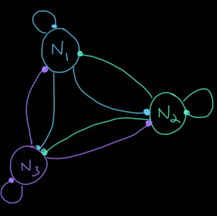


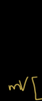
Activity at time t:

$$\bar{u}_t = \begin{bmatrix} u_{1,t} \\ u_{2,t} \\ u_{3,t} \end{bmatrix}$$ ← Response of N_1 ... of N_2 ... of N_3

Synaptic weights between neurons:

W_{ij} = weight of synapse from j to i

Peak depolarization that a post-synaptic cell produces



$$u_{1,t+1} = w_{11}u_{1,t} + w_{12}u_{2,t} + w_{13}u_{3,t}$$
$$u_{2,t+1} = w_{21}u_{1,t} + w_{22}u_{2,t} + w_{23}u_{3,t}$$
$$u_{3,t+1} = w_{31}u_{1,t} + w_{32}u_{2,t} + w_{33}u_{3,t}$$

$$\bar{u}_{t+1} = W\bar{u}_t$$
$$W = \begin{bmatrix} w_{11} & w_{12} & w_{13} \\ w_{21} & w_{22} & w_{23} \\ w_{31} & w_{32} & w_{33} \end{bmatrix}$$

Start with initial condition $\bar{u}_0$

$$\bar{u}_1 = W\bar{u}_0$$
$$\bar{u}_2 = W\bar{u}_1 = W(W\bar{u}_0) = W^2\bar{u}_0$$
$$\bar{u}_3 = W\bar{u}_2 = W(W^2\bar{u}_0) = W^3\bar{u}_0$$
$$\vdots$$
$$\boxed{\bar{u}_t = W^t\bar{u}_0}$$

$|\lambda_i| < 1$ for all λ of W

$|\lambda_i| > 1$ for all λ of W

$\bar{u}_t = W^t\bar{u}_0$

If $\bar{u}_0$ is an eigenvector:

$$\bar{u}_1 = W\bar{u}_0 = \lambda\bar{u}_0$$
$$\bar{u}_2 = W\bar{u}_1 = W(\lambda\bar{u}_0) = \lambda W\bar{u}_0 = \lambda^2\bar{u}_0$$
$$\vdots$$
$$\bar{u}_t = \lambda^t\bar{u}_0$$

If $|\lambda| < 1$, $|\lambda^t|$ will get smaller

If $|\lambda| > 1$, $|\lambda^t|$ will get larger

For given W:

$\lambda_1, \lambda_2, \lambda_3$ eigenvalues with corresponding eigenvectors $\underbrace{\bar{v}_1, \bar{v}_2, \bar{v}_3}_{\text{Form basis}}$

$\bar{u}_0 = a\bar{v}_1 + b\bar{v}_2 + c\bar{v}_3$ for any $\bar{u}_0$

$$\bar{u}_t = W^t\bar{u}_0$$
$$= W^t(a\bar{v}_1 + b\bar{v}_2 + c\bar{v}_3)$$
$$= a(W^t\bar{v}_1) + b(W^t\bar{v}_2) + c(W^t\bar{v}_3)$$
$$= a\lambda_1^t\bar{v}_1 + b\lambda_2^t\bar{v}_2 + c\lambda_3^t\bar{v}_3$$

$|\lambda_i| > 1$ for $i = 1,2,3$: system will diverge to ∞ for any $\bar{u}_0$

$|\lambda_i| < 1$ for $i = 1,2,3$: system will converge to 0 for any $\bar{u}_0$

$\lambda_i = 1$ for $i = 1,2,3$: system will stay at $\bar{u}_0$

Mix of λ_i (some > 1, some < 1): more dependent on $\bar{u}_0$ but will diverge to ∞ for most

Caveats:

- Discrete dynamical system
- Complex eigenvals exist ⇒ if W has complex eigenvals, performs rotation

$\lambda = 1$: sustained amplitude

$|\lambda| < 1$: shrinking amplitude

$|\lambda| > 1$: expanding amplitude