Julian Salazar Partas

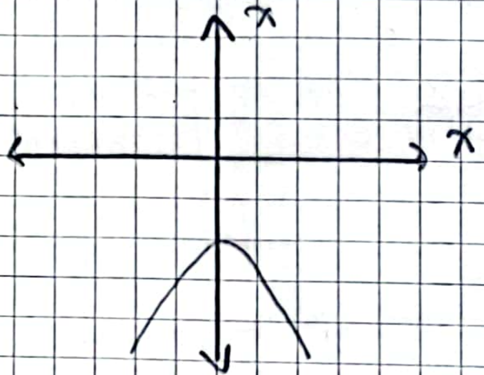
Taller 2

Ejercicios Capitulo 3

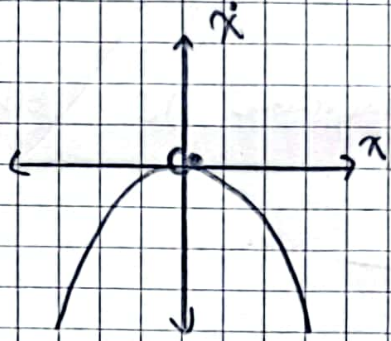
3.1.2 $\dot{x} = r - \cosh(x)$ $\qquad 0 = r - \cosh(x) \rightarrow r = \cosh(x)$

$r = 1$

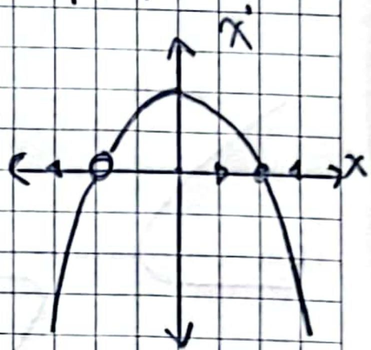
- $r < 1$



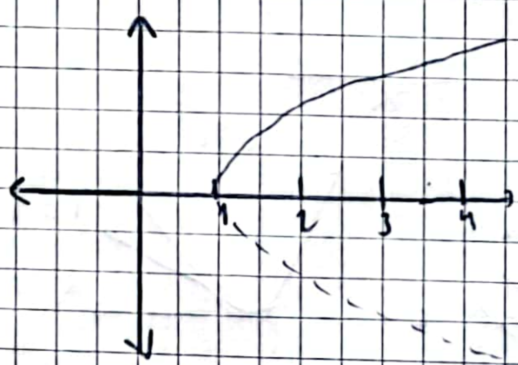
- $r = 1$



- $r > 1$



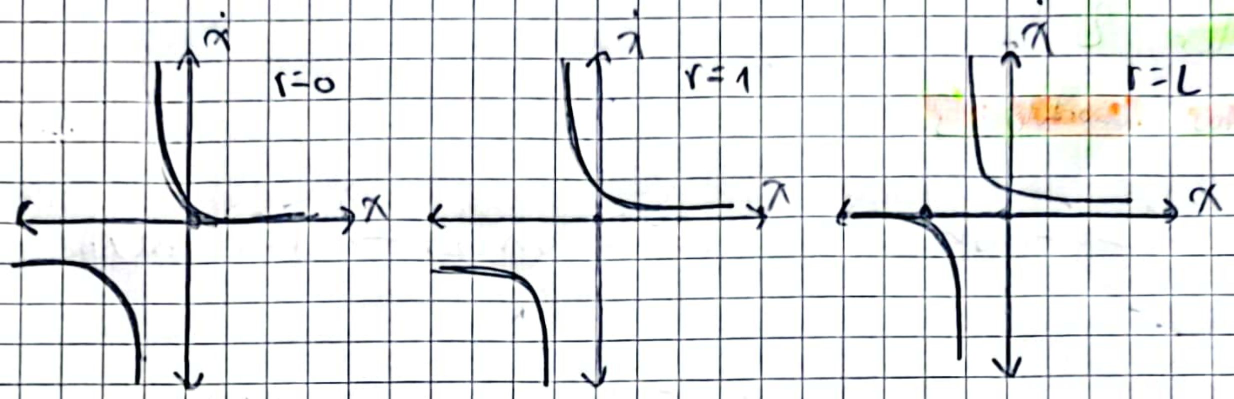
$r = 1$ Bifurcación silla nodo (Diagrama de bifurcación)



3.1.4 $\dot{x} = r + \frac{1}{2}x - \frac{x}{(1+x)}$

$$x = \frac{1}{2} - r \pm \sqrt{r^2 - 3r + 1/4}$$

No negativo para $|r - 3/2| > \sqrt{2}$



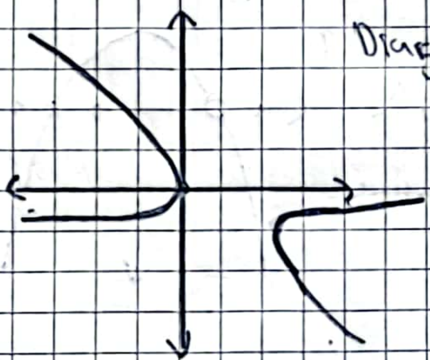
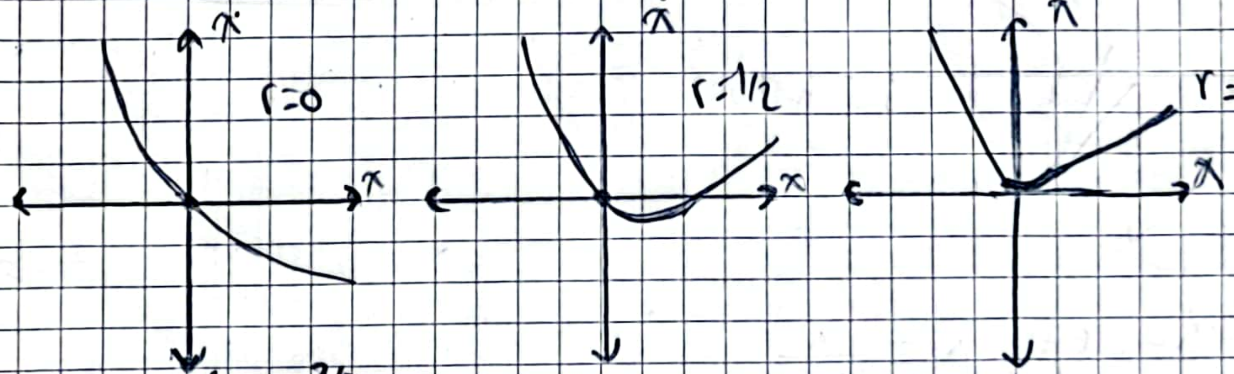
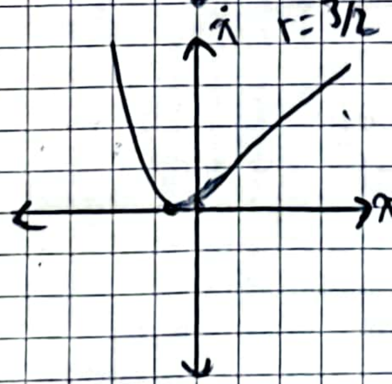


Diagrama de bifurcación

3.2.2 $\dot{x} = rx - \ln(1+x)$ $\qquad r = \dfrac{\ln(1+x)}{x}$





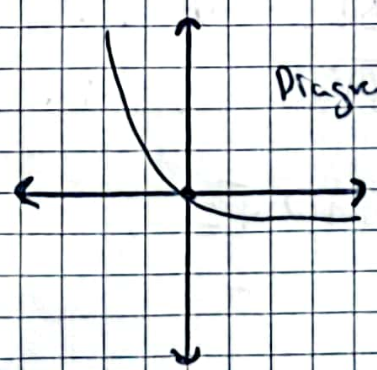
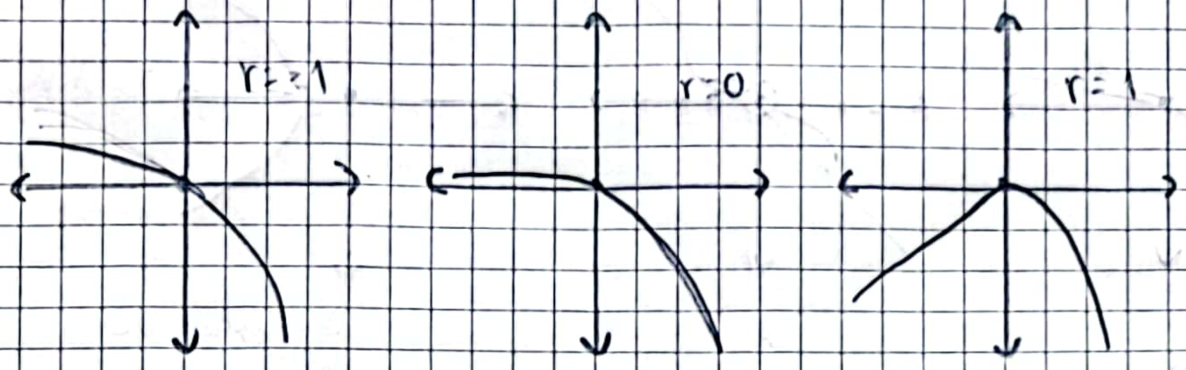


Diagrama de bifurcación

3.2.4 $\dot{x} = x(r - e^{x})$



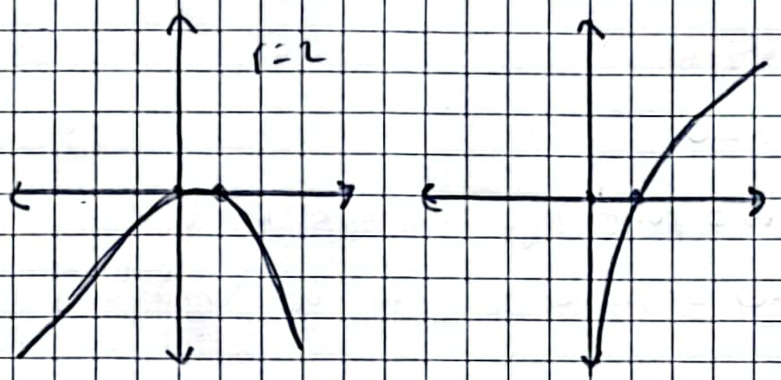
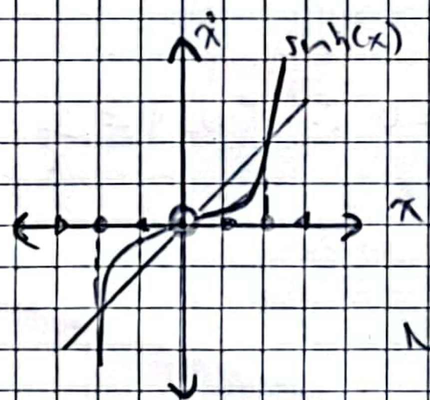


Diagrama de bifurcación

3.4.2 $\dot{x} = rx - \operatorname{senh}(x) \rightarrow x = 0$ $\quad 0 = rx - \operatorname{senh}(x)$

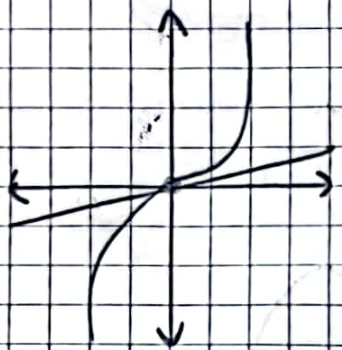
$rx = \operatorname{senh}(x) \quad \therefore \quad r = \dfrac{\operatorname{senh}(x)}{x}$

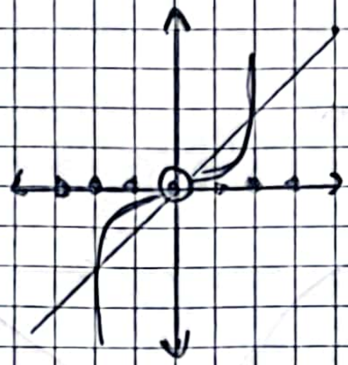


$x^* = 0$. Es el punto de equilibrio para cualquier r, su estabilidad varía cuando rx es tangente

Nuevos puntos $r = \dfrac{d\,\operatorname{senh}(x)}{dx} = \cosh(x)$

$r|_{x=0} = 1$





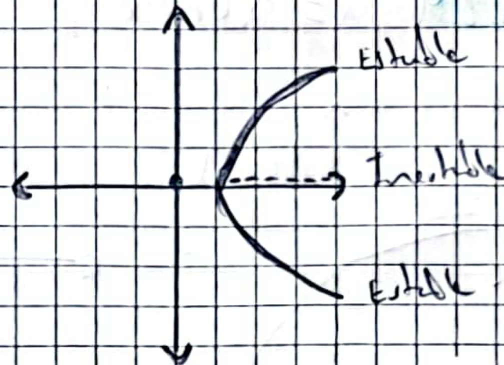


Diagrama de bifurcación

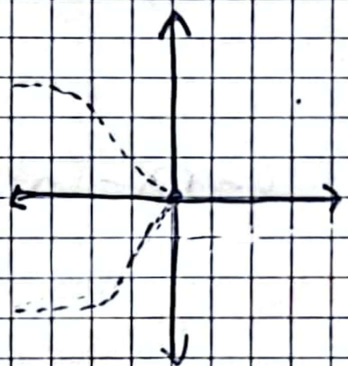
$r = \frac{\operatorname{senh}(x^*)}{x^*}$ = Puntos de equilibrio

3.4.4 $\dot{x} = x + \frac{rx}{1+x^2} \rightarrow \dot{x} = 0$

$x = \frac{-rx}{1+x^2}$

$r > 0$ = punto fijo inestable en 0

$r < 0$ = estable y otros dos inestables



3.5.2 $\frac{d\phi}{dt} = \operatorname{sen}\phi\,(r\cos\phi - 1)$ $\quad \operatorname{sen}\phi = 0 \therefore \cos\phi = \frac{1}{r}$

$r > 1$ = punto fijo cuando $\phi^* = \pm\cos(1/8)$ estable

$r < 0$ = punto fijo inestable para ϕ^*

3.5.8. $\dot{u} = au + bu^3 - cu^5$, donde $b, c > 0$, tiene bifurcación en $a = 0$ y la ecuación se puede reescribir como

$\frac{dx}{d\tau} = rx + x^3 - x^5$

donde $x = \frac{u}{U}$, $\tau = \frac{t}{T}$, U, T y r se deben determinar en terminos de a, b, c.

Teniendo que $u = xU \therefore \frac{du}{dt} = U\frac{dx}{dt}$ si $t = T\tau$ $dt = T.d\tau$

$\Rightarrow \frac{U dx}{T d\tau} = a.x.U + bx^3U^3 - Cx^5U^5$

$\Rightarrow \frac{dx}{d\tau} = T(ax + bx^3U^2 - Cx^5U^4)$

$\frac{dx}{d\tau} = aTx + (bTU^2)x^3 - (C\,TU^4)x^5$

Resolviendo

$C\,TU^4 = 1$ $\quad bTU^2 = 1$ $\quad \Rightarrow b\,T\,b/c = 1$ $\quad T = c/b^2$

$C\,TU^2 = 1$ $\quad \frac{CU^2}{b} = 1$ $\quad U^2 = b/c$ $\quad U = \sqrt{\frac{b}{c}}$

$\Rightarrow aT = r$ $\quad r = \frac{ac}{b^2}$ $\Rightarrow U = \sqrt{\frac{b}{c}}$, $T = \frac{c}{b^2}$, $r = \frac{ac}{b^2} = \frac{dx}{d\tau} = rx + x^3 - x^5$

Ejercicios Capitulo 5

S.1.4 $\dot{x} = 3x - 2y$, $\dot{y} = 2y - x$

$$\Rightarrow \begin{pmatrix} \dot{x} \\ \dot{y} \end{pmatrix} = \begin{bmatrix} 3 & -2 \\ -1 & 2 \end{bmatrix} \begin{pmatrix} x \\ y \end{pmatrix}$$

S.1.6 $\dot{x} = x$, $\dot{y} = 5x + y$

$$\begin{pmatrix} \dot{x} \\ \dot{y} \end{pmatrix} = \begin{bmatrix} 1 & 0 \\ 5 & 1 \end{bmatrix} \begin{pmatrix} x \\ y \end{pmatrix}$$