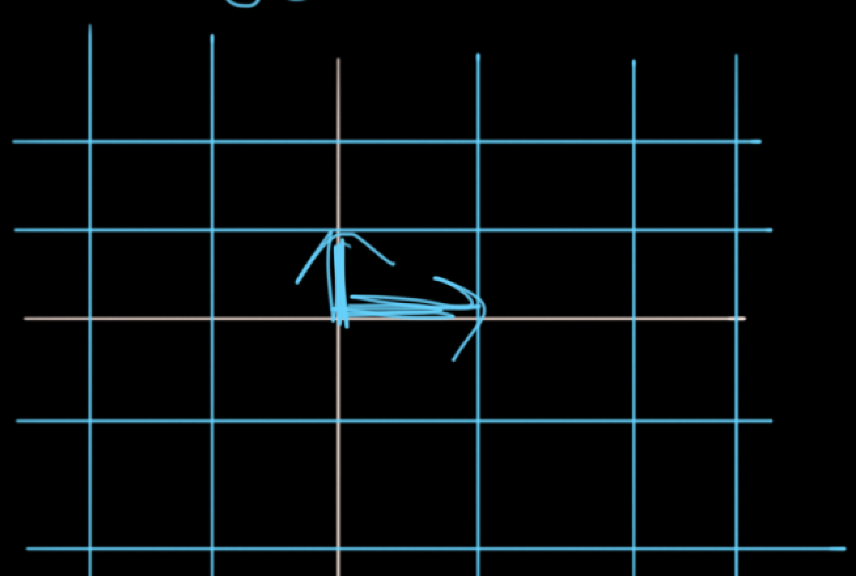
Diagonal matrices

Only nonzero entries on diagonal (row=col) $\begin{bmatrix} 3 & 0 \\ 0 & 2 \end{bmatrix}$
zeros elsewhere

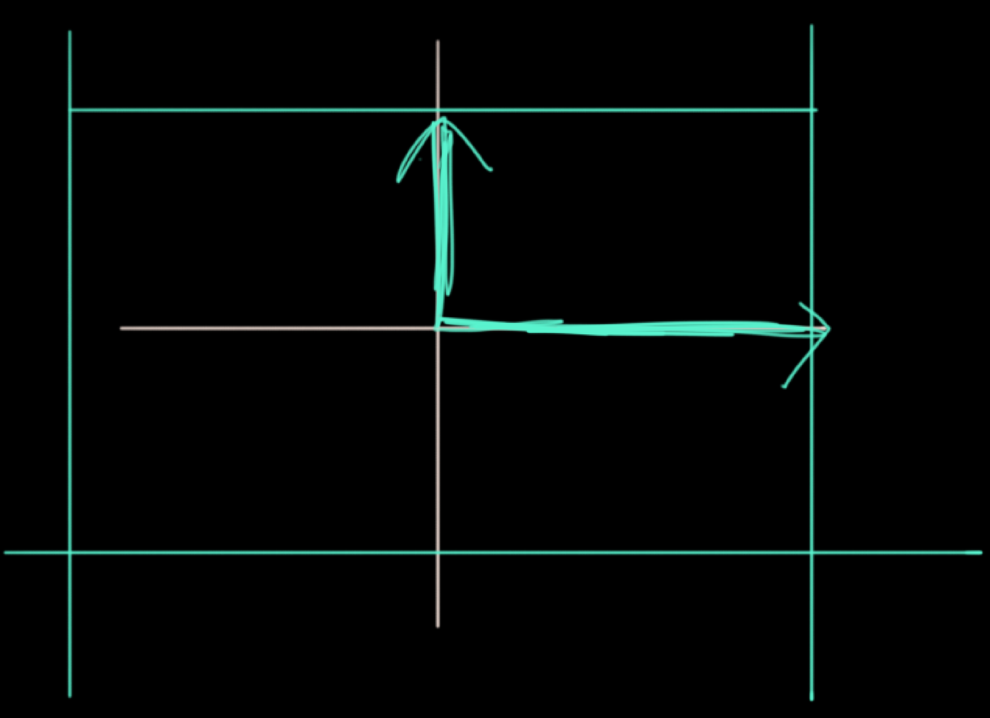
Scales space
Can be rectangular
Adds or removes dimensions

3D → 2D $\begin{bmatrix} 3 & 0 & 0 \\ 0 & 2 & 0 \end{bmatrix}$ 2D → 3D $\begin{bmatrix} 3 & 0 \\ 0 & 2 \\ 0 & 0 \end{bmatrix}$

Before



After



Inverse = diagonal matrix with inverse entries

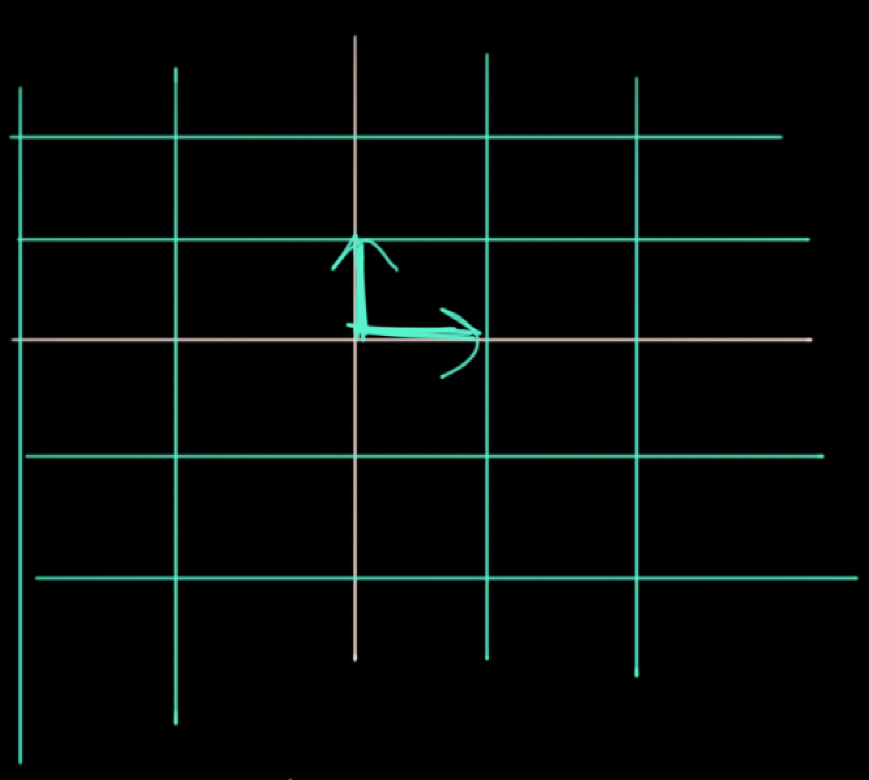
$D = \begin{bmatrix} 3 & 0 \\ 0 & 2 \end{bmatrix}$ $D^{-1} = \begin{bmatrix} 1/3 & 0 \\ 0 & 1/2 \end{bmatrix}$

Orthogonal matrices

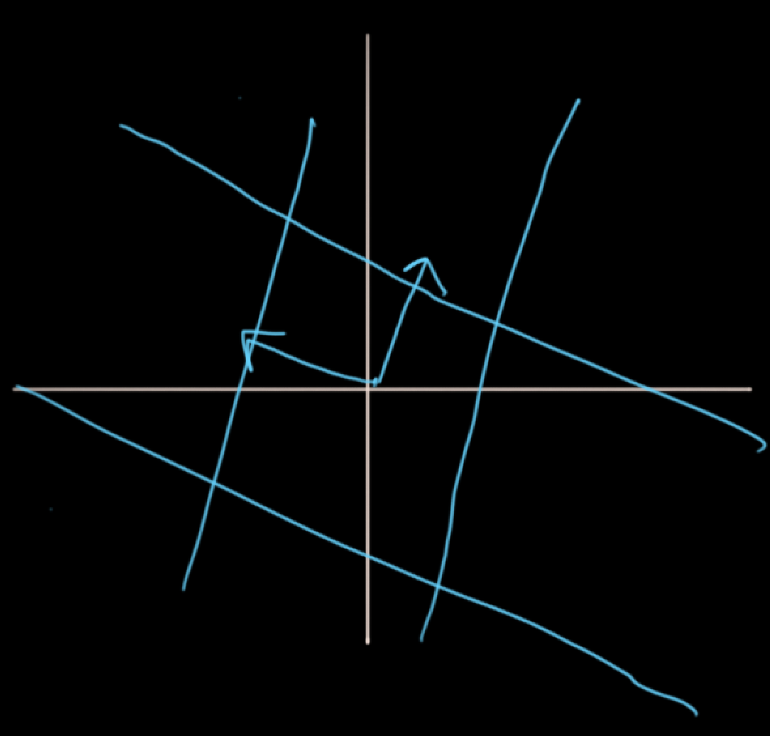
Square matrix where every column is a unit vector and every pair of columns is orthogonal

$A = [\bar{c}_1 \ \bar{c}_2 \ldots]$ $\|\bar{c}_i\| = 1$ for all i
$\bar{c}_i \cdot \bar{c}_j = 0$ for all i/j where $i \neq j$

Before



After



Rotates space

Inverse = the transpose $O^{-1} = O^T$
↳ Flips matrix over its diagonal
aka flips rows + columns

$O = \begin{bmatrix} 1/\sqrt{2} & -1/\sqrt{2} \\ 1/\sqrt{2} & 1/\sqrt{2} \end{bmatrix}$ $O^T = \begin{bmatrix} 1/\sqrt{2} & 1/\sqrt{2} \\ -1/\sqrt{2} & 1/\sqrt{2} \end{bmatrix}$

$OO^T = O^TO = I$

$\underbrace{[\bar{c}_1 \ \bar{c}_2 \ \bar{c}_3]}_{O} \underbrace{\begin{bmatrix} \bar{c}_1^T \\ \bar{c}_2^T \\ \bar{c}_3^T \end{bmatrix}}_{O^T} = \begin{bmatrix} \bar{c}_1\bar{c}_1^T & \bar{c}_1\bar{c}_2^T & \bar{c}_1\bar{c}_3^T \\ \bar{c}_2\bar{c}_1^T & \bar{c}_2\bar{c}_2^T & \bar{c}_2\bar{c}_3^T \\ \bar{c}_3\bar{c}_1^T & \bar{c}_3\bar{c}_2^T & \bar{c}_3\bar{c}_3^T \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$

Symmetric matrix

Square matrix where $a_{ij} = a_{ji}$ for every i/j

$A = A^T$ $\begin{bmatrix} 3 & 5 & 6 \\ 5 & 2 & 4 \\ 6 & 4 & 1 \end{bmatrix}$

Eigenvalues are always real (not complex)
Eigenvectors associated with different eigenvalues are orthogonal

Assume $\bar{v}_1, \bar{v}_2$ are eigenvectors with eigenvalues λ_1, λ_2

$(AB)^T = B^TA^T$

$\lambda_1(\bar{v}_1 \cdot \bar{v}_2) = (\lambda \bar{v}_1) \cdot \bar{v}_2$
$= (A\bar{v}_1) \cdot \bar{v}_2$
$= (A\bar{v}_1)^T \bar{v}_2$
$= \bar{v}_1^T A^T \bar{v}_2$
$= \bar{v}_1^T A \bar{v}_2$
$= \bar{v}_1^T \lambda_2 \bar{v}_2$
$= \lambda_2(\bar{v}_1 \cdot \bar{v}_2)$

$\lambda_1(\bar{v}_1 \cdot \bar{v}_2) = \lambda_2(\bar{v}_1 \cdot \bar{v}_2)$
$\cancel{(\lambda_1 - \lambda_2)}(\bar{v}_1 \cdot \bar{v}_2) = 0$
$\bar{v}_1 \cdot \bar{v}_2 = 0 \Rightarrow$ Orthogonal! ▽

Identity matrix is diagonal, orthogonal, and symmetric