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Ejercicios capitulo 2

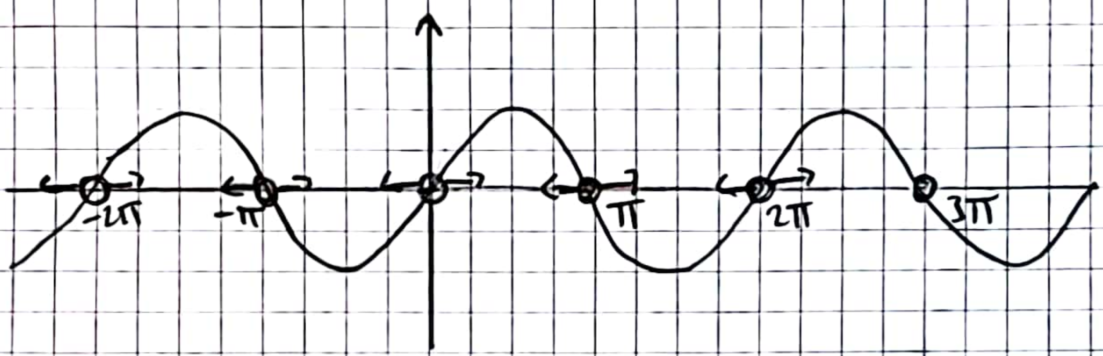
2.1 $\dot{x} = sen(x)$

2.1.1 Encuentre los puntos fijos del flujo

Punto fijo $\to \dot{x}=0 \to sen(x)=0$

$sen(\pi)=0$; $sen(-\pi)=0$; $sen(-2\pi)=0$; $sen(2\pi)=0$

$x^* = n\pi \quad \forall n \in E$



2.2

2.2.1 $\dot{x} = 4x^2 - 16 \to f(x) = 4x^2 - 16$

$\dot{x} \to f(x) = 0 \qquad 4x^2 - 16 = 0 \qquad x^2 = \frac{16}{4} \qquad x = \sqrt{4} \qquad x = \pm 2$

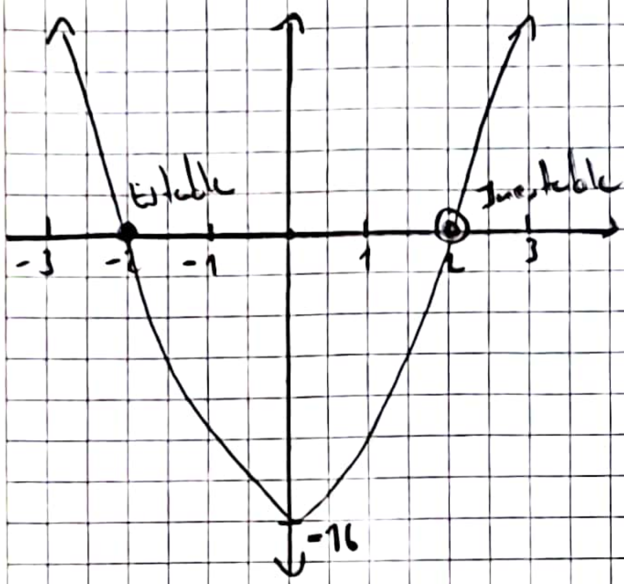
$f_x = 8x \to \cdot 2 \to f_x|_{2=x} = 16$ Inestable

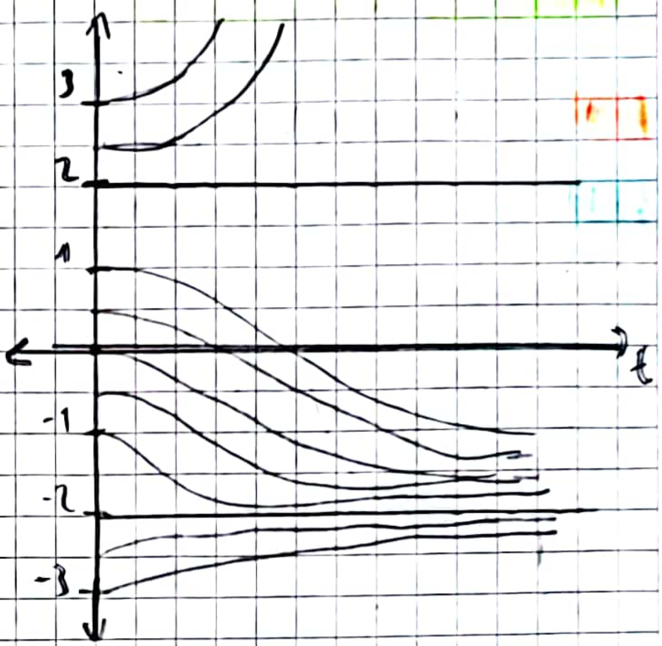
$\cdot -2 \to f_x|_{-2=x} = -16$ Estable

$\dot{x} = 4x^2 - 16$

$\int \frac{dx}{dt} = \int 4\,dt\,(x^2-4) \qquad \int \frac{dx}{x^2-4} = \int 4\,dt \qquad \frac{1}{4}\ln\left(\frac{x-2}{x+2}\right) = 4t + c$

$x = 2\,\frac{1 + C_2 e^{16t}}{1 - C_2 e^{16t}} \quad \therefore \quad C_2(t=0) = \frac{x-2}{x+2}$





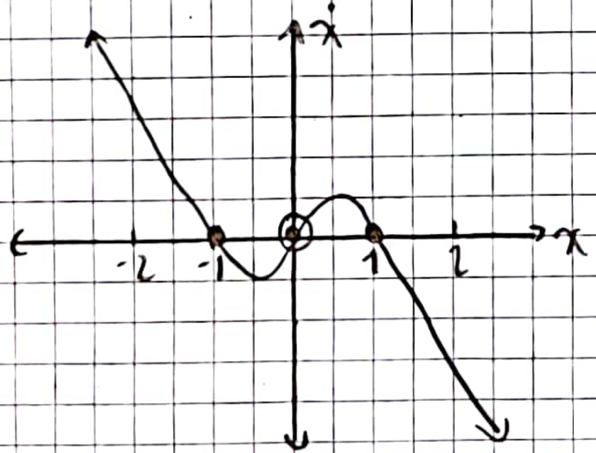
2.2.3 $\dot{x} = x - x^3 \longrightarrow f(x) = x - x^3$

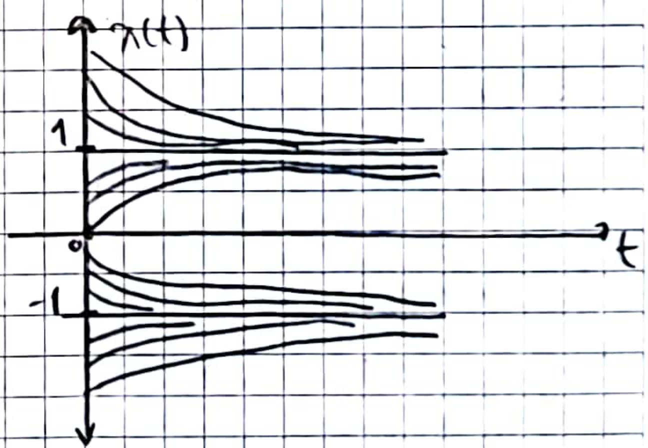
$x - x^3 = 0 \quad \therefore \quad x = \{-1, 0, 1\}$

$f_x = 1 - 3x^2$; $\quad f_x|_{x=-1} = -2$ Estable

$f_x|_{x=0} = 1$ Inestable

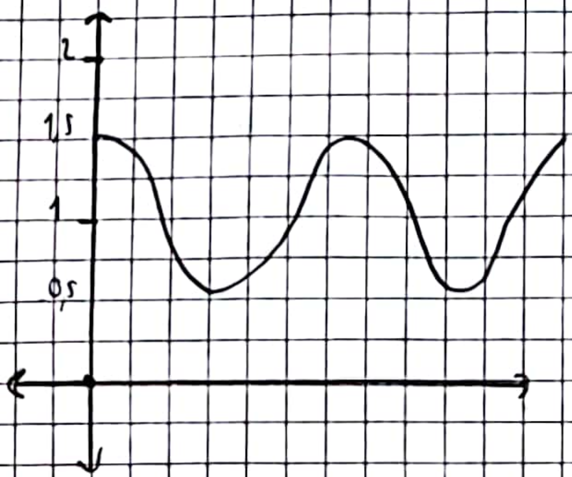
$f_x|_{x=1} = -2$ Estable





2.2.5

$\dot{x} = 1 + \frac{1}{2}\cos x \qquad f'(x) = -\frac{1}{2}\,\text{sen}(x)$



2.3

2.3.1 a) $\frac{dN}{N\left(1-\frac{N}{K}\right)} = r\,dt \qquad \frac{K\,dN}{N\,(K-N)} = r\,dt \qquad \frac{dN}{N} + \frac{dN}{K-N} = r\,dt$

$\ln N - \ln|K-N| = rt + C \qquad \ln\left|\frac{N}{K-N}\right| = rt + C$

$\frac{N}{K-N} = Ce^{rt} \qquad \frac{K-N}{N} = Ce^{-rt} \qquad \frac{K}{N} = 1 + Ce^{-rt}$

$\frac{K}{1+Ce^{-rt}} = N \qquad N(0) = n_0 = \frac{K}{1+C} \qquad C = \frac{K}{n_0} - 1$

$N = \frac{K}{1+\left(\frac{K}{n_0}-1\right)e^{-rt}}$

b) $x = \frac{1}{N} \qquad \frac{dx}{dt} = -\frac{1}{N^2}\frac{dN}{dt} \qquad -N^2\frac{dx}{dt} = r\,\frac{1}{x}\left(1-\frac{1}{xK}\right)$

$-\frac{1}{x^2}\frac{dx}{dt} = r\,\frac{1}{x^2K}\,(xK-1) \qquad -\frac{dx}{dt} = \frac{r}{K}\,(xK-1)$

$$\frac{dx}{(xk-1)} = -\frac{r\,dt}{k} \qquad \frac{1}{k}\frac{k\,dx}{(xk-1)} = -\frac{r\,dt}{k}$$

$$\frac{1}{k}\ln(xk-1) = -\frac{rt}{k} + c \qquad \ln(xk-1) = -rt + C$$

$$xk - 1 = Ce^{-rt} \qquad xk = 1 + Ce^{-rt} \qquad x = \frac{1+Ce^{-rt}}{k}$$

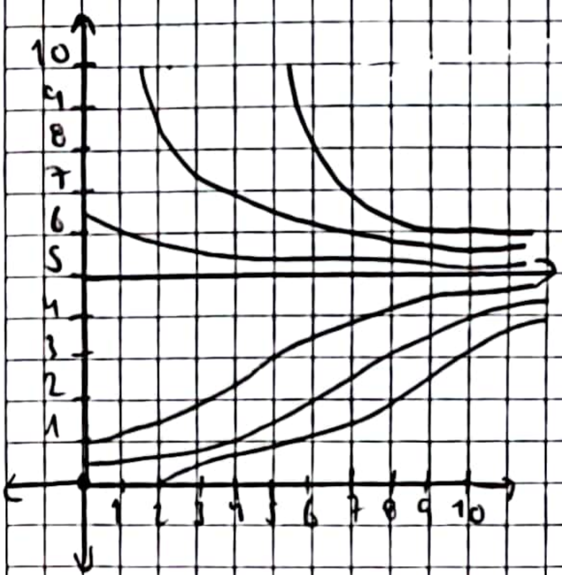
$$\frac{1}{N} = \frac{1+Ce^{-rt}}{k} \qquad N = \frac{k}{1+Ce^{-rt}} \qquad N = \frac{k}{1+\left(\frac{k}{n_0}-1\right)e^{-rt}}$$

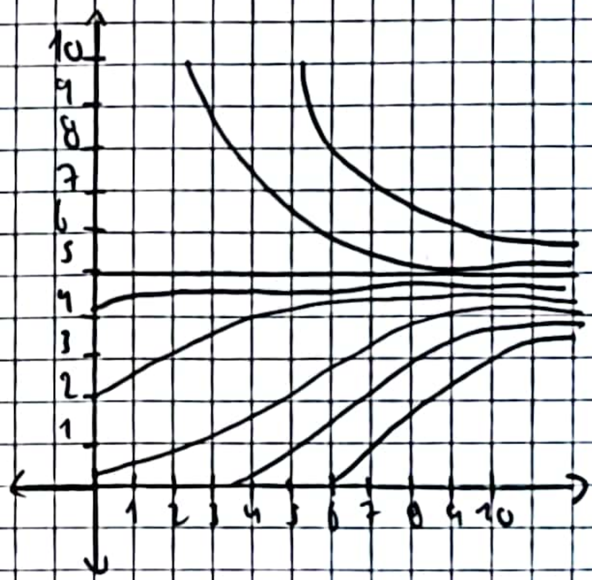
2.3.3 a) 'a' es la tasa de crecimiento intrínseco de la célula y b^{-1} es la capacidad de carga ambiental.

b) los puntos fijos para N^* son

$$-aN\ln(bN) = 0 \qquad \ln(bN) = 0 \qquad bN(t) = e^0 \qquad bN(t) = 1$$

$$N^*(t) = 0, \frac{1}{b}$$





$N^* = -aN\ln(bN)$ $a=0{,}5$; $b=0{,}2$ $\qquad$ $N^* = -aN\ln(bN)$; $a=0{,}8$; $b=0{,}2$

2.3.5

$$x(t) = \frac{x_0 . e^{(a-b)t}}{x_0 . e^{(a-b)t} + y_0}$$

numerador y denominador se dividen por $x_0 . e^{(a-b)t}$

$$\Rightarrow x(t) = \frac{1}{1 + \frac{y_0}{x_0 . e^{(a-b)t}}}$$

$a > b$. $\lim\limits_{t \to \infty} e^{(a-b)t} = \infty$

$$\Rightarrow \frac{1}{e^{(a-b)t}} = 0$$

$$\therefore \lim_{t \to \infty} x(t) = \frac{1}{1+0} = 1$$

Dado $x' = ax$ $y' = by$ condiciones iniciales $x_0, y_0 > 0$

tasas de crecimiento $a > b > 0$

$x' = ax$	$y' = by$
$\frac{dx}{dt} = ax$	$\frac{dy}{dt} = by$
$\frac{dx}{x} = a.dt$	$\frac{dy}{y} = bdt$
$\ln x = a.t + C$	$\ln y = bt + C_2$
$x(t) = k_1 . e^{at}$	$y(t) = k_2 e^{bt}$
$x(0) = x_0$	$y(0) = y_0$
$k_1 = x_0$	$k_2 = y_0$
$x(t) = x_0 e^{at}$	$y(t) = y_0 . e^{bt}$

$$\Rightarrow x = \frac{x(t)}{x(t) + y(t)} = \frac{x_0 e^{at}}{x_0 e^{at} + y_0 e^{bt}}$$

se divide numerador y denominador por e^{bt}

$$\Rightarrow \frac{x_0 e^{at} / e^{bt}}{x_0 \frac{e^{at}}{e^{bt}} + y_0 \frac{e^{bt}}{e^{bt}}}$$

2.4

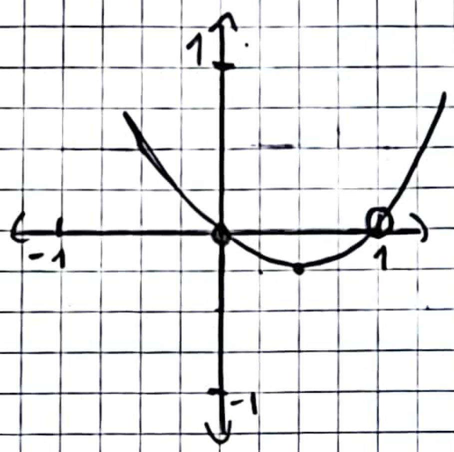
2.4.1

$\dot{x} = x(1-x) \qquad x(1-x)=0 \qquad x=0 \qquad x^* = \{0,1\}$ Puntos fijos

$x=1$

$f(x) = x - x^2 \rightarrow f_x = 1-2x$

- $f_x|_{x=0} = 1$ Inestable
- $f_x|_{x=1} = -1$ Estable



2.4.3

$\dot{x} = \tan(x) \rightarrow f(x) = \tan(x)$

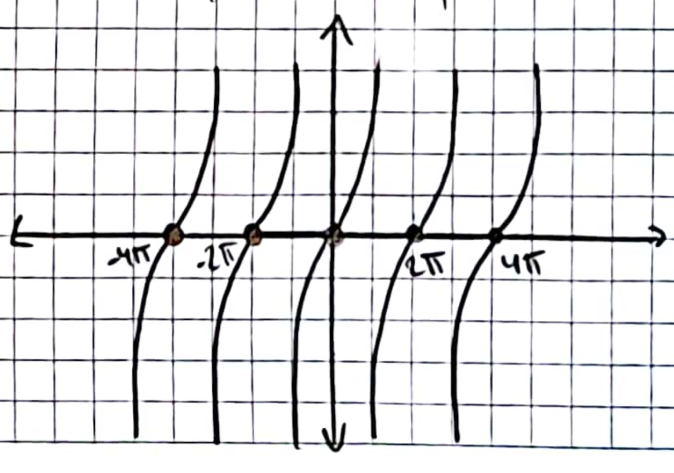
$\tan(x) = 0 \qquad x^* = 2\pi n \; ; \; n \in \mathbb{Z}$

$x = \tan^{-1}(0)$

$f(x) = \tan(x) \qquad f_x = \sec^2(x) \rightarrow f_x|_{x=2\pi n} = \sec^2(2\pi n)$

$\Rightarrow \dfrac{1}{\cos^2(2\pi n)^1} = 1$ para cualquier n (Inestable)

$x^* = 2\pi n \rightarrow$ Todos puntos de equilibrio Inestables

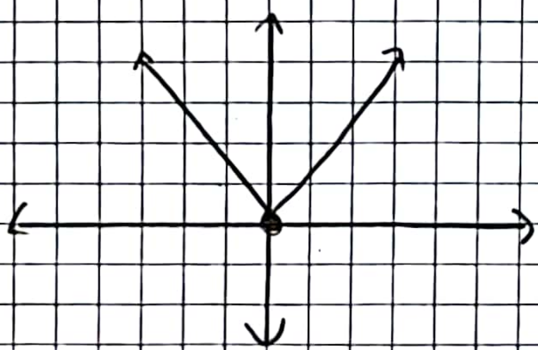


2.7.5

$\dot{x} = 1 - e^{-x^2}$ $\quad \forall x \in \mathbb{R},\ x^2 \geq 0 \quad -x^2 \leq 0$

$e^{-x^2} \leq 1 \quad 1 - e^{-x^2} \geq 0 \implies 1 - e^{-x^2} = 0 \quad e^{x^2} = 1 \quad x = 0$

0 = punto inestable



2.8

2.8.3 a) $\dot{x} = -x \quad , x(0) = 1$

$\frac{dx}{dt} = -x \qquad \int dt = \int -\frac{1}{x}\,dx \qquad t = -\ln(x) + c$

$\ln(x) = -t + c \qquad x = e^{-t+c} \qquad x = e^{-t}\,\cancel{e^{c}}\,k$

$x = k e^{-t} \qquad x(0) = k \qquad k = 1$

$\implies x(t) = e^{-t} \qquad x(1) = e^{-1} = 0{,}3678$