



CS 412 Intro. to Data Mining

Chapter 7 : Advanced Frequent Pattern Mining

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Chapter 7 : Advanced Frequent Pattern Mining

- Mining Diverse Patterns
- Sequential Pattern Mining
- Constraint-Based Frequent Pattern Mining
- Graph Pattern Mining
- Pattern Mining Application: Mining Software Copy-and-Paste Bugs
- Summary

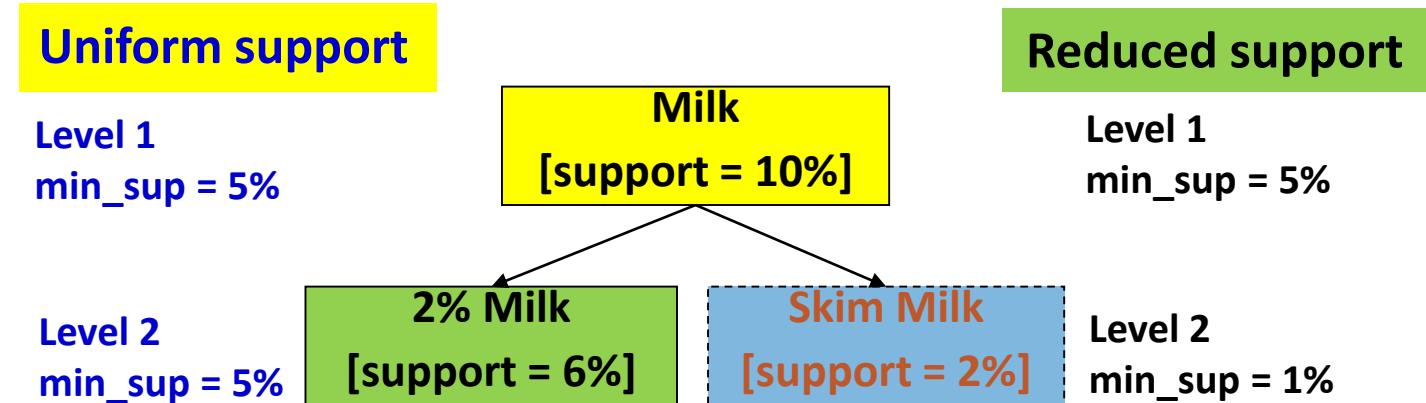


Mining Diverse Patterns

- Mining Multiple-Level Associations
- Mining Multi-Dimensional Associations
- Mining Quantitative Associations
- Mining Negative Correlations
- Mining Compressed and Redundancy-Aware Patterns

Mining Multiple-Level Frequent Patterns

- ❑ Items often form hierarchies
 - ❑ Ex.: Dairyland 2% milk;
Wonder wheat bread
 - ❑ How to set min-support thresholds?



- ❑ Uniform min-support across multiple levels (reasonable?)
- ❑ Level-reduced min-support: Items at the lower level are expected to have lower support
- ❑ Efficient mining: *Shared* multi-level mining
- ❑ Use the lowest min-support to pass down the set of candidates

Redundancy Filtering at Mining Multi-Level Associations

- Multi-level association mining may generate many redundant rules
- Redundancy filtering: Some rules may be redundant due to “ancestor” relationships between items
 - milk \Rightarrow wheat bread [support = 8%, confidence = 70%] (1)
 - 2% milk \Rightarrow wheat bread [support = 2%, confidence = 72%] (2)
 - Suppose the 2% milk sold is about $\frac{1}{4}$ of milk sold in gallons
 - (2) should be able to be “derived” from (1)
- A rule is *redundant* if its support is close to the “expected” value, according to its “ancestor” rule, and it has a similar confidence as its “ancestor”
 - Rule (1) is an ancestor of rule (2), which one to prune?

Customized Min-Supports for Different Kinds of Items

- ❑ We have used the same min-support threshold for all the items or item sets to be mined in each association mining
- ❑ In reality, some items (e.g., diamond, watch, ...) are valuable but less frequent
- ❑ It is necessary to have customized min-support settings for different kinds of items
- ❑ One Method: Use **group-based “individualized” min-support**
 - ❑ E.g., {diamond, watch}: 0.05%; {bread, milk}: 5%; ...
 - ❑ How to mine such rules efficiently?
 - ❑ Existing scalable mining algorithms can be easily extended to cover such cases

Mining Multi-Dimensional Associations

- Single-dimensional rules (e.g., items are all in “product” dimension)
 - $\text{buys}(X, \text{"milk"}) \Rightarrow \text{buys}(X, \text{"bread"})$
- Multi-dimensional rules (i.e., items in ≥ 2 dimensions or predicates)
 - Inter-dimension association rules (*no repeated predicates*)
 - $\text{age}(X, \text{"18-25"}) \wedge \text{occupation}(X, \text{"student"}) \Rightarrow \text{buys}(X, \text{"coke"})$
 - Hybrid-dimension association rules (*repeated predicates*)
 - $\text{age}(X, \text{"18-25"}) \wedge \text{buys}(X, \text{"popcorn"}) \Rightarrow \text{buys}(X, \text{"coke"})$
- Attributes can be categorical or numerical
 - Categorical Attributes (e.g., *profession*, *product*: no ordering among values): Data cube for inter-dimension association
 - Quantitative Attributes: Numeric, implicit ordering among values—discretization, clustering, and gradient approaches

Mining Quantitative Associations

- ❑ Mining associations with numerical attributes
 - ❑ Ex.: Numerical attributes: **age** and **salary**
- ❑ Methods
 - ❑ Static discretization based on predefined concept hierarchies
 - ❑ Discretization on each dimension with hierarchy
 - ❑ age: {0-10, 10-20, ..., 90-100} → {young, mid-aged, old}
 - ❑ Dynamic discretization based on data distribution
 - ❑ Clustering: Distance-based association
 - ❑ First one-dimensional clustering, then association
 - ❑ Deviation analysis:
 - ❑ Gender = female ⇒ Wage: mean=\$7/hr (overall mean = \$9)

Mining Extraordinary Phenomena in Quantitative Association Mining

- Mining extraordinary (i.e., interesting) phenomena
 - Ex.: Gender = female \Rightarrow Wage: mean=\$7/hr (overall mean = \$9)
 - LHS: a subset of the population
 - RHS: an extraordinary behavior of this subset
- The rule is accepted only if a statistical test (e.g., Z-test) confirms the inference with high confidence
- Subrule: Highlights the extraordinary behavior of a subset of the population of the super rule
 - Ex.: (Gender = female) \wedge (South = yes) \Rightarrow mean wage = \$6.3/hr
- Rule condition can be categorical or numerical (quantitative rules)
 - Ex.: Education in [14-18] (yrs) \Rightarrow mean wage = \$11.64/hr
- Efficient methods have been developed for mining such extraordinary rules (e.g., Aumann and Lindell@KDD'99)

Rare Patterns vs. Negative Patterns

- Rare patterns
 - Very low support but interesting (e.g., buying Rolex watches)
 - How to mine them? Setting individualized, group-based min-support thresholds for different groups of items
- Negative patterns
 - Negatively correlated: Unlikely to happen together
 - Ex.: Since it is unlikely that the same customer buys both a **Ford Expedition** (an SUV car) and a **Ford Fusion** (a hybrid car), buying a **Ford Expedition** and buying a **Ford Fusion** are likely negatively correlated patterns
 - How to define negative patterns?

Defining Negative Correlated Patterns

- A support-based definition
 - If itemsets A and B are both frequent but rarely occur together, i.e.,
 $\text{sup}(A \cup B) << \text{sup}(A) \times \text{sup}(B)$
 - Then A and B are negatively correlated
 - Is this a good definition for large transaction datasets?
 - Ex.: Suppose a store sold two needle packages A and B 100 times each, but only one transaction contained both A and B
 - When there are in total 200 transactions, we have
 - $s(A \cup B) = 0.005, s(A) \times s(B) = 0.25, s(A \cup B) << s(A) \times s(B)$
 - But when there are 10^5 transactions, we have
 - $s(A \cup B) = 1/10^5, s(A) \times s(B) = 1/10^3 \times 1/10^3, s(A \cup B) > s(A) \times s(B)$
 - What is the problem?—Null transactions: The support-based definition is not null-invariant!

Does this remind you the definition of *lift*?

Defining Negative Correlation: Need Null-Invariance in Definition

- A good definition on negative correlation should take care of the null-invariance problem
 - Whether two itemsets A and B are negatively correlated should not be influenced by the number of null-transactions
- A Kulczynski measure-based definition
 - If itemsets A and B are frequent but
$$(s(A \cup B)/s(A) + s(A \cup B)/s(B))/2 < \epsilon,$$
where ϵ is a negative pattern threshold, then A and B are negatively correlated
- For the same needle package problem:
 - No matter there are in total 200 or 10^5 transactions
 - If $\epsilon = 0.01$, we have
$$(s(A \cup B)/s(A) + s(A \cup B)/s(B))/2 = (0.01 + 0.01)/2 < \epsilon$$

Mining Compressed Patterns

Pat-ID	Item-Sets	Support
P1	{38,16,18,12}	205227
P2	{38,16,18,12,17}	205211
P3	{39,38,16,18,12,17}	101758
P4	{39,16,18,12,17}	161563
P5	{39,16,18,12}	161576

- Closed patterns
 - P1, P2, P3, P4, P5
 - Emphasizes too much on support
 - There is no compression
- Max-patterns
 - P3: information loss
- Desired output (a good balance):
 - P2, P3, P4

- Why mining compressed patterns?

- Too many scattered patterns but not so meaningful

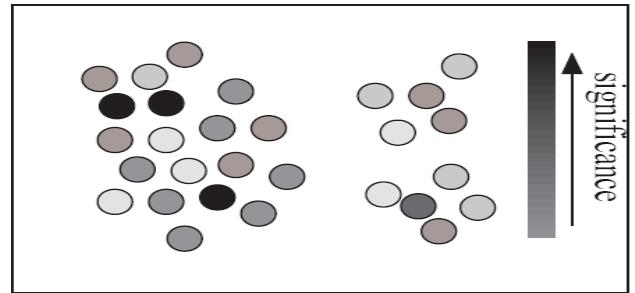
- Pattern distance measure

$$Dist(P_1, P_2) = 1 - \frac{|T(P_1) \cap T(P_2)|}{|T(P_1) \cup T(P_2)|}$$

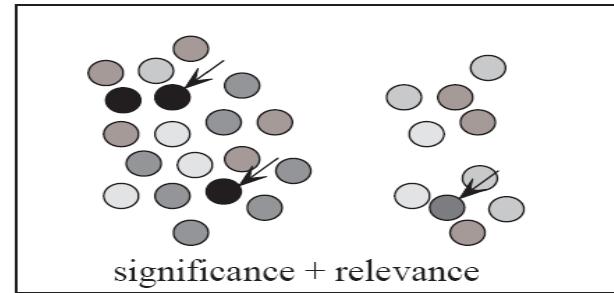
- δ -clustering: For each pattern P, find all patterns which can be expressed by P and whose distance to P is within δ (δ -cover)
 - All patterns in the cluster can be represented by P
 - Method for efficient, direct mining of compressed frequent patterns (e.g., D. Xin, J. Han, X. Yan, H. Cheng, "On Compressing Frequent Patterns", Knowledge and Data Engineering, 60:5-29, 2007)

Redundancy-Aware Top-k Patterns

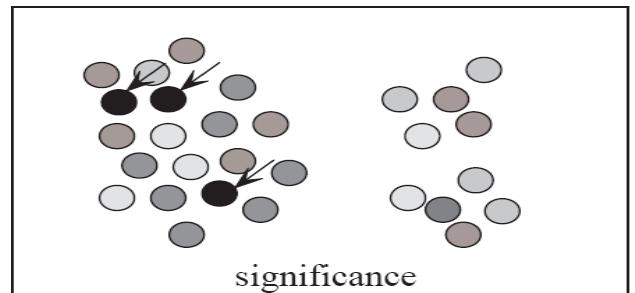
- Desired patterns: high significance & low redundancy



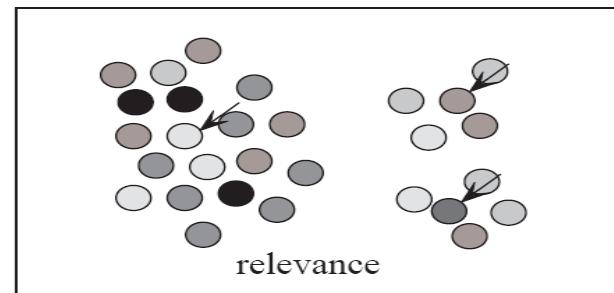
(a) a set of patterns



(b) redundancy-aware top- k



(c) traditional top- k



(d) summarization

- Method: Use MMS (Maximal Marginal Significance) for measuring the combined significance of a pattern set
- Xin et al., Extracting Redundancy-Aware Top-K Patterns, KDD'06

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Sequential Pattern Mining

- Sequential Pattern and Sequential Pattern Mining
- GSP: Apriori-Based Sequential Pattern Mining
- SPADE: Sequential Pattern Mining in Vertical Data Format
- PrefixSpan: Sequential Pattern Mining by Pattern-Growth
- CloSpan: Mining Closed Sequential Patterns

Sequence Databases & Sequential Patterns

- ❑ Sequential pattern mining has broad applications
 - ❑ Customer shopping sequences
 - ❑ Purchase a laptop first, then a digital camera, and then a smartphone, within 6 months
 - ❑ Medical treatments, natural disasters (e.g., earthquakes), science & engineering processes, stocks and markets, ...
 - ❑ Weblog click streams, calling patterns, ...
 - ❑ Software engineering: Program execution sequences, ...
 - ❑ Biological sequences: DNA, protein, ...
- ❑ Transaction DB, sequence DB vs. time-series DB
- ❑ Gapped vs. non-gapped sequential patterns
- ❑ Shopping sequences, clicking streams vs. biological sequences

Sequential Pattern and Sequential Pattern Mining

- ❑ Sequential pattern mining: Given a set of sequences, find the **complete set of frequent subsequences** (i.e., satisfying the min_sup threshold)

A sequence database

SID	Sequence
10	<a(<u>abc</u>)(ac <u>c</u> d(cf)>
20	<(ad)c(bc)(ae)>
30	<(ef)(&u2022ab)(df) <u>c</u> b>
40	<eg(af)cbc>

A sequence: < (ef) (ab) (df) c b >



- ❑ An element may contain a set of items (also called events)
- ❑ Items within an element are unordered and we list them alphabetically

<a(bc)dc> is a subsequence of <a(abc)(ac)d(cf)>

- ❑ Given support threshold $\text{min_sup} = 2$, <(ab)c> is a sequential pattern

Sequential Pattern Mining Algorithms

- Algorithm requirement: Efficient, scalable, finding complete set, incorporating various kinds of user-specific constraints
- The Apriori property still holds: If a subsequence s_1 is infrequent, none of s_1 's super-sequences can be frequent
- Representative algorithms
 - **GSP** (Generalized Sequential Patterns): Srikant & Agrawal @ EDBT'96)
 - Vertical format-based mining: **SPADE** (Zaki@Machine Learning'00)
 - Pattern-growth methods: **PrefixSpan** (Pei, et al. @TKDE'04)
- Mining closed sequential patterns: **CloSpan** (Yan, et al. @SDM'03)
- Constraint-based sequential pattern mining (to be covered in the constraint mining section)

GSP: Apriori-Based Sequential Pattern Mining

- Initial candidates: All 8-singleton sequences
 - <a>, , <c>, <d>, <e>, <f>, <g>, <h>
- Scan DB once, count support for each candidate
- Generate length-2 candidate sequences

min_sup = 2

Cand.	sup
<a>	3
	5
<c>	4
<d>	3
<e>	3
<f>	2
<g>	1
<h>	1

	<a>		<c>	<d>	<e>	<f>
<a>	<aa>	<ab>	<ac>	<ad>	<ae>	<af>
	<ba>	<bb>	<bc>	<bd>	<be>	<bf>
<c>	<ca>	<cb>	<cc>	<cd>	<ce>	<cf>
<d>	<da>	<db>	<dc>	<dd>	<de>	<df>
<e>	<ea>	<eb>	<ec>	<ed>	<ee>	<ef>
<f>	<fa>	<fb>	<fc>	<fd>	<fe>	<ff>

	<a>		<c>	<d>	<e>	<f>
<a>		<(ab)>	<(ac)>	<(ad)>	<(ae)>	<(af)>
			<(bc)>	<(bd)>	<(be)>	<(bf)>
<c>				<(cd)>	<(ce)>	<(cf)>
<d>					<(de)>	<(df)>
<e>						<(ef)>
<f>						

SID	Sequence
10	<(bd)cb(ac)>
20	<(bf)(ce)b(fg)>
30	<(ah)(bf)abf>
40	<(be)(ce)d>
50	<a(bd)bcb(ade)>

- Without Apriori pruning:
(8 singletons) $8*8+8*7/2 = 92$ length-2 candidates
- With pruning, length-2 candidates: $36 + 15 = 51$

GSP (Generalized Sequential Patterns): Srikant & Agrawal @ EDBT'96)

GSP Mining and Pruning

5th scan: 1 cand. 1 length-5 seq. pat.

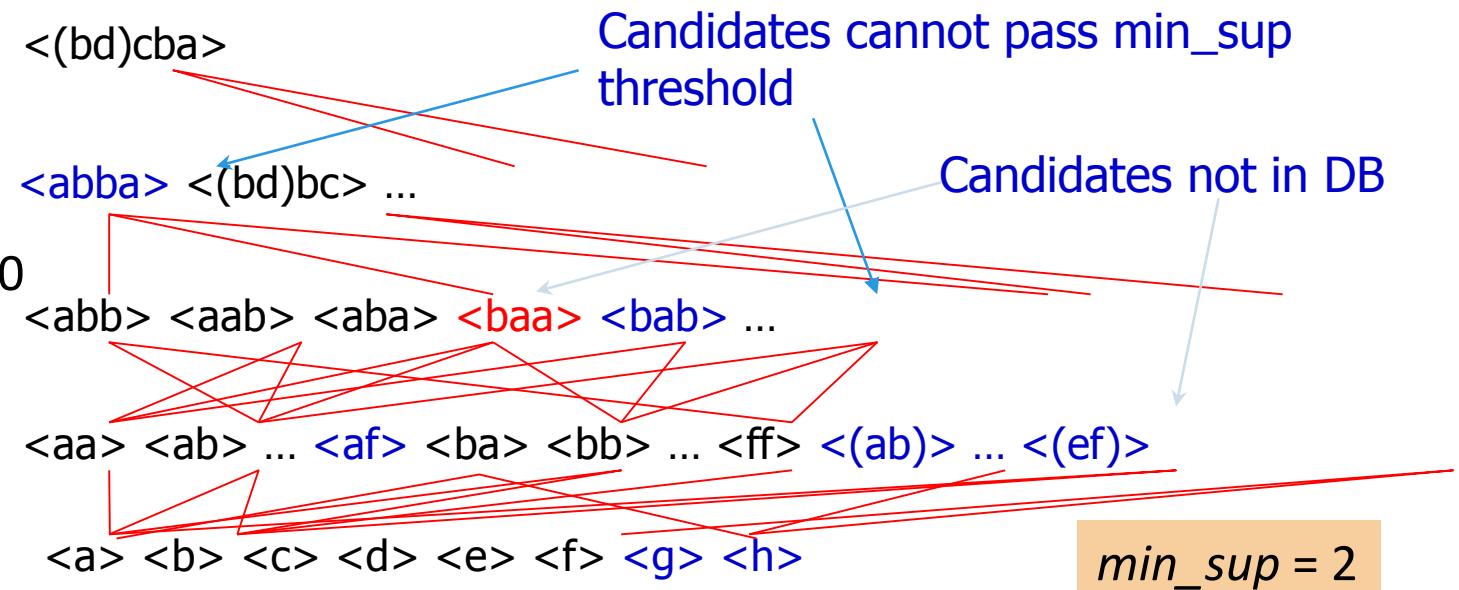
4th scan: 8 cand. 7 length-4 seq. pat.

3rd scan: 46 cand. 20 length-3 seq. pat. 20 cand. not in DB at all

2nd scan: 51 cand. 19 length-2 seq. pat.
10 cand. not in DB at all

1st scan: 8 cand. 6 length-1 seq. pat.

- Repeat (for each level (i.e., length-k))
- Scan DB to find length-k frequent sequences
- Generate length-(k+1) candidate sequences from length-k frequent sequences using Apriori
- set k = k+1
- Until no frequent sequence or no candidate can be found



min_sup = 2	SID	Sequence
10		<(bd)cb(ac)>
20		<(bf)(ce)b(fg)>
30		<(ah)(bf)abf>
40		<(be)(ce)d>
50		<a(bd)bcb(ade)>

Sequential Pattern Mining in Vertical Data Format: The SPADE Algorithm

- A sequence database is mapped to: <SID, EID>
- Grow the subsequences (patterns) one item at a time by Apriori candidate generation

SID	Sequence
1	<a(<u>bc</u>)(ac) <u>d</u> (cf)>
2	<(ad)c(bc)(ae)>
3	<(ef)(ab)(df) <u>cb</u> >
4	<eg(af)cbc>
<i>min_sup = 2</i>	

Ref: SPADE (Sequential
Pattern Discovery
using Equivalent Class)
[M. Zaki 2001]

SID	EID	Items
1	1	a
1	2	abc
1	3	ac
1	4	d
1	5	cf
2	1	ad
2	2	c
2	3	bc
2	4	ae
3	1	ef
3	2	ab
3	3	df
3	4	c
3	5	b
4	1	e
4	2	g
4	3	af
4	4	c
4	5	b
4	6	c

a		b		...
SID	EID	SID	EID	...
1	1	1	2	
1	2	2	3	
1	3	3	2	
2	1	3	5	
2	4	4	5	
3	2			
4	3			

ab		ba		...		
SID	EID (a)	EID(b)	SID	EID (b)	EID(a)	...
1	1	2	1	2	3	
2	1	3	2	3	4	
3	2	5				
4	3	5				

aba				...
SID	EID (a)	EID(b)	EID(a)	...
1	1	2	3	
2	1	3	4	

PrefixSpan: A Pattern-Growth Approach

SID	Sequence
10	<a(<u>abc</u>)(ac)d(cf)>
20	<(ad)c(bc)(ae)>
30	<(ef)(<u>ab</u>)(df) <u>c</u> b>
40	<eg(af)cbc>

min_sup = 2	
Prefix	Suffix (Projection)
<a>	<(abc)(ac)d(cf)>
<aa>	<(_bc)(ac)d(cf)>
<ab>	<(_c)(ac)d(cf)>

- PrefixSpan Mining: Prefix Projections
 - Step 1: Find length-1 sequential patterns
 - <a>, , <c>, <d>, <e>, <f>
 - Step 2: Divide search space and mine each projected DB
 - <a>-projected DB,
 - -projected DB,
 - ...
 - <f>-projected DB, ...

- Prefix and suffix
 - Given <a(abc)(ac)d(cf)>
 - Prefixes: <a>, <aa>, <a(ab)>, <a(abc)>, ...
 - Suffix: Prefixes-based projection

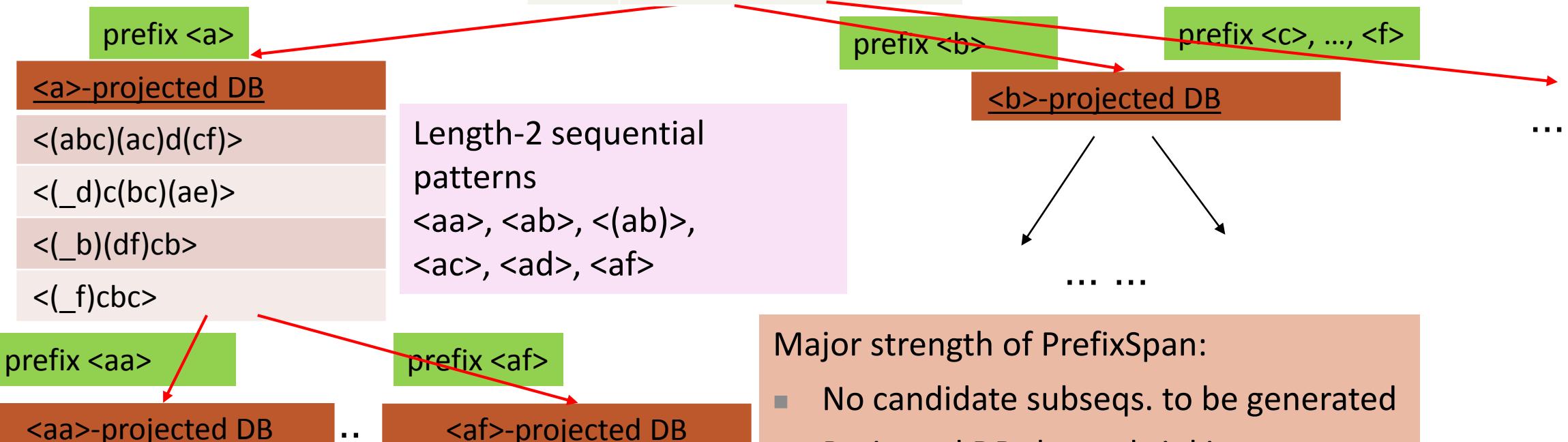
PrefixSpan (Prefix-projected Sequential pattern mining)
Pei, et al. @TKDE'04

PrefixSpan: Mining Prefix-Projected DBs

SID	Sequence
10	<a(<u>abc</u>)(ac <u>c</u>)d(cf)>
20	<(ad)c(bc)(ae)>
30	<(ef)(<u>ab</u>)(df) <u>cb</u> >
40	<eg(af)cbc>

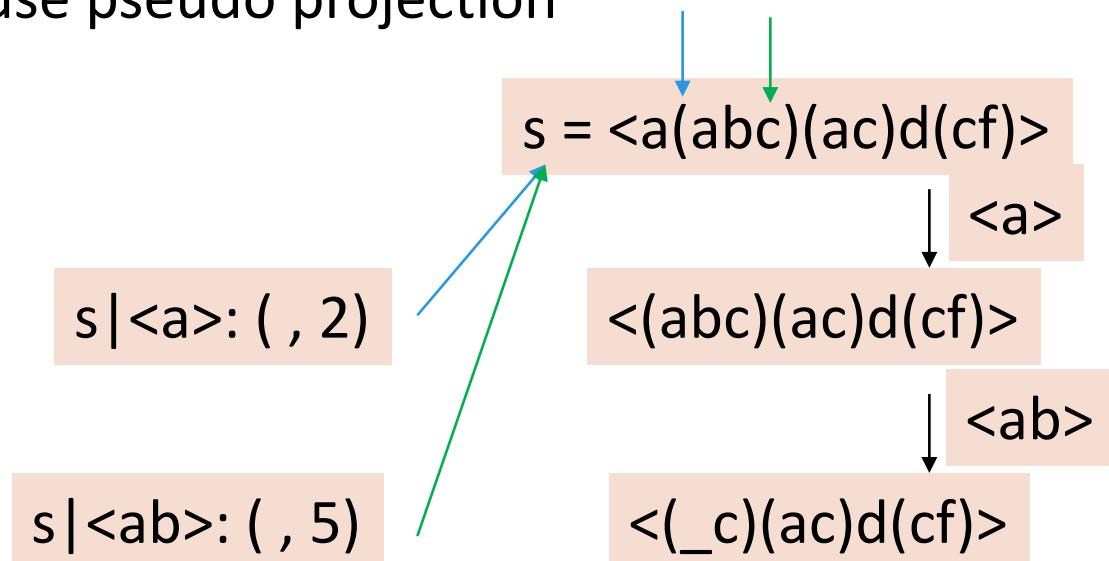
min_sup = 2

Length-1 sequential patterns
<a>, , <c>, <d>, <e>, <f>



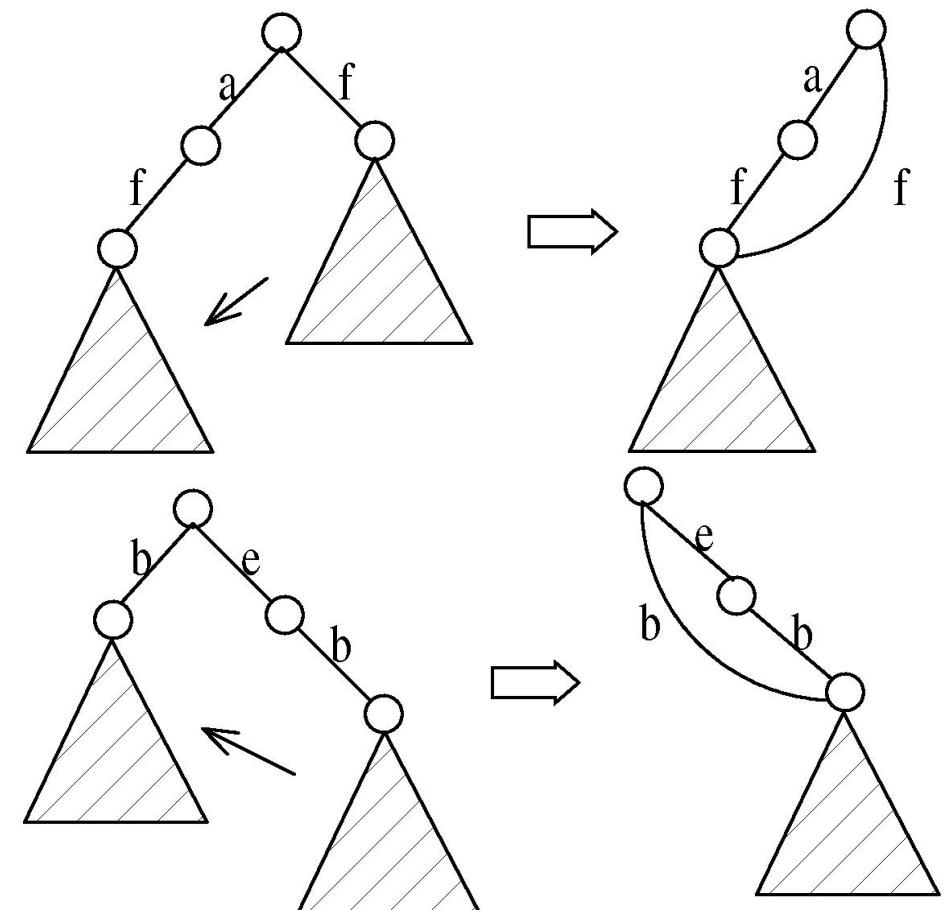
Implementation Consideration: Pseudo-Projection vs. Physical Projection

- Major cost of PrefixSpan: Constructing projected DBs
 - Suffixes largely repeating in recursive projected DBs
- When DB can be held in main memory, use pseudo projection
 - No physically copying suffixes
 - Pointer to the sequence
 - Offset of the suffix
- But if it does not fit in memory
 - Physical projection
- Suggested approach:
 - Integration of physical and pseudo-projection
 - Swapping to pseudo-projection when the data fits in memory



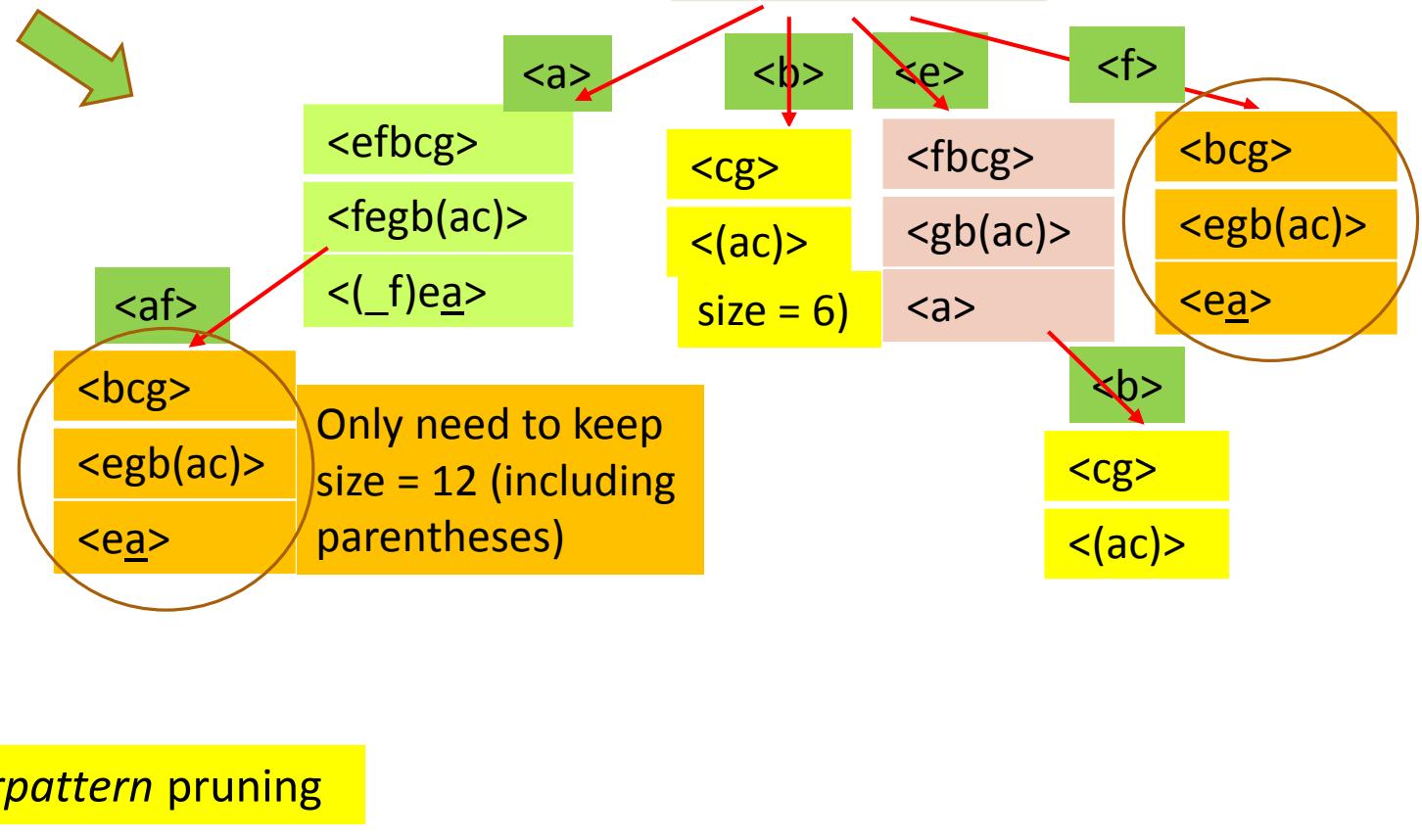
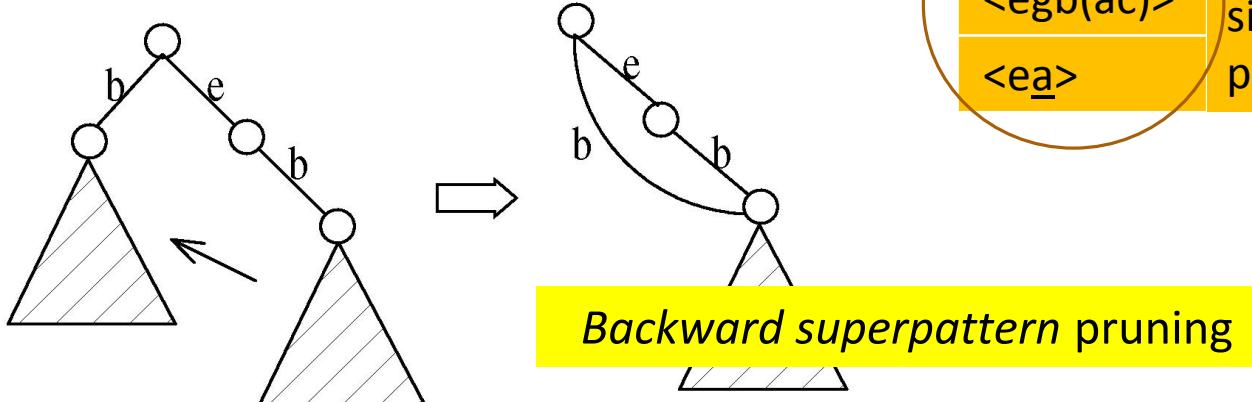
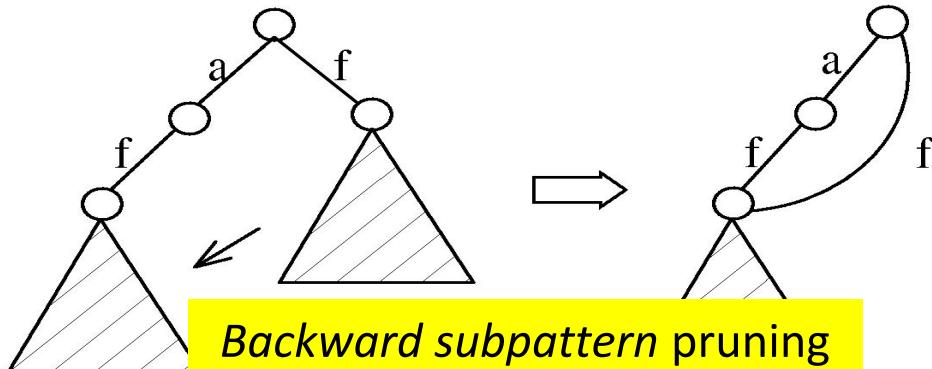
CloSpan: Mining Closed Sequential Patterns

- A **closed sequential pattern** s : There exists no superpattern s' such that $s' \supset s$, and s' and s have the same support
- Which ones are closed? $\langle abc \rangle: 20$, $\langle abcd \rangle: 20$, $\langle abcde \rangle: 15$
- Why directly mine closed sequential patterns?
 - Reduce # of (redundant) patterns
 - Attain the same expressive power
- Property P_1 : If $s \supset s_1$, s is closed iff two project DBs have the same size
- Explore *Backward Subpattern* and *Backward Superpattern* pruning to prune redundant search space
- Greatly enhances efficiency (Yan, et al., SDM'03)



CloSpan: When Two Projected DBs Have the Same Size

- If $s \supset s_1$, s is closed iff two project DBs have the same size
- When two projected sequence DBs have the same size?
- Here is one example:



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Constraint-Based Pattern Mining

- Why Constraint-Based Mining?
- Different Kinds of Constraints: Different Pruning Strategies
- Constrained Mining with Pattern Anti-Monotonicity
- Constrained Mining with Pattern Monotonicity
- Constrained Mining with Data Anti-Monotonicity
- Constrained Mining with Succinct Constraints
- Constrained Mining with Convertible Constraints
- Handling Multiple Constraints
- Constraint-Based Sequential-Pattern Mining

Why Constraint-Based Mining?

- Finding **all** the patterns in a dataset **autonomously**?—unrealistic!
 - Too many patterns but not necessarily user-interested!
- Pattern mining in practice: Often a user-guided, **interactive** process
 - User directs what to be mined using a **data mining query language** (or a graphical user interface), **specifying various kinds of constraints**
- What is constraint-based mining?
 - Mine together with user-provided constraints
- Why constraint-based mining?
 - User flexibility: User provides **constraints** on what to be mined
 - Optimization: System explores such constraints for mining efficiency
 - E.g., Push constraints deeply into the mining process

Various Kinds of User-Specified Constraints in Data Mining

- **Knowledge type constraint**—Specifying what kinds of knowledge to mine
 - Ex.: Classification, association, clustering, outlier finding, ...
- **Data constraint**—using SQL-like queries
 - Ex.: Find products sold together in **NY** stores this year
- **Dimension/level constraint**—similar to projection in relational database
 - Ex.: In relevance to **region**, price, brand, customer category
- **Interestingness constraint**—various kinds of thresholds
 - Ex.: Strong rules: $\text{min_sup} \geq 0.02$, $\text{min_conf} \geq 0.6$, $\text{min_correlation} \geq 0.7$
- **Rule (or pattern) constraint**  **The focus of this study**
 - Ex.: Small sales (price < \$10) triggers big sales (sum > \$200)

Pattern Space Pruning with Pattern Anti-Monotonicity

TID	Transaction
10	a, b, c, d, f, h
20	b, c, d, f, g, h
30	b, c, d, f, g
40	a, c, e, f, g

min_sup = 2

Item	Price	Profit
a	100	40
b	40	0
c	150	-20
d	35	-15
e	55	-30
f	45	-10
g	80	20
h	10	5

- A constraint c is **anti-monotone**
 - If an itemset S **violates** constraint c , so does any of its superset
 - That is, mining on itemset S can be terminated
- Ex. 1: $c_1: \text{sum}(S.\text{price}) \leq v$ is **anti-monotone**
- Ex. 2: $c_2: \text{range}(S.\text{profit}) \leq 15$ is **anti-monotone**
 - Itemset ab violates c_2 ($\text{range}(ab) = 40$)
 - So does every superset of ab
- Ex. 3. $c_3: \text{sum}(S.\text{Price}) \geq v$ is **not anti-monotone**
- Ex. 4. Is $c_4: \text{support}(S) \geq \sigma$ anti-monotone?
 - Yes! Apriori pruning is essentially pruning with an anti-monotonic constraint!

Note: item.price > 0

Profit can be negative

Pattern Monotonicity and Its Roles

TID	Transaction
10	a, b, c, d, f, h
20	b, c, d, f, g, h
30	b, c, d, f, g
40	a, c, e, f, g

min_sup = 2

Item	Price	Profit
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d	35	-15
e	55	-30
f	45	-10
g	80	20
h	10	5

- A constraint c is *monotone*: If an itemset S **satisfies** the constraint c , so does any of its superset
 - That is, we do not need to check c in subsequent mining
- Ex. 1: $c_1: \text{sum}(S.\text{Price}) \geq v$ is **monotone**
- Ex. 2: $c_2: \text{min}(S.\text{Price}) \leq v$ is **monotone**
- Ex. 3: $c_3: \text{range}(S.\text{profit}) \geq 15$ is **monotone**
 - Itemset ab satisfies c_3
 - So does every superset of ab

Note: item.price > 0
Profit can be negative

Data Space Pruning with Data Anti-Monotonicity

TID	Transaction
10	a, b, c, d, f, h
20	b, c, d, f, g, h
30	b, c, d, f, g
40	a, c, e, f, g

min_sup = 2

Item	Price	Profit
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e	55	-30
f	45	-10
g	80	20
h	10	5

- ❑ A constraint c is **data anti-monotone**: In the mining process, if a data entry t cannot satisfy a pattern p under c , t cannot satisfy p 's superset either
 - ❑ Data space pruning: Data entry t can be pruned
- ❑ Ex. 1: $c_1: \text{sum}(S.\text{Profit}) \geq v$ is **data anti-monotone**
 - ❑ Let constraint c_1 be: $\text{sum}(S.\text{Profit}) \geq 25$
 - ❑ $T_{30}: \{b, c, d, f, g\}$ can be removed since none of their combinations can make an S whose sum of the profit is ≥ 25
- ❑ Ex. 2: $c_2: \text{min}(S.\text{Price}) \leq v$ is **data anti-monotone**
 - ❑ Consider $v = 5$ but every item in a transaction, say T_{50} , has a price higher than 10
- ❑ Ex. 3: $c_3: \text{range}(S.\text{Profit}) > 25$ is **data anti-monotone**

Note: item.price > 0
Profit can be negative

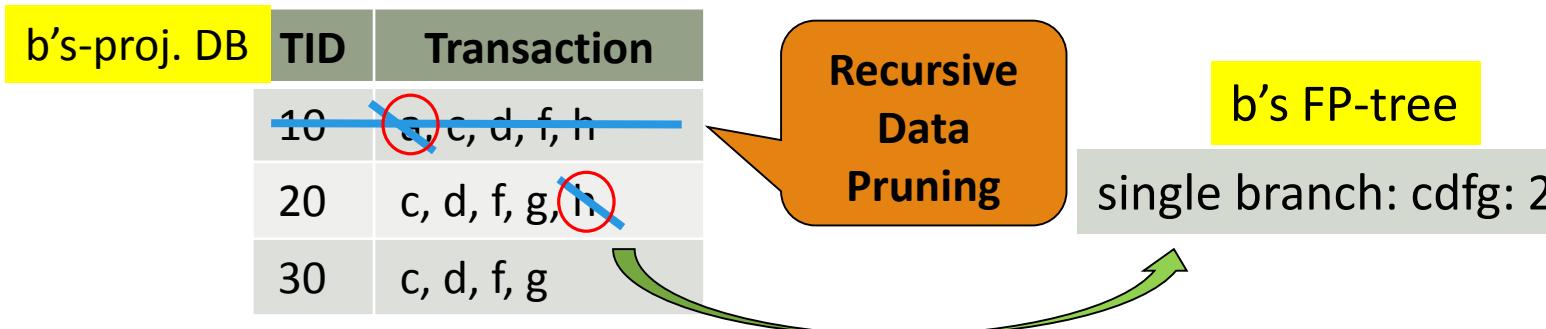
Data Space Pruning Should Be Explored Recursively

- Example. $c_3: \text{range}(S.\text{Profit}) > 25$
- We check b's projected database →
- But item "a" is infrequent ($\text{sup} = 1$)
- After removing "a (40)" from T_{10}
- T_{10} cannot satisfy c_3 any more
 - Since "b (0)" and "c (-20), d (-15), f (-10), h (5)"
- By removing T_{10} , we can also prune "h" in T_{20}

b's-proj. DB

TID	Transaction	b's-proj. DB	
TID	Transaction	Item	Profit
10	a, c, d, f, h	a	40
20	c, d, f, g, h	b	0
30	c, d, f, g	c	-20
		d	-15
		e	-30
		f	-10
		g	20
		h	5

min_sup = 2
 price(item) > 0
 Constraint:
 $\text{range}\{S.\text{profit}\} > 25$



- Note: c_3 prunes T_{10} effectively only after "a" is pruned (by min-sup) in b's projected DB

Succinctness: Pruning Both Data and Pattern Spaces

- Succinctness: If the constraint c can be enforced by directly manipulating the data
- Ex. 1: To find those patterns without item i
 - Remove i from DB and then mine (pattern space pruning)
- Ex. 2: To find those patterns containing item i
 - Mine only i -projected DB (data space pruning)
- Ex. 3: $c_3: \min(S.\text{Price}) \leq v$ is succinct
 - Start with only items whose price $\leq v$ and remove transactions with high-price items only (pattern + data space pruning)
- Ex. 4: $c_4: \sum(S.\text{Price}) \geq v$ is not succinct
 - It cannot be determined beforehand since sum of the price of itemset S keeps increasing

Convertible Constraints: Ordering Data in Transactions

TID	Transaction
10	a, b, c, d, f, h
20	a, b, c, d, f, g, h
30	b, c, d, f, g
40	a, c, e, f, g

min_sup = 2

Item	Price	Profit
a	100	40
b	40	0
c	150	-20
d	35	-15
e	55	-30
f	45	-5
g	80	30
h	10	5

- Convert tough constraints into (anti-)monotone by proper ordering of items in transactions
- Examine $c_1: \text{avg}(S.\text{profit}) > 20$
 - Order items in (profit) value-descending order
 - $\langle a, g, f, b, h, d, c, e \rangle$
 - An itemset ab violates c_1 ($\text{avg}(ab) = 20$)
 - So does ab^* (i.e., ab -projected DB)
 - C_1 : **anti-monotone if patterns grow in the right order!**
 - Can item-reordering work for Apriori?
 - Level-wise candidate generation requires multi-way checking!
 - $\text{avg}(agf) = 21.7 > 20$, but $\text{avg}(gf) = 12.5 < 20$
 - Apriori will not generate “agf” as a candidate

Different Kinds of Constraints Lead to Different Pruning Strategies

- In summary, constraints can be categorized as
 - Pattern space pruning constraints vs. data space pruning constraints
- Pattern space pruning constraints
 - Anti-monotonic: If constraint c is violated, its further mining can be terminated
 - Monotonic: If c is satisfied, no need to check c again
 - Succinct: If the constraint c can be enforced by directly manipulating the data
 - Convertible: c can be converted to monotonic or anti-monotonic if items can be properly ordered in processing
- Data space pruning constraints
 - Data succinct: Data space can be pruned at the initial pattern mining process
 - Data anti-monotonic: If a transaction t does not satisfy c , then t can be pruned to reduce data processing effort

How to Handle Multiple Constraints?

- It is beneficial to use multiple constraints in pattern mining
- But different constraints may require potentially conflicting item-ordering
 - If there exists conflict ordering between c_1 and c_2
 - Try to sort data and enforce *one constraint* first (which one?)
 - Then enforce the other constraint when mining the projected databases
- Ex. c_1 : $\text{avg}(S.\text{profit}) > 20$, and c_2 : $\text{avg}(S.\text{price}) < 50$
 - Assume c_1 has more pruning power
 - Sort in profit descending order and use c_1 first
 - For each project DB, sort trans. in price ascending order and use c_2 at mining

Constraint-Based Sequential-Pattern Mining

- Share many similarities with constraint-based itemset mining
- **Anti-monotonic:** If S violates c , the super-sequences of S also violate c
 - $\text{sum}(S.\text{price}) < 150; \text{min}(S.\text{value}) > 10$
- **Monotonic:** If S satisfies c , the super-sequences of S also do so
 - $\text{element_count}(S) > 5; S \supseteq \{\text{PC}, \text{digital_camera}\}$
- **Data anti-monotonic:** If a sequence s_1 with respect to S violates c_3 , s_1 can be removed
 - $c_3: \text{sum}(S.\text{price}) \geq v$
- **Succinct:** Enforce constraint c by explicitly manipulating data
 - $S \supseteq \{\text{i-phone}, \text{MacAir}\}$
- **Convertible:** Projection based on the sorted value not sequence order
 - $\text{value_avg}(S) < 25; \text{profit_sum}(S) > 160$
 - $\text{max}(S)/\text{avg}(S) < 2; \text{median}(S) - \text{min}(S) > 5$

Timing-Based Constraints in Seq.-Pattern Mining

- **Order constraint:** Some items must happen before the other
 - {algebra, geometry} → {calculus} (where “→” indicates ordering)
 - Anti-monotonic: Constraint-violating sub-patterns pruned
- **Min-gap/max-gap constraint:** Confines two elements in a pattern
 - E.g., mingap = 1, maxgap = 4
 - Succinct: Enforced directly during pattern growth
- **Max-span constraint:** Maximum allowed time difference between the 1st and the last elements in the pattern
 - E.g., maxspan (S) = 60 (days)
 - Succinct: Enforced directly when the 1st element is determined
- **Window size constraint:** Events in an element do not have to occur at the same time: Enforce max allowed time difference
 - E.g., window-size = 2: Various ways to merge events into elements

Episodes and Episode Pattern Mining

- ❑ Episodes and regular expressions: Alternative to seq. patterns
 - ❑ Serial episodes: AB  a total order relationship: first A then B
 - ❑ Parallel episodes: A|B  a partial order relationship: A and B can be in any order
 - ❑ Regular expressions: (A|B)C*(DE)  (DE) means D, E happen in the same time window
- ❑ Ex. Given a large shopping sequence database, one may like to find
 - ❑ Suppose the pattern order follows the template (A|B)C*(D E), and
 - ❑ Sum of the prices of A, B, C*, D, and E is greater than \$100, where C* means C appears *-times
 - ❑ How to efficiently mine such episode patterns?

Summary: Constraint-Based Pattern Mining

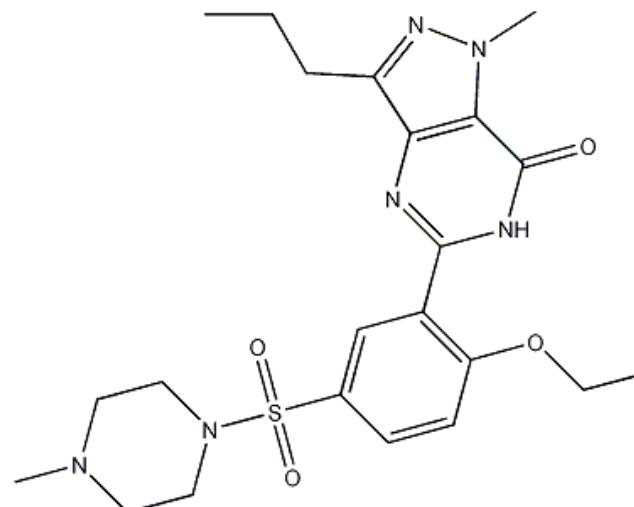
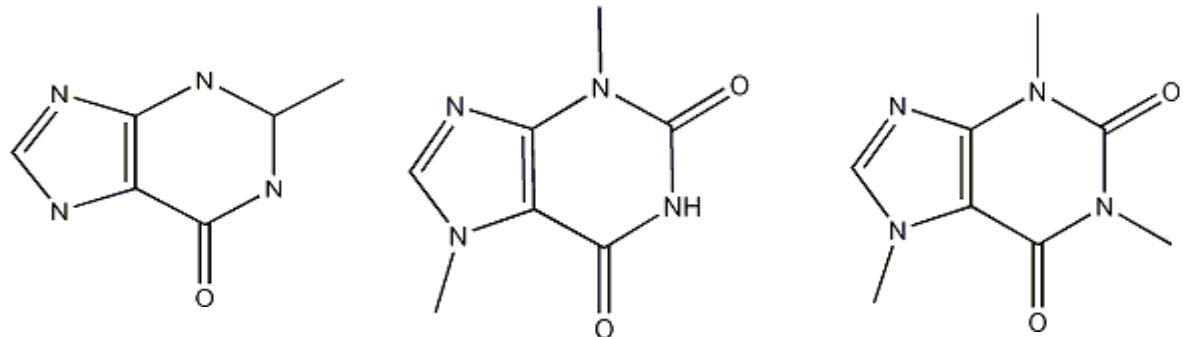
- Why Constraint-Based Mining?
- Different Kinds of Constraints: Different Pruning Strategies
- Constrained Mining with Pattern Anti-Monotonicity
- Constrained Mining with Pattern Monotonicity
- Constrained Mining with Data Anti-Monotonicity
- Constrained Mining with Succinct Constraints
- Constrained Mining with Convertible Constraints
- Handling Multiple Constraints
- Constraint-Based Sequential-Pattern Mining

Chapter 7 : Advanced Frequent Pattern Mining

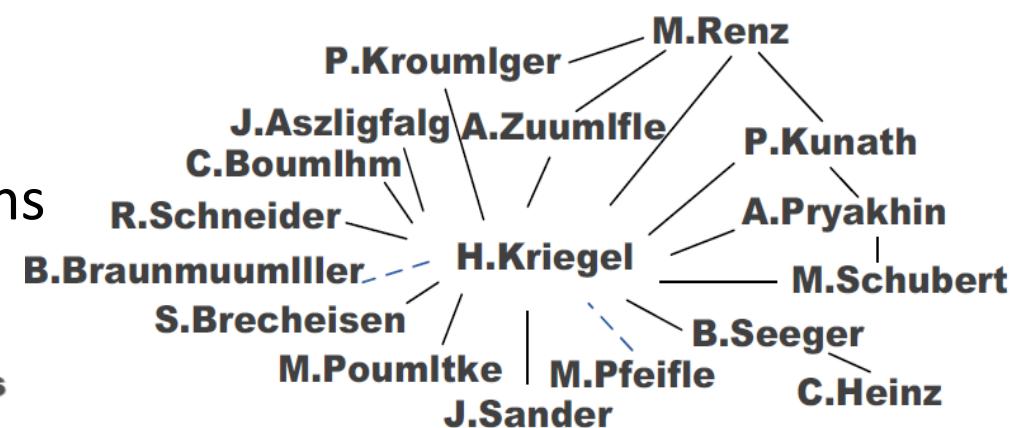
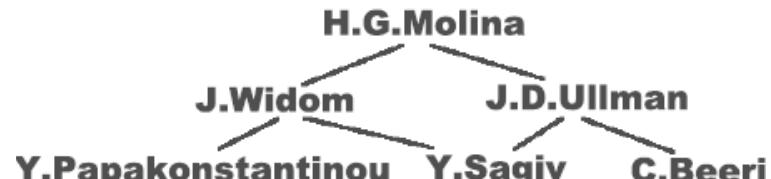
- ❑ Mining Diverse Patterns
- ❑ Sequential Pattern Mining
- ❑ Constraint-Based Frequent Pattern Mining
- ❑ Graph Pattern Mining 
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What Is Graph Pattern Mining?

- Chem-informatics:
 - Mining frequent chemical compound structures



- Social networks, web communities, tweets, ...
 - Finding frequent research collaboration subgraphs



Frequent (Sub)Graph Patterns

- Given a labeled graph dataset $D = \{G_1, G_2, \dots, G_n\}$, the supporting graph set of a subgraph g is $D_g = \{G_i \mid g \subseteq G_i, G_i \in D\}$
 - $\text{support}(g) = |D_g| / |D|$
 - A (sub)graph g is **frequent** if $\text{support}(g) \geq \text{min_sup}$
 - Ex.: Chemical structures
 - Alternative:
 - Mining frequent subgraph patterns from a single large graph or network

Graph D

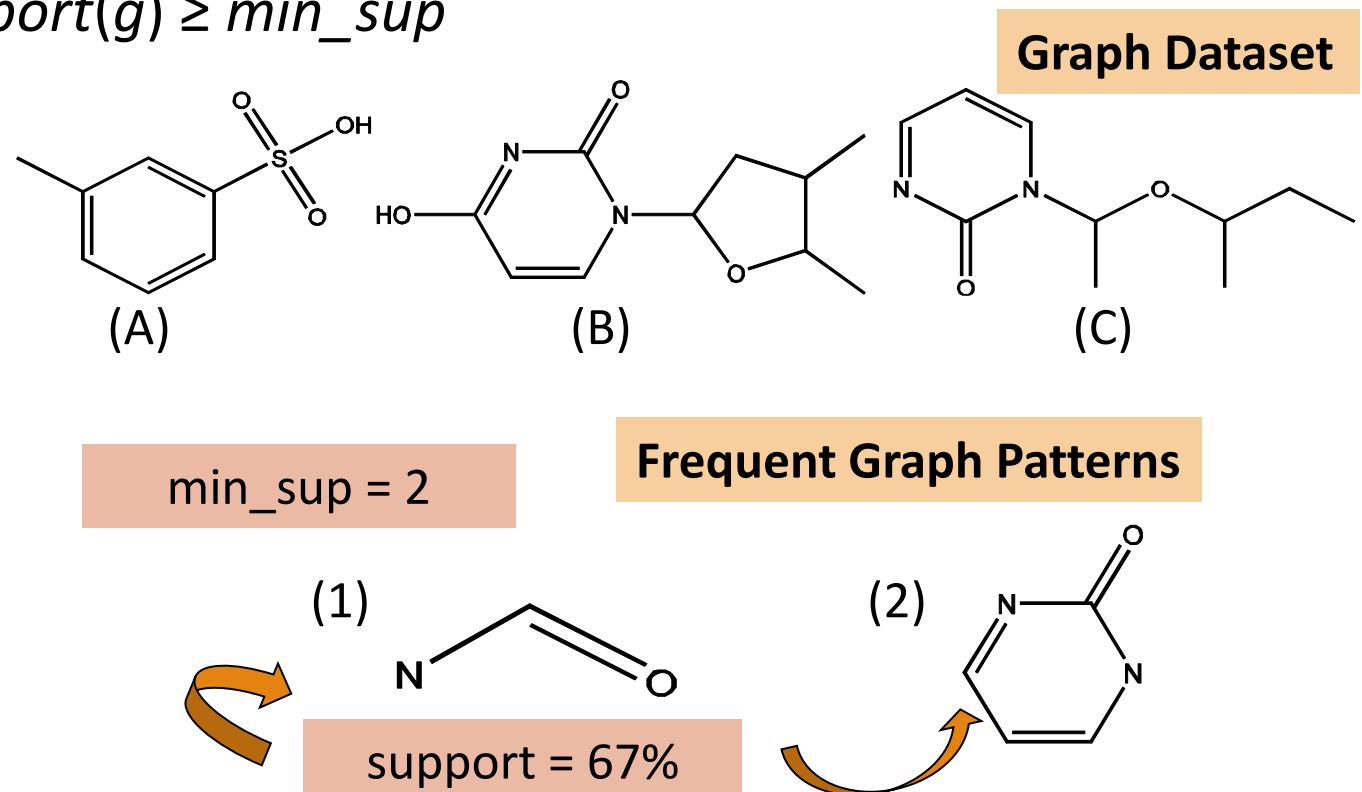
(A)

(B)

(C)

min_sup = 2

Frequent Graph Patterns



Applications of Graph Pattern Mining

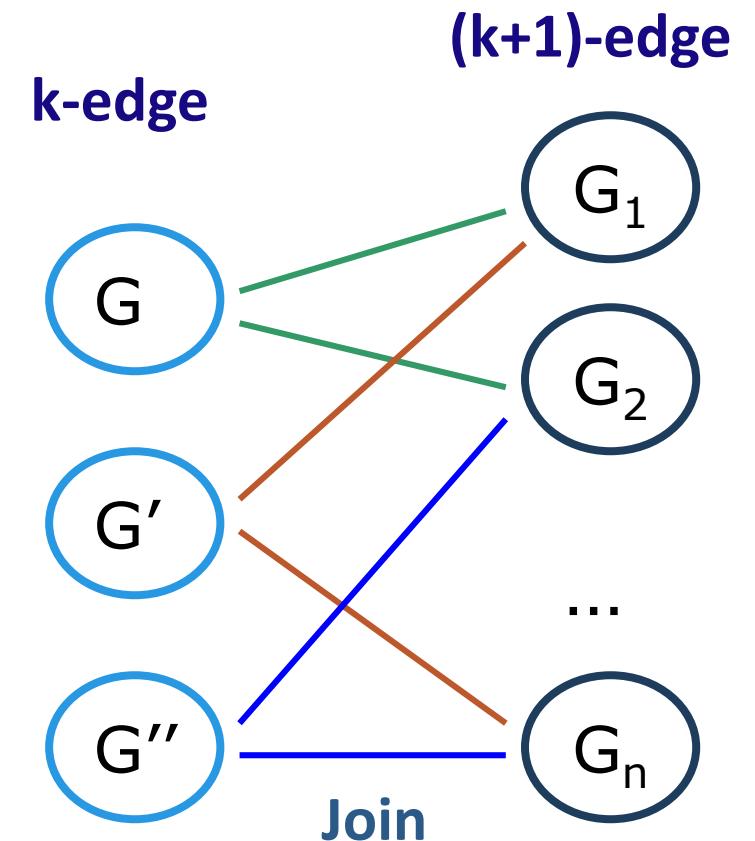
- Bioinformatics
 - Gene networks, protein interactions, metabolic pathways
- Chem-informatics: Mining chemical compound structures
- Social networks, web communities, tweets, ...
- Cell phone networks, computer networks, ...
- Web graphs, XML structures, Semantic Web, information networks
- Software engineering: Program execution flow analysis
- Building blocks for graph classification, clustering, compression, comparison, and correlation analysis
- Graph indexing and graph similarity search

Graph Pattern Mining Algorithms: Different Methodologies

- Generation of candidate subgraphs
 - Apriori vs. pattern growth (e.g., FSG vs. gSpan)
- Search order
 - Breadth vs. depth
- Elimination of duplicate subgraphs
 - Passive vs. active (e.g., gSpan [Yan & Han, 2002])
- Support calculation
 - Store embeddings (e.g., GASTON [Nijssen & Kok, 2004], FFSM [Huan, Wang, & Prins, 2003], MoFa [Borgelt & Berthold, ICDM'02])
- Order of pattern discovery
 - Path → tree → graph (e.g., GASTON [Nijssen & Kok, 2004])

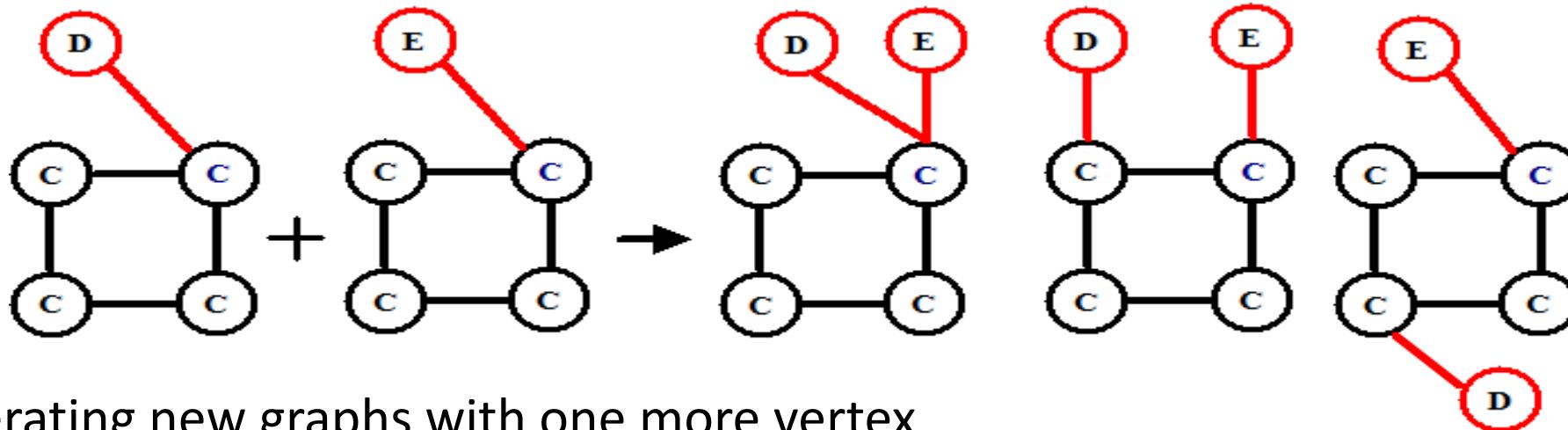
Apriori-Based Approach

- The Apriori property (anti-monotonicity): A size- k subgraph is frequent if and only if all of its subgraphs are frequent
- A candidate size- $(k+1)$ edge/vertex subgraph is generated if its corresponding two k -edge/vertex subgraphs are frequent
- Iterative mining process:
 - Candidate-generation → candidate pruning → support counting → candidate elimination



Candidate Generation: Vertex Growing vs. Edge Growing

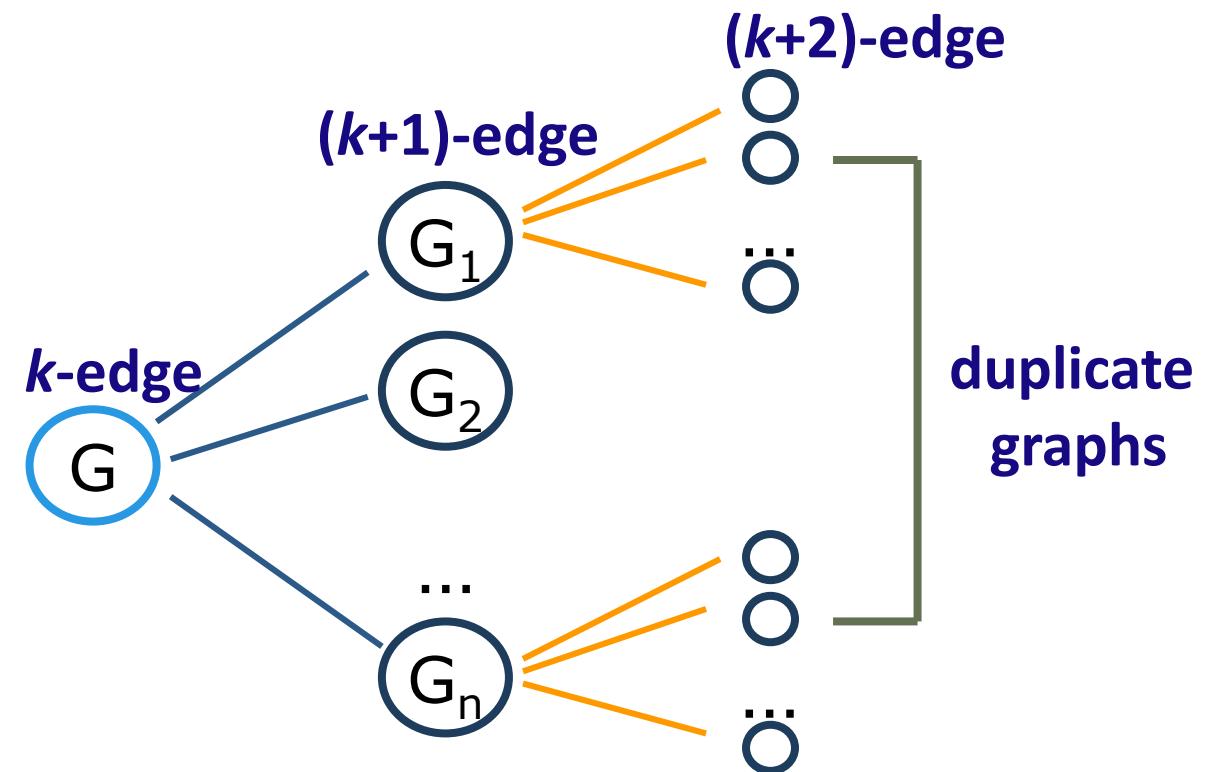
- Methodology: Breadth-search, Apriori joining two size- k graphs
 - Many possibilities at generating size- $(k+1)$ candidate graphs



- Generating new graphs with one more vertex
 - AGM (Inokuchi, Washio, & Motoda, PKDD'00)
- Generating new graphs with one more edge
 - FSG (Kuramochi & Karypis, ICDM'01)
- Performance shows *via edge growing* is more efficient

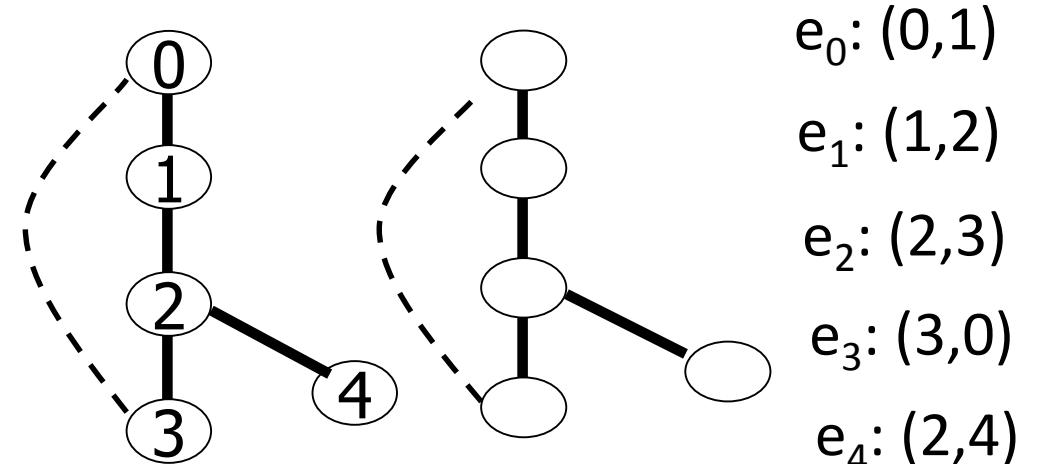
Pattern-Growth Approach

- Depth-first growth of subgraphs from k -edge to $(k+1)$ -edge, then $(k+2)$ -edge subgraphs
- Major challenge
 - Generating many duplicate subgraphs
- Major idea to solve the problem
 - Define an order to generate subgraphs
 - DFS spanning tree: Flatten a graph into a sequence using depth-first search
 - gSpan (Yan & Han, ICDM'02)



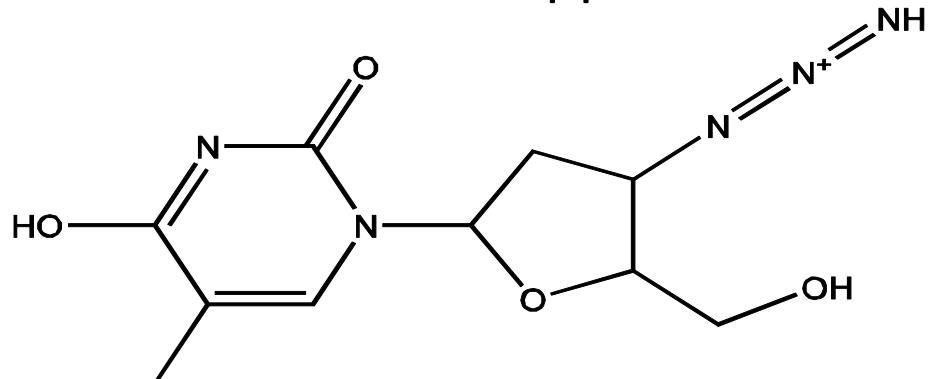
gSPAN: Graph Pattern Growth in Order

- **Right-most path extension** in subgraph pattern growth
- Right-most path: The path from root to the right-most leaf (choose the vertex with the smallest index at each step)
- Reduce generation of duplicate subgraphs
- **Completeness:** The enumeration of graphs using right-most path extension is complete
- DFS code: Flatten a graph into a sequence using depth-first search



Why Mine Closed Graph Patterns?

- Challenge: An n -edge frequent graph may have 2^n subgraphs
- Motivation: Explore *closed frequent subgraphs* to handle graph pattern explosion problem
- A frequent graph G is *closed* if there exists no supergraph of G that carries the same support as G

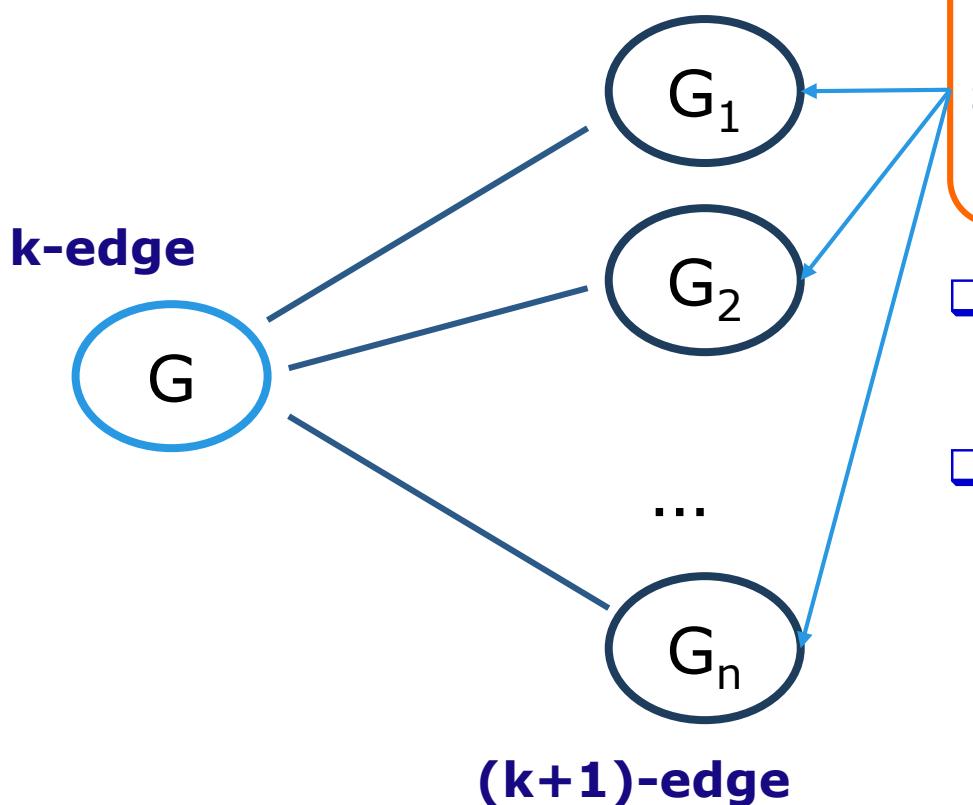


If this subgraph is *closed* in the graph dataset, it implies that none of its frequent super-graphs carries the same support

- *Lossless compression*: Does not contain non-closed graphs, but still ensures that the mining result is complete
- Algorithm CloseGraph: Mines closed graph patterns directly

CloseGraph: Directly Mining Closed Graph Patterns

- CloseGraph: Mining closed graph patterns by extending gSpan (Yan & Han, KDD'03)

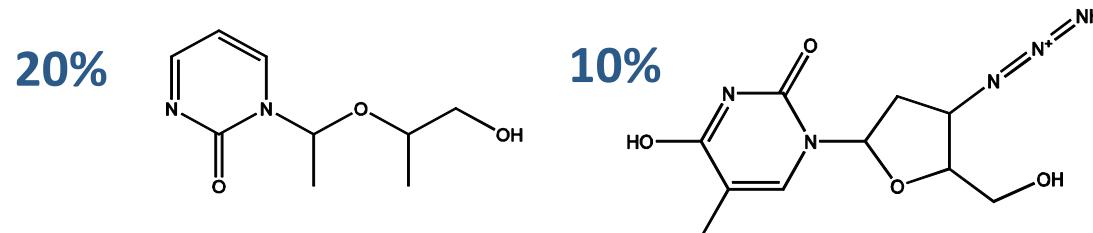


At what condition can we stop searching their children, i.e., early termination?

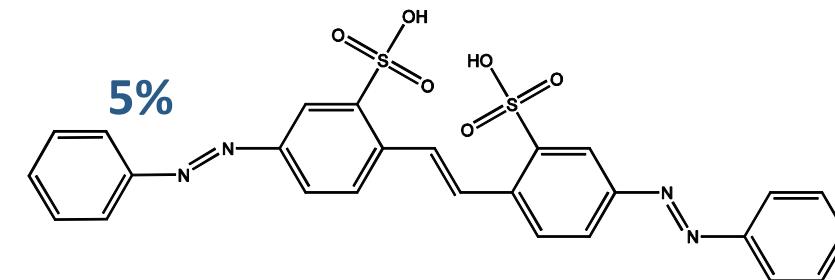
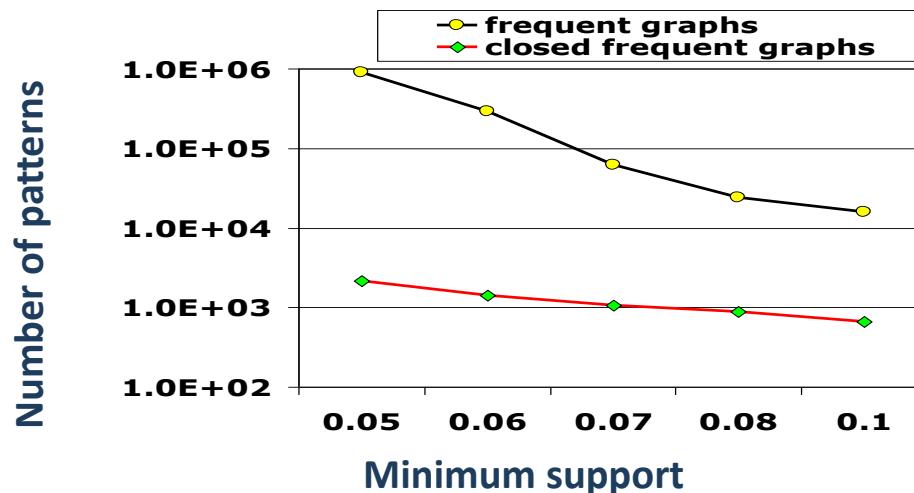
- Suppose G and G_1 are frequent, and G is a subgraph of G_1
- If **in any part of the graph in the dataset where G occurs, G_1 also occurs**, then we need not grow G (except some special, subtle cases), since none of G 's children will be closed except those of G_1

Experiment and Performance Comparison

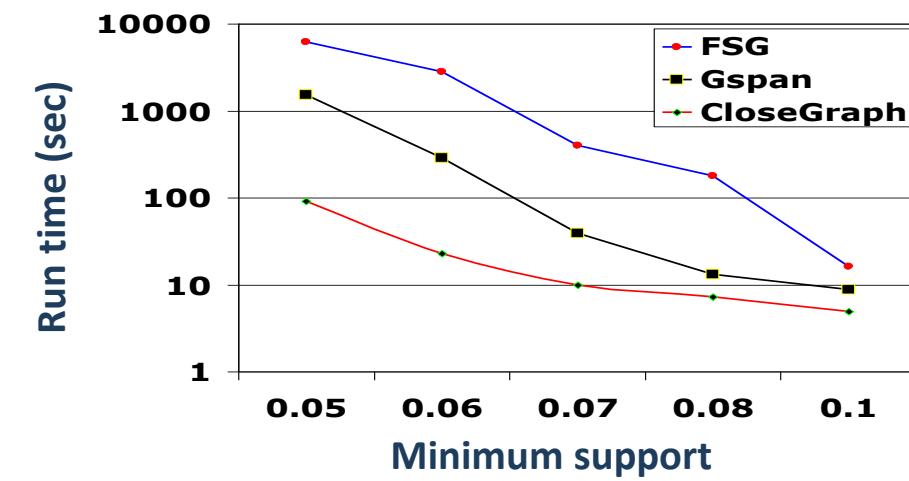
- The AIDS antiviral screen compound dataset from NCI/NIH
- The dataset contains 43,905 chemical compounds
- Discovered patterns: The smaller minimum support, the bigger and more interesting subgraph patterns discovered



of Patterns: Frequent vs. Closed



Runtime: Frequent vs. Closed



Chapter 7 : Advanced Frequent Pattern Mining

- ❑ Mining Diverse Patterns
- ❑ Sequential Pattern Mining
- ❑ Constraint-Based Frequent Pattern Mining
- ❑ Graph Pattern Mining
- ❑ Pattern Mining Application: Mining Software Copy-and-Paste Bugs
- ❑ Summary



Pattern Mining Application: Software Bug Detection

- **Mining rules from source code**
 - Bugs as deviant behavior (e.g., by statistical analysis)
 - Mining programming rules (e.g., by frequent itemset mining)
 - Mining function precedence protocols (e.g., by frequent subsequence mining)
 - Revealing neglected conditions (e.g., by frequent itemset/subgraph mining)
- **Mining rules from revision histories**
 - By frequent itemset mining
- **Mining copy-paste patterns from source code**
 - Find copy-paste bugs (e.g., CP-Miner [Li et al., OSDI'04]) (to be discussed here)
 - Reference: Z. Li, S. Lu, S. Myagmar, Y. Zhou, “[CP-Miner](#): A Tool for Finding Copy-paste and Related Bugs in Operating System Code”, OSDI’04

Application Example: Mining Copy-and-Paste Bugs

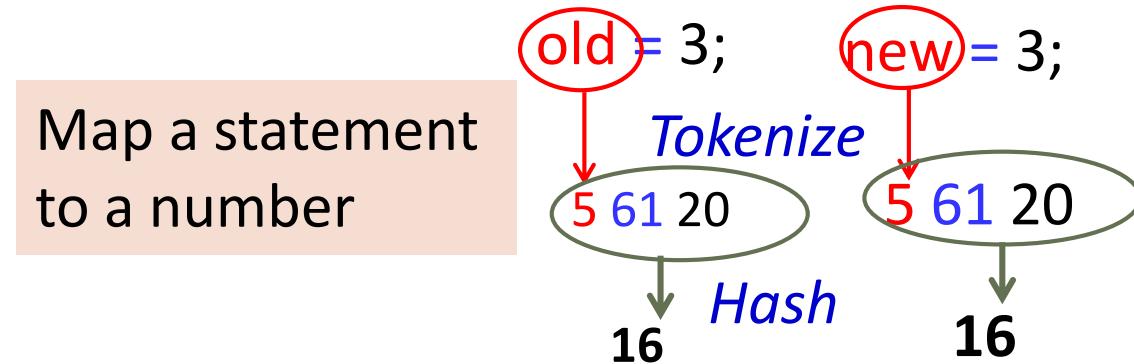
- ❑ Copy-pasting is common
 - ❑ 12% in Linux file system
 - ❑ 19% in X Window system
- ❑ Copy-pasted code is error-prone
- ❑ Mine “*forget-to-change*” bugs by sequential pattern mining
 - ❑ Build a sequence database from source code
 - ❑ Mining sequential patterns
 - ❑ Finding mismatched identifier names & bugs

```
void __init prom_meminit(void)
{
    .....
    for (i=0; i<n; i++) {
        total[i].adr = list[i].addr;
        total[i].bytes = list[i].size;
        total[i].more = &total[i+1];
    }
    .....
    for (i=0; i<n; i++) {
        taken[i].adr = list[i].addr;
        taken[i].bytes = list[i].size,
        taken[i].more = &total[i+1];
    }
}
```

Code copy-and-pasted but **forget to change “id”!**

Building Sequence Database from Source Code

- Statement $\xrightarrow{\text{(mapped to)}}$ number
- Tokenize each component
 - Different operators, constants, key words \rightarrow different tokens
 - Same type of identifiers \rightarrow same token
- Program \rightarrow A long sequence
- Cut the long sequence by blocks



Hash values

65
16
16
71
...
65
16
16
71

```
for (i=0; i<n; i++) {  
    total[i].adr = list[i].addr;  
    total[i].bytes = list[i].size;  
    total[i].more = &total[i+1];  
}
```

.....

```
for (i=0; i<n; i++) {  
    taken[i].adr = list[i].addr;  
    taken[i].bytes = list[i].size;  
    taken[i].more = &total[i+1];  
}
```

Final sequence DB:

(65)
(16, 16, 71)
...
(65)
(16, 16, 71)

Sequential Pattern Mining & Detecting “Forget-to-Change” Bugs

□ Modification to the *sequence pattern mining algorithm*

- Constrain the max gap

(16, 16, 71)

.....

(16, 16, 10, 71)

Allow a maximal gap:
inserting statements
in copy-and-paste

□ Composing Larger Copy-Pasted Segments

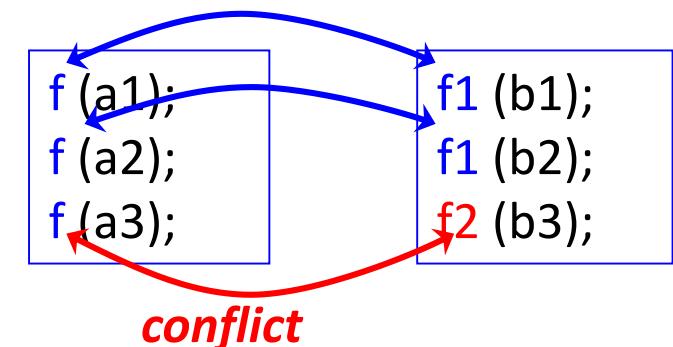
- Combine the neighboring copy-pasted segments repeatedly

□ Find conflicts: Identify names that cannot be mapped to the corresponding ones

- E.g., 1 out of 4 “**total**” is unchanged, *unchanged ratio* = 0.25

- If $0 < \text{unchanged ratio} < \text{threshold}$, then report it as a bug

- CP-Miner reported many C-P bugs in Linux, Apache, ... out of millions of LOC (lines of code)



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Summary: Advanced Frequent Pattern Mining

- ❑ Mining Diverse Patterns
 - ❑ Mining Multiple-Level Associations
 - ❑ Mining Multi-Dimensional Associations
 - ❑ Mining Quantitative Associations
 - ❑ Mining Negative Correlations
 - ❑ Mining Compressed and Redundancy-Aware Patterns
- ❑ Sequential Pattern Mining
 - ❑ Sequential Pattern and Sequential Pattern Mining
 - ❑ GSP: Apriori-Based Sequential Pattern Mining
 - ❑ SPADE: Sequential Pattern Mining in Vertical Data Format
 - ❑ PrefixSpan: Sequential Pattern Mining by Pattern-Growth
 - ❑ CloSpan: Mining Closed Sequential Patterns
- ❑ Constraint-Based Frequent Pattern Mining
 - ❑ Why Constraint-Based Mining?
 - ❑ Constrained Mining with Pattern Anti-Monotonicity
 - ❑ Constrained Mining with Pattern Monotonicity
 - ❑ Constrained Mining with Data Anti-Monotonicity
 - ❑ Constrained Mining with Succinct Constraints
 - ❑ Constrained Mining with Convertible Constraints
 - ❑ Handling Multiple Constraints
 - ❑ Constraint-Based Sequential-Pattern Mining
- ❑ Graph Pattern Mining
 - ❑ Graph Pattern and Graph Pattern Mining
 - ❑ Apriori-Based Graph Pattern Mining Methods
 - ❑ gSpan: A Pattern-Growth-Based Method
 - ❑ CloseGraph: Mining Closed Graph Patterns
- ❑ Pattern Mining Application: Mining Software Copy-and-Paste Bugs

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