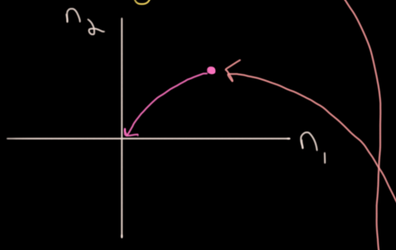
Dynamical System

System concerned with the evolution of something over time

Already seen a dynamical system!

$\bar{u}_{t+1} = W\bar{u}_t$

W: Synaptic weights; $\bar{u}_t$: Neural activities



State of DS: Complete description of the system at time t

$\bar{u}_t = \begin{bmatrix} u_1 \\ u_2 \end{bmatrix}$

State space: appropriate geometrical space in which to describe the state

Initial condition: initial state at time 0

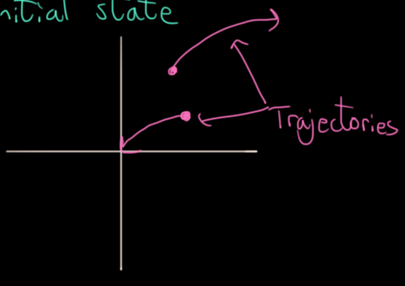
Predefined rules for the evolution of the system, dynamics, how the state now changes in the future

System in which some mathematical function describes the movement of a point in some geometrical space as a function of time

→ Defined by state space and rules for evolution

→ Specific trajectories based on initial state

Used in many fields!



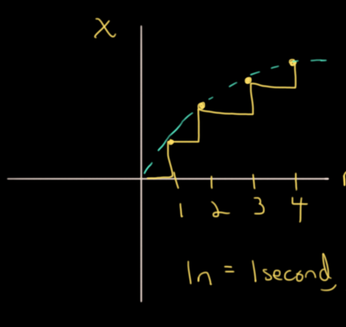
Dynamical System

How dynamics are defined

Discrete

$x_{n+1} = F(x_n)$

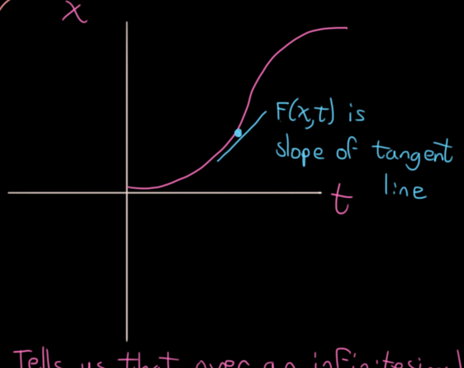
↑ Some Function



n = 1 second/year...

Continuous

$\frac{\partial x}{\partial t} = F(x,t)$



Tells us that over an infinitesimal instant, x changes by $F(x,t)$

Differential equation

Equation that relates one or more functions and their derivatives

Continuous dynamics described by systems of differential equations

$x(t) \rightarrow \frac{\partial x}{\partial t}$