1P)/\lambda$

The electric field produced by first slit at point P, $E_1 = A\dfrac{\sin(\beta)}{\beta}\cos(\omega t - \beta)$

The field produced by second slit at point P, $E_2 = A\dfrac{\sin(\beta)}{\beta}\cos(\omega t - \beta - \Phi_1)$

where $\Phi_1 = \dfrac{2\pi}{\lambda} d\sin(\theta)$

Φ_1 gives phase difference between disturbances reaching point P from pair of points such *as* (A_1, B_1), (A_2, B_2)... separated by distance d.

Calculation of intensity patterns due to double slits

The resultant field will be given by:

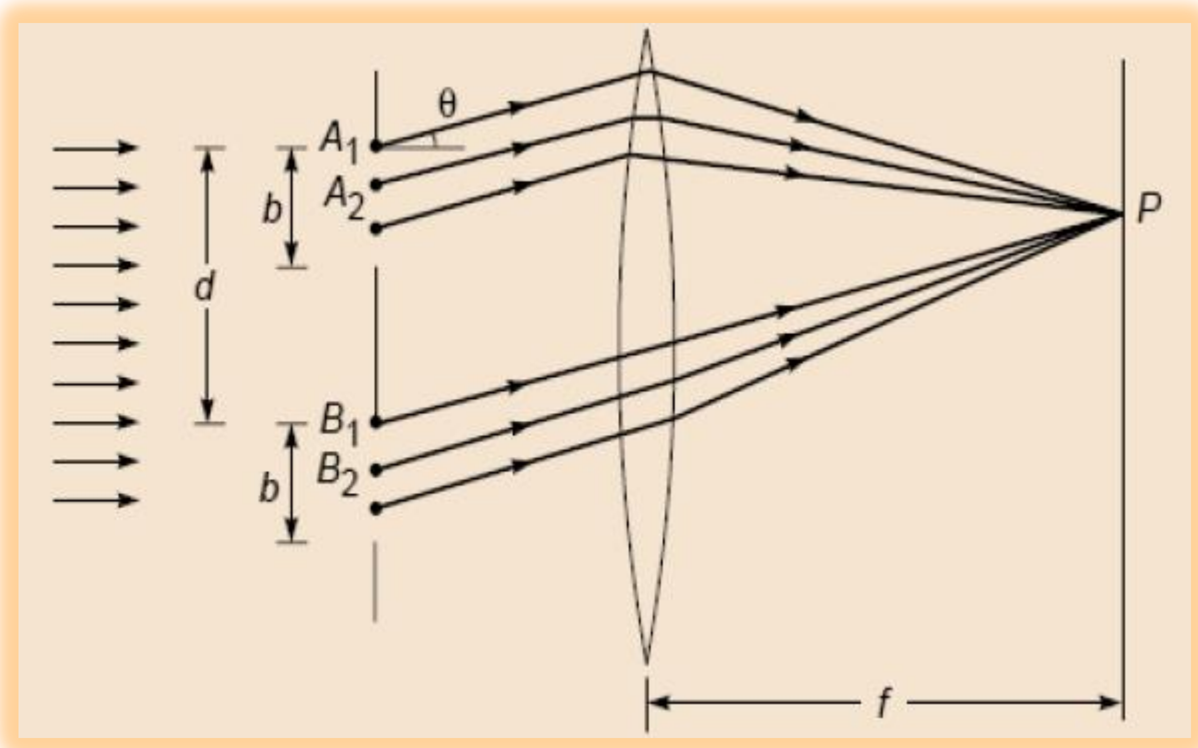
$$
\begin{aligned}
E &= E_1 + E_2 \\
&= A\frac{\sin(\beta)}{\beta}\cos(\omega t-\beta) + A\frac{\sin(\beta)}{\beta}\cos(\omega t-\beta-\Phi_1) \\
&= A\frac{\sin(\beta)}{\beta}\left(\cos(\omega t-\beta)+\cos(\omega t-\beta-\Phi_1)\right) \\
&= 2A\frac{\sin(\beta)}{\beta}\cos\left(\frac{\Phi_1}{2}\right)\cos\left(\omega t-\beta-\frac{\Phi_1}{2}\right)
\end{aligned}
$$

The intensity distribution will be given by (squaring the amplitude of electric field)

$$\boxed{I = 4I_0\frac{\sin^2\beta}{\beta^2}\cos^2\left(\frac{\Phi_1}{2}\right);\ where\ I_0 = A^2}$$

In this expression, $(\sin^2\beta)/\beta^2$ represents diffraction pattern produced by single slit of width b. Second term $\cos^2(\Phi_1/2)$ represents interference produced by two slits sources separated by distance d.

Intensity patterns due to double slits



$$I = 4 I_0 \frac{\sin^2\beta}{\beta^2} \cos^2\left(\frac{\Phi_1}{2}\right)$$

$$\beta = \frac{\pi b \sin(\theta)}{\lambda}$$

If slit widths are very small (b→0, sin(β)→β) so that there is no variation of the term $(\sin^2\beta)/\beta^2$ with θ, then one simply obtains Young's interference pattern.

Positions of minima

Intensity in double slit

$$I = 4 I_0 \frac{\sin^2\beta}{\beta^2}\cos^2\left(\frac{\Phi_1}{2}\right) \quad \text{where} \quad \beta = \frac{\pi b \sin(\theta)}{\lambda}; \Phi_1 = \frac{2\pi}{\lambda} d \sin(\theta)$$

Intensity is zero wherever $\beta = \pi, 2\pi, 3\pi \ldots$..
which means $b\sin(\theta) = m\lambda$, $m = 1,2,3\ldots$….

or

When $\Phi_1 = \pi, 3\pi, 5\pi \ldots$.., it means $d\sin(\theta) = (n-1/2)\lambda$, with $n = 1,2,3\ldots$….

Positions of maxima

$$I = 4 I_0 \frac{\sin^2\beta}{\beta^2}\cos^2\left(\frac{\Phi_1}{2}\right) \quad \text{where} \quad \beta = \frac{\pi b \sin(\theta)}{\lambda} ; \Phi_1 = \frac{2\pi}{\lambda} d \sin(\theta)$$

Interference maxima:

When $\boldsymbol{\Phi_1/2 = 0, \pi, 2\pi}$ **…..** or $\boldsymbol{(\pi/\lambda)d\sin(\theta) = 0, \pi, 2\pi}$**……**

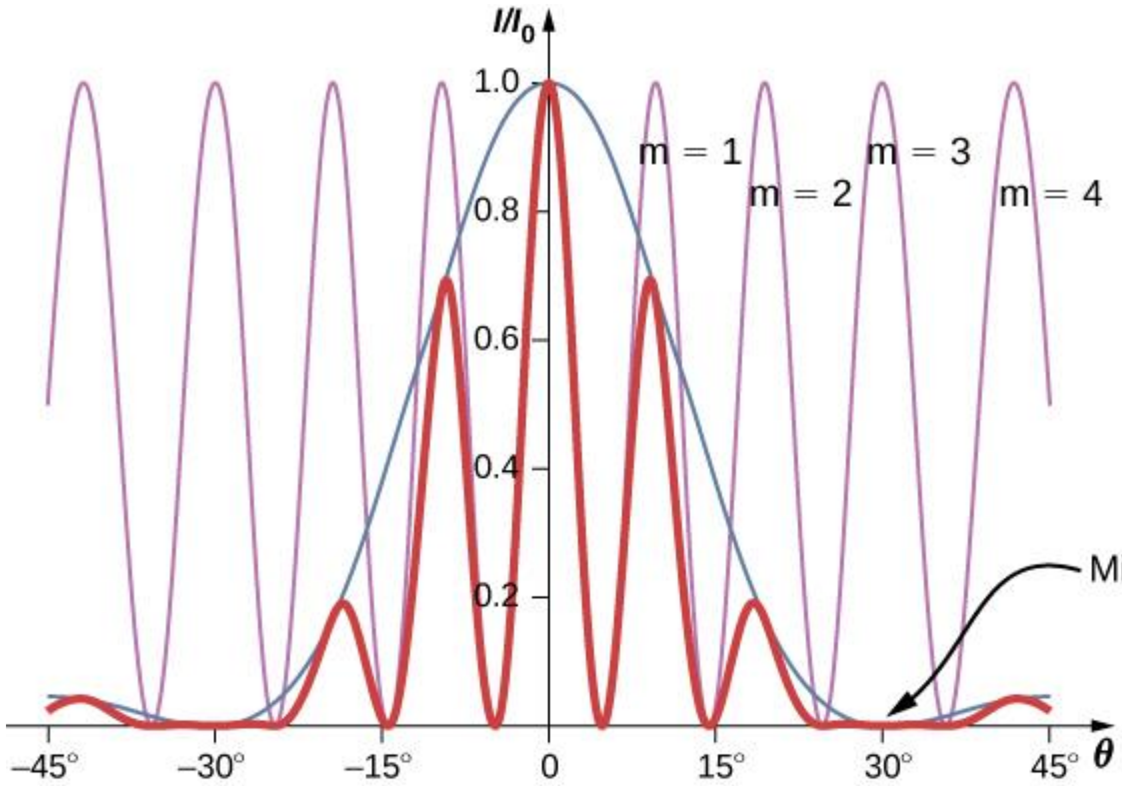
which means $\boldsymbol{d\sin(\theta) = 0, \lambda, 2\lambda, 3\lambda}$**…….**

This can be used to calculate fringe width for double slit interference maxima.

This will be approximate positions of maxima provided variation of diffraction pattern is not too rapid.

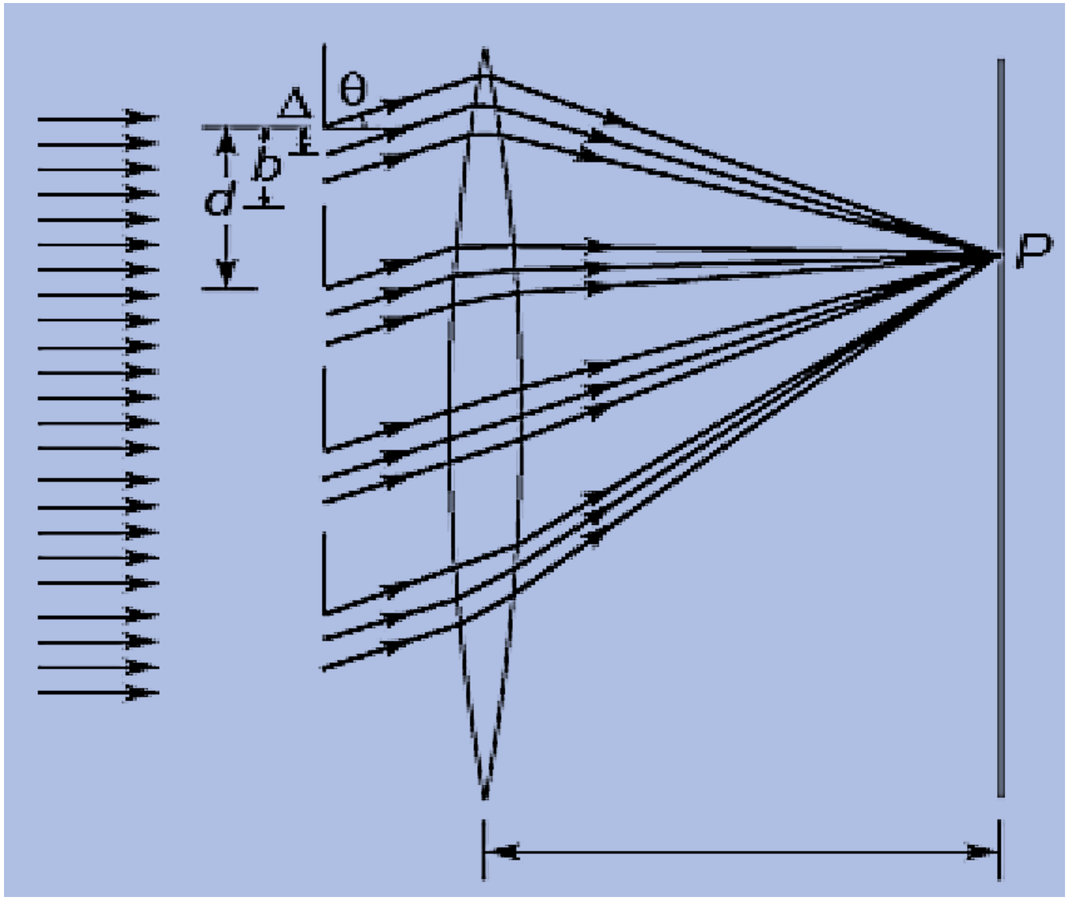
A maximum may not occur at all if θ corresponds to diffraction minimum ($\boldsymbol{b \sin(\theta) = \lambda, 2\lambda, 3\lambda}$**…..**). ***These are called missing orders.*** Missing order will occur when d is integral multiple of b ***so that the diffraction minimum coincides with interference maximum.***

Missing order of interference maxima



Slit width $b = 2\lambda$, slit separation $d = 6\lambda$, Since $d = 3b$, the third order interference maxima will be missing as it coincides with the first order minima, similarly 6th order maxima, 9th order maxima….. so on will be missing.

Calculation of intensity patterns due to N slits



b is the slit size

d is the slit separation

Calculation of intensity patterns due to N slits

The resultant field at any arbitrary point P will be :

$$E = A\frac{\sin(\beta)}{\beta}\cos(\omega t-\beta) + A\frac{\sin(\beta)}{\beta}\cos(\omega t-\beta-\Phi_1) + A\frac{\sin(\beta)}{\beta}$$

$$\cos(\omega t-\beta-2\Phi_1) + \ldots\ldots + A\frac{\sin(\beta)}{\beta}\cos(\omega t-\beta-(N-1)\Phi_1)$$

$$= A\frac{\sin(\beta)}{\beta}\left(\cos(\omega t-\beta)+\cos(\omega t-\beta-\Phi_1)+\cos(\omega t-\beta-2\Phi_1)+\ldots\ldots+\cos(\omega t-\beta-(N-1)\Phi_1)\right)$$

Using same triganometric relation as used in single slit

$$= A\frac{\sin(\beta)}{\beta}\frac{\sin\left(\frac{N\Phi_1}{2}\right)}{\sin\left(\frac{\Phi_1}{2}\right)}\cos\left(\omega t-\beta-\frac{(N-1)}{2}\Phi_1\right)$$

where $\beta = \frac{\pi b\sin(\theta)}{\lambda}; \Phi_1 = \frac{2\pi}{\lambda}d\sin(\theta)$

Intensity distribution due N slits

Intensity due *N* Slits:

$$I = I_0 \frac{\sin^2(\beta)}{\beta^2} \frac{\sin^2\left(\frac{N\Phi_1}{2}\right)}{\sin^2\left(\frac{\Phi_1}{2}\right)}$$

In this expression, $\sin^2\beta/\beta^2$ represents diffraction pattern produced by single slit of width b.

Second term $\sin^2(N\Phi_1/2)/\sin^2(\Phi_1/2))$ represents interference produced by N equally spaced point sources separated by distance d.

Special cases

Intensity distribution due to *N* slits: $$I = I_0 \frac{\sin^2(\beta)}{\beta^2} \frac{\sin^2(\frac{N\Phi_1}{2})}{\sin^2(\frac{\Phi_1}{2})}$$

Single slit result

$$\boxed{\text{Substituting } N = 1: \qquad I = I_0 \frac{\sin^2(\beta)}{\beta^2}}$$

Double slit result

Substituting $N = 2$: $$I = I_0 \frac{\sin^2(\beta)}{\beta^2} \frac{\sin^2(\frac{2\Phi_1}{2})}{\sin^2(\frac{\Phi_1}{2})}$$

$$= I_0 \frac{\sin^2(\beta)}{\beta^2} \frac{(2\sin(\frac{\Phi_1}{2})\cos(\frac{\Phi_1}{2}))^2}{\sin^2(\frac{\Phi_1}{2})}$$

$$\boxed{= 4 I_0 \frac{\sin^2(\beta)}{\beta^2} \cos^2(\frac{\Phi_1}{2})}$$

Position of principal maxima due to N slits

From intensity distribution due to N slits:

$$I = I_0 \frac{\sin^2(\beta)}{\beta^2} \frac{\sin^2\left(\frac{N\Phi_1}{2}\right)}{\sin^2\left(\frac{\Phi_1}{2}\right)}$$

The condition for principal maxima (maximum possible intensity) is :

$$\frac{\Phi_1}{2} = m\pi\,;\, m = 0,1,2,3\ldots$$

$$\Phi_1 = \frac{2\pi}{\lambda} d\sin(\theta)$$

$$\Rightarrow \frac{\pi}{\lambda} d\sin(\theta) = m\pi$$

$$\boxed{\Rightarrow d\sin(\theta) = m\lambda\,;\, m = 0,1,2,3\ldots}$$

Intensity at Maxima

Resultant electric field due to N slits

$$E = A\frac{\sin(\beta)}{\beta}\frac{\sin\left(\frac{N\Phi_1}{2}\right)}{\sin\left(\frac{\Phi_1}{2}\right)}\cos\left(\omega t-\beta-\frac{(N-1)}{2}\Phi_1\right)$$

L'Hospital's rule

$$\lim_{x\to a} f(x) = 0 \text{ and } \lim_{x\to a} g(x) = 0$$

$$\lim_{x\to a}\frac{f(x)}{g(x)} = \lim_{x\to a}\frac{f'(x)}{g'(x)}.$$

But

$$\lim_{\frac{\Phi_1}{2}\to m\pi}\frac{\sin\left(\frac{N\Phi_1}{2}\right)}{\sin\left(\frac{\Phi_1}{2}\right)} = \lim_{\frac{\Phi_1}{2}\to m\pi}\frac{N\cos\left(\frac{N\Phi_1}{2}\right)}{\cos\left(\frac{\Phi_1}{2}\right)} = \pm N$$

where we used L' Hospital's rule.

Thus, the net electric field at Maxima

$$E = N\frac{A\sin(\beta)}{\beta}\cos\left(\omega t-\beta-\frac{(N-1)}{2}\Phi_1\right)$$

Physically, at these maxima fields produced by each of the slits are in phase and hence resultant field (E) is N times of field produced by single slit

Intensity at Maxima

Intensity at maximum positions

$$\boxed{I = I_0 N^2 \frac{\sin^2(\beta)}{\beta^2}}$$

where

$$\beta = \frac{\pi b \sin(\theta)}{\lambda} = \frac{\pi b}{\lambda}\frac{m\lambda}{d} = \frac{\pi b m}{d};\, m = 0,1,2,3.....$$

Intensity has large value unless $\sin^2\beta/\beta^2$ itself is small.

➢ Since $\sin(\theta) \leq 1$, m can not be greater than d/λ.

➢ It means more is the number of slit ***(N)***, intensity of maxima will be more.

➢ This concept is used in diffraction grating where you have 10000 or so many slits per inch!

Minima

Recalling the intensity due to N slits

$$I = I_0 \frac{\sin^2(\beta)}{\beta^2} \frac{\sin^2\left(\frac{N\Phi_1}{2}\right)}{\sin^2\left(\frac{\Phi_1}{2}\right)} \quad \text{where} \quad \beta = \frac{\pi b \sin(\theta)}{\lambda} ; \Phi_1 = \frac{2\pi}{\lambda} d \sin(\theta)$$

Intensity is zero when

$\beta = \pi, 2\pi, 3\pi$ ….. (minima for single slit!)

$b\sin(\theta) = n\lambda$, n $= 1,2,3$…….

Or

Intensity is also zero when $N\Phi_1/2 = p\pi$, but $p \neq N, 2N, 3N$……

(Note: intensity has indeterminate form for $p \neq N, 2N, 3N$……

As numerator and denominator both vanish)

$$\frac{N\Phi_1}{2} = p\pi \Rightarrow \frac{N \frac{2\pi}{\lambda} d \sin(\theta)}{2} = p\pi$$

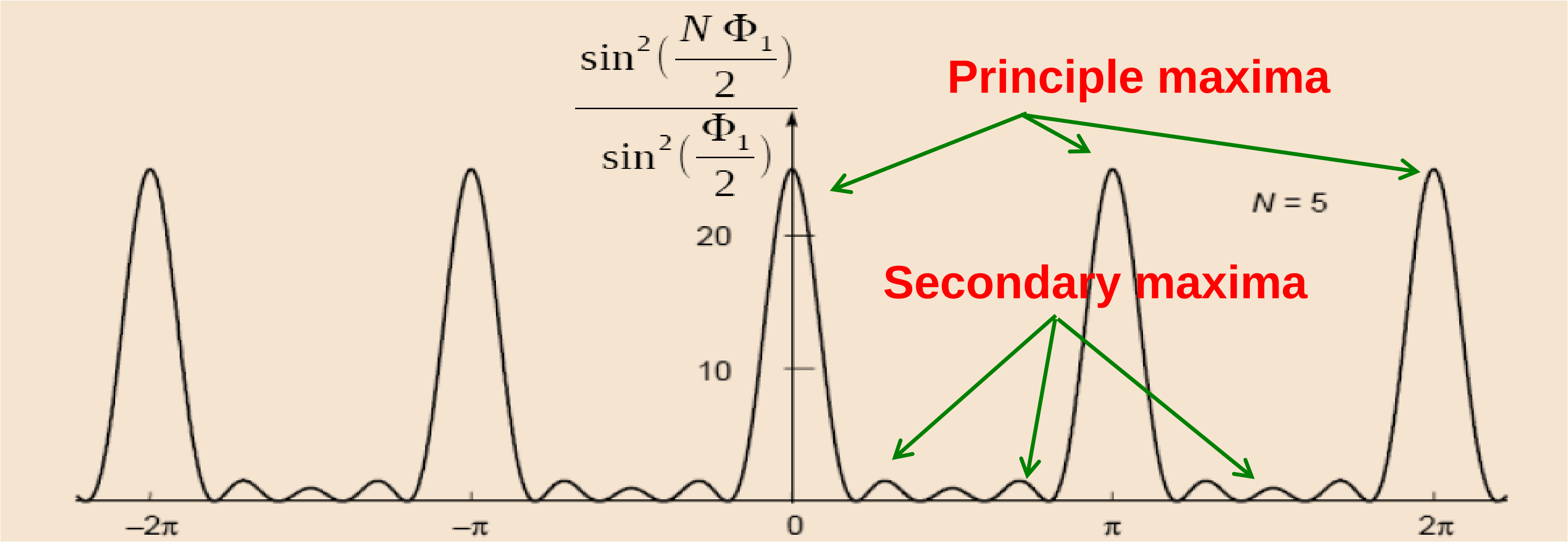
$$\Rightarrow d\sin(\theta) = p\frac{\lambda}{N}$$

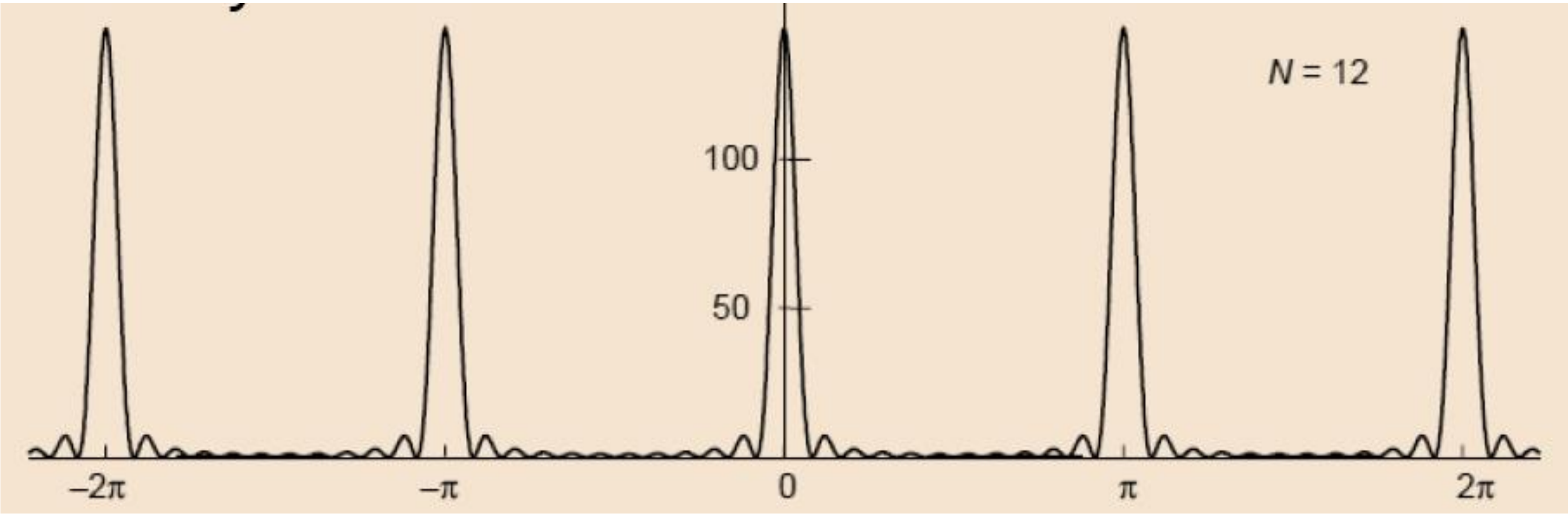
$$d\sin(\theta)=\frac{p\lambda}{N};\ p\neq N, 2N, 3N.....$$

Or $$d\sin(\theta)=\frac{\lambda}{N}, \frac{2\lambda}{N}, \frac{3\lambda}{N}....\frac{(N-1)\lambda}{N}, \frac{(N+1)\lambda}{N}, \frac{(N+2)\lambda}{N}.....$$

$$\frac{(2N-1)\lambda}{N}, \frac{(2N+1)\lambda}{N}, \frac{(2N+2)\lambda}{N}$$

- Notice the missing terms! They correspond to principal maxima. There are (N-1) minima between 2 principal maxima.
- Between two consecutive minima, intensity has to have a maximum. These maxima are called secondary maxima. There will be (N-2) secondary maxima between two principal maxima.





How many secondary maxima for double slit?

Missing order

What if angle of principal maxima is same as diffraction minima?

In that situation, following conditions are satisfied simultaneously,

$d \sin(\theta) = m\lambda, \quad m=0,1,2,\ldots\ldots$ (Principal maxima)

$b\sin(\theta) = \lambda, 2\lambda, 3\lambda\ldots\ldots$ (Diffraction minima)

Thus, certain principal maxima cannot be observed when distance or separation between the slits is integral multiple of slit width. These maxima coincide with diffraction minimum and thus cannot be observed and therefore are referred as **missing order.**

Diffraction grating:

A system of very large number of equidistant slits is called diffraction grating. The diffraction pattern formed due to diffraction grating is defined as the grating spectrum.
It is used as monochromators, spectrometers etc.

- Principal maxima can be obtained using $d \sin(\theta) = m\lambda$, where $m=0,1,2,\ldots$(This is the same expression for N slits.)

- Principal maxima $(m \neq 0)$ for different λ are observed at different angles.

- The order of principle maxima and the angle at which it is observed can be used to measure the wavelength.

- More is number of slits, narrower will be principal maxima.
(Usually 15,000 per inch slits are there. Lines should be as equally spaced as much as possible.)

Dispersive power of grating

Dispersive power of a grating is defined as the diffraction or spread in angle for a unit spread in the wavelength.
This is an important quantity when light consists of a spectrum.

We know that the principal maxima is given by

$d \sin(\theta) = m\lambda, \quad m=0,1,2,\ldots\ldots$

Differentiating with respect to λ on both side of this equation

$$d\cos(\theta)\frac{d\theta}{d\lambda} = m$$

$$d\cos(\theta)\Delta\theta = m\Delta\lambda$$

$$\Rightarrow \frac{\Delta\theta}{\Delta\lambda} = \frac{m}{d\cos(\theta)}$$

Dispersive power is defined as

$$\boxed{\frac{\Delta\theta}{\Delta\lambda} = \frac{m}{d\cos(\theta)}}$$

Points to note:

- Dispersive power is proportional to "*m*" (order of principal maximum). Higher is *m*, well separated will be maxima corresponding to 2 close wavelengths like sodium doublet. Zeroth order principal maxima will overlap.

- Dispersive power is inversely proportional to "*d*" (the grating element). Smaller is "*d*", larger will be angular dispersion.

- Dispersive power is inversely proportional to cos(θ). If θ is very small then cos(θ)≃1,

Such spectrum is known as normal spectrum. For this dθ is directly proportional to dλ.

$$\frac{d\theta}{d\lambda} = \frac{m}{d\cos(\theta)} \quad \text{becomes} \quad \frac{d\theta}{d\lambda} = \frac{m}{d}$$

Sodium doublet consists of wavelengths 589.0 nm and 589.6 nm

Grating Spectrum

- Principal maxima: $d \sin(\theta) = m\lambda$, *where* $m=0,1,2,\ldots\ldots..$
  This equation is also called the grating equation.

- The zeroth-order principal maxima occurs at $\theta=0$ irrespective of wavelength.

- Thus for white light, central maximum will be white. For $m\neq0$, θ are different for different λ, various spectral components appear at different locations.

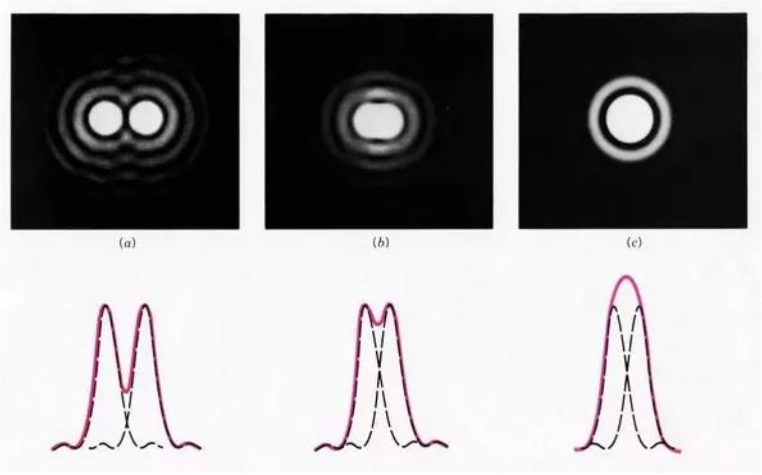
Resolving power of grating

- The capacity of an instrument to resolve two closely spaced objects or spectrum is called it's resolving power.

- Minimum separation at which two objects look separate is called "limit of resolution".

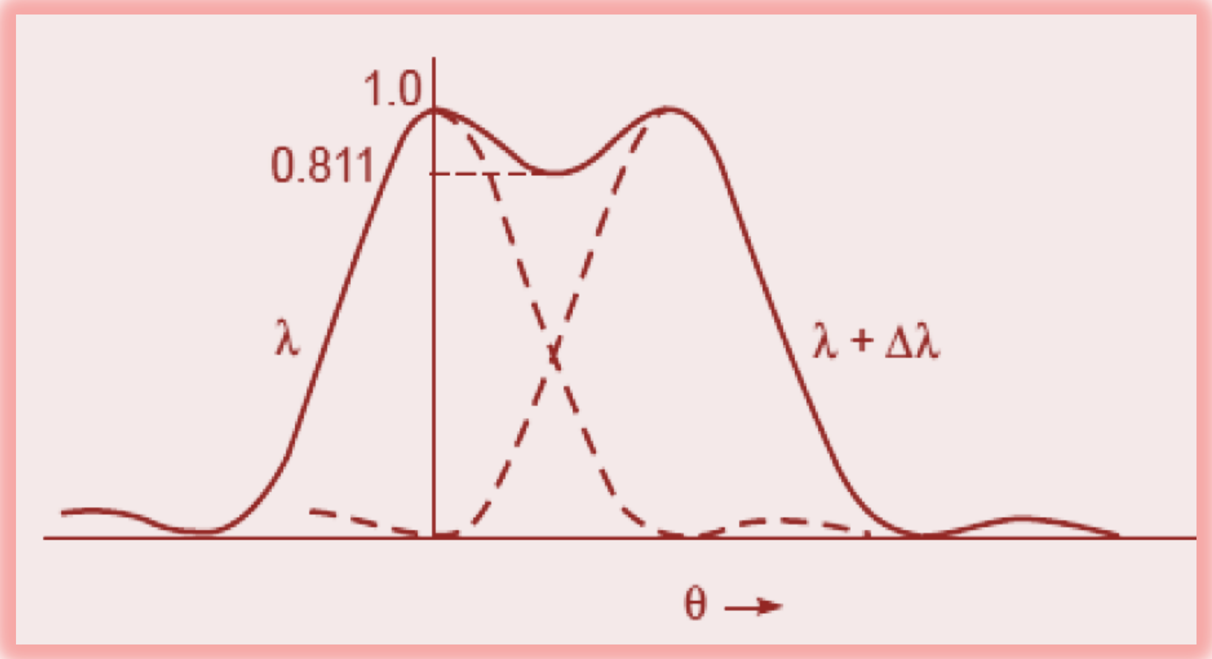
Smaller is separation between 2 objects an instrument can resolve, higher is its resolving power and better is the Instrument.

- In case of diffraction grating, resolving power is power of distinguishing two nearby spectral lines.

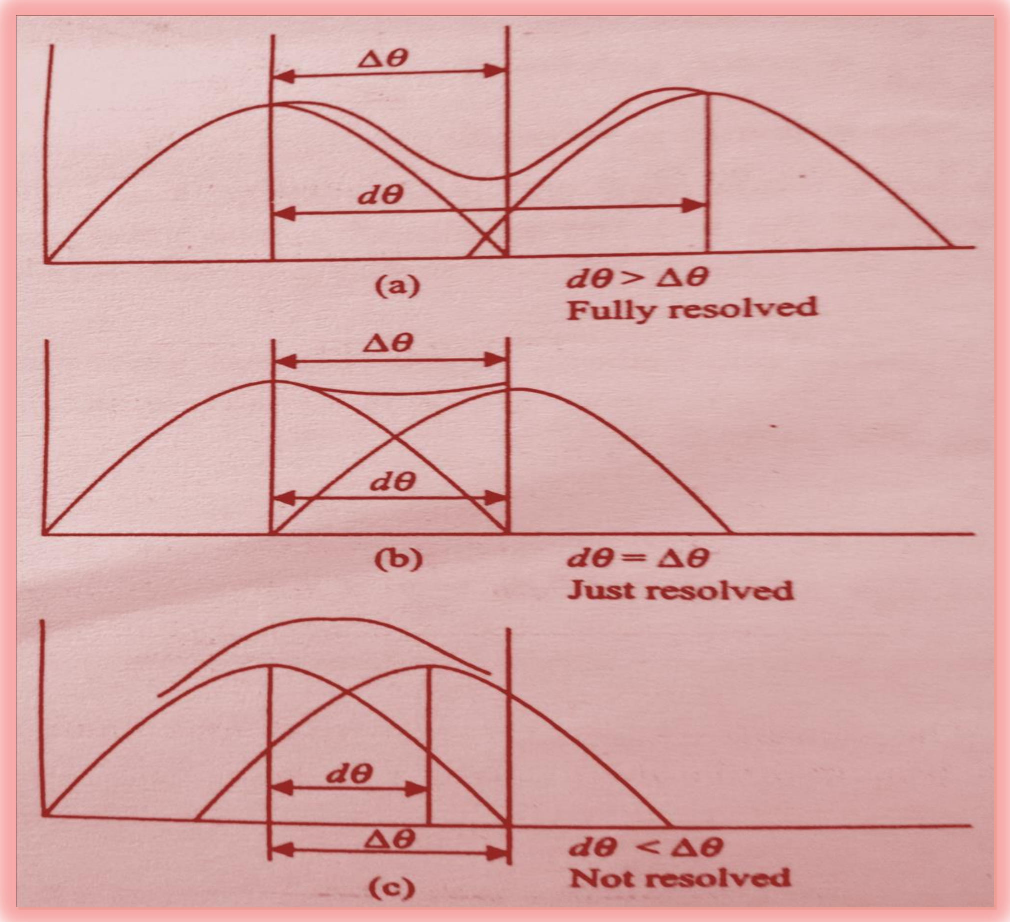


Rayleigh's Criterion

According to Rayleigh's Criterion, if the principal maximum corresponding to wavelength $\lambda + \Delta\lambda$ falls on first minimum (on either side) of the wavelength λ, then the two wavelengths λ and $\lambda + \Delta\lambda$ are said to be just resolved.



dθ = Angular separation between the principal maxima of 2 patterns
Δθ = half angular width of principal maxima of each pattern



Resolving power of grating

- If angle θ is the angle corresponding to mth order spectrum then these conditions are satisfied simultaneously :

- Principal maximum for wavelength $\lambda+\Delta\lambda$:
  $d \sin(\theta) = m(\lambda+\Delta\lambda)$

- Minimum for wavelength λ:
  $d \sin(\theta) = m\lambda+\lambda/N$

- Equating both sides: $m(\lambda+\Delta\lambda) = m\lambda+\lambda/N$
  or $m \Delta\lambda = \lambda/N$
  or $\lambda/\Delta\lambda = mN$
  $\lambda/\Delta\lambda$ is called the resolving power of a grating.

Resolving power $\lambda/\Delta\lambda = mN$

- Resolving power depends on total number of lines in grating exposed to incident light (N).
- Resolving power is proportional to "order of spectrum".