17/01/23

① A pair of dice is thrown. If sum of the nos is an odd no. then what is the prob. that sum is divisible by 3?

Sol^n :- Let $A \equiv$ sum is an odd no.

A has already occured.

Let $B \equiv$ sum is divisible by 3.

To find $p(B|A)$.

$$S = \{(1,1),(1,2),(1,3),(1,4),(1,5),(1,6),$$
$$(2,1),(2,2),(2,3),(2,4),(2,5),(2,6),$$
$$\vdots$$
$$(6,1),(6,2),(6,3),(6,4),(6,5),(6,6)\}$$

$$A = \{(1,2),(1,4),(1,6),(2,1),(2,3),(2,5),$$
$$(3,2),(3,4),(3,6),(4,1),(4,3),(4,5),$$
$$(5,2),(5,4),(5,6),(6,1),(6,3),(6,5)\}$$

$\Rightarrow n(A) = 18$

$$B = \{(1,2),(2,1),(3,6),(4,5),(5,4),(6,3)\}$$

$n(B) = 6$

$$P(B|A) = \frac{n(B)}{n(A)}$$
$$= \frac{6}{18} = \frac{1}{3}$$

② Find the probability that a single

② Find the probability that a single toss of a die will result in a number less than 4 if it is given that the toss resulted in an odd no.

Sol^n:- Let $A \equiv$ toss resulted in an odd no.

A has already occurred.

$B \equiv$ no. is less than 4.

$S = \{1, 2, 3, 4, 5, 6\}$

$A = \{1, 3, 5\} \Rightarrow n(A) = 3$

$B = \{1, 3\} \Rightarrow n(B) = 2$

$$P(B \backslash A) = \frac{n(B)}{n(A)}$$

$$= \frac{2}{3}$$

③ A die is thrown twice and the sum of the nos. appearing is observed to be 6. What is the prob. that the no. 4 has appeared at least once?

Sol^n:- Let $A \equiv$ Sum is 6.

A has already occurred.

$B \equiv$ 4 has appeared atleast once.

$B \equiv$ 4 has appeared atleast once.

To find $P(B \backslash A)$.

$S = \{(1,1), (1,2), (1,3), (1,4), (1,5), (1,6),$
$\vdots$
$(6,1), (6,2), (6,3), (6,4), (6,5), (6,6)\}$

$A = \{(1,5), (2,4), (3,3), (4,2), (5,1)\}$

$n(A) = 5$

$B = \{(2,4), (4,2)\}$

$n(B) = 2$

$$P(B \backslash A) = \frac{n(B)}{n(A)}$$
$$= \frac{2}{5}$$

(4) A pair of dice is thrown. If sum of the nos. is an even no., what is the probability that it is perfect square?

Sol^n:- $A \equiv$ Sum of the nos. is even
A has already occured.
$B \equiv$ No. is perfect square.

$A = \{(1,1), (1,3), (1,5), (2,2), (2,4), (2,6),$
$(3,1), (3,3), (3,5), (4,2), (4,4), (4,6),$
$(5,1), (5,3), (5,5), (6,2), (6,4), (6,6)\}$

$n(A) = 18$

$(5,1), (5,3), (5,5), (6,2), (6,4), (6,6)\}$

$n(A) = 18$

$B = \{(1,3), (2,2), (3,1)\}$

$\Rightarrow n(B) = 3$

$$P(B \backslash A) = \frac{n(B)}{n(A)}$$

$$= \frac{3}{18}$$

⑤ Weather records show that the prob. of high barometric pressure is 0.82 and the probability of rain and high barometric pressure is 0.20. Find the prob. of rain, given high barometric pressure?

Solⁿ:- $A \equiv$ Rain

$B \equiv$ High barometric pressure

$A \cap B \equiv$ Rain and high barometric pressure

To find $P(A \backslash B)$.

So $$P(A \backslash B) = \frac{P(A \cap B)}{P(B)}$$

$$= \frac{0.2}{0.82}$$

$$= 0.2446$$

0.82

$= 0.2446$

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(6) Suppose a box contains 3 red marbles and 2 black marbles. We Select 2 marbles. What is the probability that second marble is red given that the first marble is red?

Soln :- Let $A \equiv$ first marble is red.

A has already occured.

$B \equiv$ Second marble is also red.

To find $p(B \backslash A)$.

we have $p(B \backslash A) = \dfrac{p(A \cap B)}{p(A)}$

So $p(A) = \dfrac{{}^3C_1}{{}^5C_2}$ $\qquad \dfrac{n(A)}{n(S)}$

$= \dfrac{3}{5}$

So $A \cap B \equiv$ both the marbles are red.

$p(A \cap B) = \dfrac{{}^3C_2}{{}^5C_2}$ $\qquad \dfrac{n(A \cap B)}{n(S)}$

$= \dfrac{3}{10}$

So $p(B \backslash A) = \dfrac{p(A \cap B)}{p(A)}$

$$\text{So } P(B\backslash A) = \frac{}{P(A)}$$

$$= \frac{3/10}{3/5} = \frac{1}{2}$$

(7) A family has 2 children. Given that one of the children is a boy. What is the probability that the other child is also a boy?

Sol^n:- Let $A \equiv$ atleast one child is a boy.

A has already occured.

$B \equiv$ other childer is also a boy.

To find $P(B\backslash A)$.

We have $P(B\backslash A) = \dfrac{P(A\cap B)}{P(A)}$

$$S = \{BB, GG, BG, GB\}$$

$$P(A) = \frac{3}{4}$$

$A\cap B \equiv$ both the children are boys.

$$P(A\cap B) = \frac{1}{4}$$

$$\text{So } P(B\backslash A) = \frac{P(A\cap B)}{P(A)}$$

$1/$

$P(A)$

$$= \frac{1/4}{3/4}$$

$$= \frac{1}{3}$$

H.W. (8) From a bag containing 4 white and 6 black balls, two balls are drawn at random. If the balls are drawn one after another without replacement, find the probability that first ball is white and second ball is black.

(9) A bag contains 6 white and 9 black balls. Four balls are drawn at random at a time. Find the probability for the first draw to give four white and second ~~ball~~ draw to give four black balls if the balls are not replaced before the second draw.

Random Variables.

Def^n:- A random variable is a function X which takes any real value from a sample space S.

Sample space S.

i.e. $X : S \longrightarrow \mathbb{R}$

e.g. Let E be the experiment of tossing two coins.

Then $S = \{HH, TH, HT, TT\}$

Let us define a random variable X as $X \stackrel{def}{=}$ no. of heads.

$\Rightarrow X(HH) = 2, \quad X(TH) = 1,$

$X(HT) = 1, \quad X(TT) = 0$

There are two types of random variable (r.v.)

(1) Discrete random variable

Def^n :- A random variable X is said to be discrete if it takes either finite or countably infinite values.

(i.e. X will not take decimal values.)

e.g. ① No. of children in a family.

② No. of stars in the sky.

② No. of stars in the sky.

③ A page in a book can have at most 300 words.

$X \equiv$ No. of misprint on a page.

② Continous random variable:-

Def :- A random variable X is said to be continous if it takes any value in a given interval.

(i.e. It can even take a decimal value.)

e.g. ① Height of a person in cm.

② Weight of a bag in kg.

③ Temperature of a city in degree celsius.

23/01/23

Discrete probability distibution

Suppose X is discrete r.v.

and X takes values $x_1, x_2, \ldots, x_n$

Suppose $P(X=x_1)=p_1$

$P(X=x_2)=p_2$

$\vdots$

$$P(X = x_n) = P_n$$

If the following two conditions are Satisfied

(i) $P_1 \geqslant 0$, $P_2 \geqslant 0$, $\cdots$, $P_n \geqslant 0$

(ii) $P_1 + P_2 + \cdots + P_n = 1$

then P_i are called as probability function or probability mass function or probability density function. and (x_i, P_i) is called probability distribution of X.

① From a lot of 10 items containing 3 defectives. A sample of 4 items is drawn at random. Let the random variable X denote the no. of defective items in the Sample. Find the probability distribution of X.

Sol^n :- Let $X \equiv$ no. of defective items in a Sample of 4

There are total 10 items.
out of 10 3 defective and 7 are good.

good.

So X can take valus 0, 1, 2 or 3.

$$P(X=0) = P(\text{no defective item})$$
$$= P(\text{Good items})$$
$$= \frac{{}^{7}C_4}{{}^{10}C_4}$$
$$= \frac{1}{6}$$

$$P(X=1) = P(\text{No. of one defective item and 3 good items})$$
$$= P(1 \text{ defe.} + 3 \text{ good})$$
$$= \frac{{}^{3}C_1 \times {}^{7}C_3}{{}^{10}C_4}$$
$$= \frac{1}{2}$$

$$P(X=2) = P(2 \text{ defective} + 2 \text{ good})$$
$$= \frac{{}^{3}C_2 \times {}^{7}C_2}{\quad}$$
$$= \frac{3}{10}$$

$$P(X=3) = P(3 \text{ defective} + 1 \text{ good})$$

$$P(X=3) = P(3 \text{ defective} + 1 \text{ good})$$

$$= \frac{{}^3C_3 \times {}^7C_1}{{}^{10}C_4}$$

$$= \frac{1}{30}$$

$X=x_i$	0	1	2	3
$P(X=x_i)$	$1/6$	$1/2$	$3/10$	$1/30$

(2) A man draws 2 balls from a bag containing 3 white and 5 black balls. If he is to receive ₹14 for every white ball which is drawn and ₹7 for every black balls. Write the probability distribution

Solⁿ :- Let $X \equiv$ money receive by man.

So X can take values 28, 14, 21.

$P(X=28) = P$(both the balls are white).

$$= \frac{{}^3C_2}{{}^8C_2}$$

$$= \frac{3}{28}$$

$$= \frac{3}{28}$$

$P(X=14) = P(\text{Both the balls are black})$

$$= \frac{{}^5C_2}{{}^8C_2}$$

$$= \frac{5}{14}$$

$P(X=21) = P(\text{one black + one white})$

$$= \frac{{}^5C_1 \times {}^3C_1}{{}^8C_2}$$

$$= \frac{15}{28}$$

probability distribution is

X	28	14	21
P	3/28	5/14	15/28

(3) If X denote the largest of the two numbers that appear when a pair of dice is thrown, find probability distribution of X.

Soln:- Let $X \equiv$ the largest no. appears on a pair dice.

X can take values 1, 2, 3, 4, 5 or 6

X take values 1, 2, 3, 4, 5 or 6

$$S=\{(1,1),(1,2),(1,3),(1,4),(1,5),(1,6),$$
$$(2,1),(2,2),(2,3),(2,4),(2,5),(2,6),$$
$$(3,1),(3,2),(3,3),(3,4),(3,5),(3,6),$$
$$(4,1),(4,2),(4,3),(4,4),(4,5),(4,6),$$
$$(5,1),(5,2),(5,3),(5,4),(5,5),(5,6),$$
$$(6,1),(6,2),(6,3),(6,4),(6,5),(6,6)\}$$

$\Rightarrow n(S) = 36$

$P(X=1) = p(\text{largest of two nos is } 1)$

$$= \frac{1}{36}$$

$$= \frac{}{}$$

$$= \frac{1}{12}$$

$P(X=3) = P(\text{largest of two nos is } 3)$

$$= \frac{5}{36}$$

$P(X=4) = 7$

$$P(X=4) = \frac{7}{36}$$

$$P(X=5) = \frac{9}{36} = \frac{1}{4}$$

$$P(X=6) = \frac{11}{36}$$

X	1	2	3	4	5	6
P	$1/36$	$1/12$	$5/36$	$7/36$	$1/4$	$11/36$

(4) If 3 cars are selected from a lot of 6 cars containing 2 defective cars, find the probability distribution of number of defective cars.

Sol^n :- Let X be the r.v. which gives no. of defective cars out of 3 cars.

X can take values 0, 1, 2.

$$P(X=0) = P(\text{no defective car}).$$

$$= P(\text{all non-defective cars})$$

$$= \frac{{}^{4}C_3}{{}^{6}C_3}$$

$$= \frac{1}{5}$$

P(X=1) = P(...)

$$P(X=1) = P(\text{one defective})$$
$$= P(1 \text{ defe.} + 2 \text{ non def.})$$
$$= \frac{{}^2C_1 \times {}^4C_2}{{}^6C_3}$$
$$= \frac{3}{5}$$

$$P(X=2) = P(\text{two cars are def.})$$
$$= P(2 \text{ def.} + 1 \text{ non def}).$$
$$= \frac{{}^2C_2 \times {}^4C_1}{{}^6C_3}$$
$$= \frac{1}{5}$$

prob. distribution of X is

X	0	1	2
P	1/5	3/5	1/5

(5) The prob. distribution of a r.v. X is

X	0	1	2	3	4	5	6
P	K	3K	5K	7K	9K	11K	13K

Find the value of K and $P(X<4)$.

Sol^n :- Since p is a p.d.f (p.m.f)

$\therefore p_1 + p_2 + p_3 + p_4 + p_5 + p_6 = 1$

$\Rightarrow K + 3K + 5K + 7K + 9K + 11K + 13K = 1$

$\Rightarrow 49K = 1$

0	1	2	3	4	5	6
$\frac{1}{49}$	$\frac{3}{49}$	$\frac{5}{49}$	$\frac{7}{49}$	$\frac{9}{49}$	$\frac{11}{49}$	$\frac{13}{49}$

Now $P(X<4) = P(X=0) + P(X=1) + P(X=2) + P(X=3)$

$$= \frac{1}{49} + \frac{3}{49} + \frac{5}{49} + \frac{7}{49}$$

$$= \frac{16}{49}$$

⑥ A random variable X has the following prob. distributi.

X	0	1	2	3	4	5	6	7
P	0	K	2K	2K	3K	K^2	$2K^2$	$7K^2+K$

Find K and smallest value of λ for which $P(X \le \lambda) > 1/2$

Sol^n :- $0 + K + 2K + 2K + 3K + K^2 + 2K^2 + 7K^2 + K = 1$

$\Rightarrow 9K + 10K^2 = 1$

$\Rightarrow 10K^2 + 9K - 1 = 0$

$\Rightarrow K = \frac{1}{10}, K = -1$

But $K \neq -1$

$\therefore K = 0.1$

X	0	1	2	3	4	5	6	7
P	0	0.1	0.2	0.2	0.3	0.01	0.02	0.17

$0 + 0.1 = 0.1$

$0 + 0.1 + 0.2 = 0.3$

$0 + 0.1 + 0.2 + 0.2 = 0.5$

$0 + 0.1 + 0.2 + 0.2 + 0.3 = 0.8 > \frac{1}{2}$

$\Rightarrow$ Smallset value of x is 4.

H.W. (7) The p.m.f (p.d.f.) of a r.v. X is zero except at $X = 0, 1, 2$ and at these points it has values $P(0) = 3c^3$, $P(1) = 4c - 10c^2$, $P(2) = 5c - 1$
Determine c, $P(X < 1)$, $P(1 < X \leq 2)$, $P(0 < X \leq 2)$.

24/01/23

Expectation, Variance and S.D.

(1) From an urn containing 3 red balls

and 2 white balls. A man is to draw 2 balls at random. He gets ₹ 20 for each red ball and ₹ 10 for each white ball he draws. Find his expectation (mean).

Solⁿ :- Mean $= \mu = \sum x_i p_i$

Let $X \stackrel{def}{=}$ money the man earns after drawing the balls.

$\therefore$ X can take values 40, 20, 30

$$p(X=40) = p(\text{both red balls})$$

$$= \frac{^3C_2}{^5C_2}$$

$$P_1 = \frac{3}{10}$$

$$p(X=20) = p(\text{both white balls})$$

$$= \frac{^2C_2}{^5C_2}$$

$$P_2 = \frac{1}{10}$$

$$p(X=30) = p(\text{one white and one red ball})$$

$$= \frac{^2C_1 \times {^3C_1}}{^5C_2}$$

$$P_3 = \frac{3}{5}$$

$\therefore$ Mean $= \mu = E(X) = x_1 p_1 + x_2 p_2 + x_3 p_3$

$$= 40 \times \frac{3}{10} + 20 \times \frac{1}{10} + 30 \times \frac{3}{5}$$

$$= 32$$

(2) A random variable X has the following probability function.

X	0	1	2	3	4	5
P	K	2K	3K	4K	5K	6K

Find the value of K and mean variance and S.D. of X.

Sol^n :— Since p is the p.d.f. of X

$\therefore$ $P_1 + P_2 + P_3 + P_4 + P_5 + P_6 = 1$

$\Rightarrow K + 2K + 3K + 4K + 5K + 6K = 1$

$\Rightarrow 21K = 1$

$\Rightarrow K = \frac{1}{21}$

X	0	1	2	3	4	5
P	$\frac{1}{21}$	$\frac{2}{21}$	$\frac{3}{21}$	$\frac{4}{21}$	$\frac{5}{21}$	$\frac{6}{21}$

Mean $= \mu = E(X) = \sum x_i p_i$

$\therefore$ Mean $= x_1 p_1 + x_2 p_2 + x_3 p_3 + x_4 p_4 + x_5 p_5$

$+ x_6 p_6$

$$\text{Mean} = (0)\left(\frac{1}{21}\right) + (1)\left(\frac{2}{21}\right) + (2)\left(\frac{3}{21}\right) + (3)\left(\frac{4}{21}\right) + (4)\left(\frac{5}{21}\right) + (5)\left(\frac{6}{21}\right)$$

$$\mu = \frac{10}{3}$$

$$\text{Variance} = \sum x_i^2 p_i - \left(\sum x_i p_i\right)^2$$

$$= \sum x_i^2 p_i - \mu^2$$

$$\text{So } \sum x_i^2 p_i = x_1^2 p_1 + x_2^2 p_2 + x_3^2 p_3 + x_4^2 p_4 + x_5^2 p_5 + x_6^2 p_6$$

$$= (0)^2\left(\frac{1}{21}\right) + (1)^2\left(\frac{2}{21}\right) + (2)^2\left(\frac{3}{21}\right) + (3)^2\left(\frac{4}{21}\right) + (4)^2\left(\frac{5}{21}\right) + (5)^2\left(\frac{6}{21}\right)$$

$$= \frac{40}{3}$$

$$\text{Variance} = \sum x_i^2 p_i - (\mu)^2$$

$$= \frac{40}{3} - \left(\frac{10}{3}\right)^2$$

$$= \frac{40 \times 3}{9} - \frac{100}{9}$$

$$\text{Variance} = \frac{20}{9}$$

$$S.D. = \sqrt{\text{Variance}}$$

$$= \sqrt{\frac{20}{9}} = \sqrt{\frac{4 \times 5}{9}} = \frac{2\sqrt{5}}{3}$$

③ Three bags contain respectively
3 green and 2 white balls.
5 green and 6 white balls.
2 green and 4 white balls.
one ball is drawn from each bag. Find the expected value of white ball drawn.

Sol^n:- Let $X \stackrel{def}{=}$ no. of white ball.

Then X can take value 0 or 1.

We calculate expected value for each bage.

For First bag.

$$P(X=0) = P(\text{no white ball})$$
$$= P(\text{green ball})$$
$$= \frac{{}^3C_1}{{}^5C_1}$$
$$= \frac{3}{5}$$

$$P(X=1) = P(\text{white ball})$$

$$= \frac{{}^2C_1}{{}^5C_1}$$

$$= \frac{2}{5}$$

Expected value $= x_1 p_1 + x p_2$

$$= (0)\left(\frac{3}{5}\right) + (1)\left(\frac{2}{5}\right)$$

$$= \frac{2}{5}$$

For Second bag

$$P(X=0) = P(\text{no white ball})$$

$$= P(\text{green ball})$$

$$= \frac{{}^5C_1}{{}^{11}C_1} = \frac{5}{11}$$

$$P(X=1) = P(\text{one white ball})$$

$$= \frac{{}^6C_1}{{}^{11}C_1}$$

$$= \frac{6}{11}$$

Expected value $= x_1 p_1 + x_2 p_2$

$$= (0)\left(\frac{5}{11}\right) + (1)\left(\frac{6}{11}\right)$$

$$= \frac{6}{11}$$

For third bag.

for third bag.

$$P(X=0) = \frac{1}{3}$$

$$P(X=1) = \frac{2}{3}$$

Expected value $= x_1p_1 + x_2p_2$

$= (0)\left(\frac{1}{3}\right) + (1)\left(\frac{2}{3}\right)$

$= 2/3$

$\therefore$ Expected value $= \frac{2}{5} + \frac{6}{11} + \frac{2}{3}$

$= \frac{266}{165} = 1.61$

30/01/23

① The probability distribution of a r.v. X is

X	-2	3	1
P(X)	$1/3$	$1/2$	$1/6$

Find (i) m.g.f about origin.
(ii) first four moment about origin.

Sol^n:- (i) m.g.f about origin is

Sol^n :- (i) m.g.f about origin is given by

$$M_0(t) = \sum p_i e^{tx_i}$$

$$= p_1 e^{tx_1} + p_2 e^{tx_2} + p_3 e^{tx_3}$$

$$M_0(t) = \frac{1}{3} e^{-2t} + \frac{1}{2} e^{3t} + \frac{1}{6} e^{t}$$

(ii) we use following formula to calculate moments.

$$\mu_r' = \left[\frac{d^r}{dt^r} M_a(t)\right]_{t=0}$$

$$\Rightarrow \mu_r' = \left[\frac{d^r}{dt^r} M_0(t)\right]_{t=0}$$

we have to calculate $\mu_1', \mu_2', \mu_3', \mu_4'$

$$\mu_1' = \left(\frac{d}{dt} M_0(t)\right]_{t=0} \qquad \frac{1}{3} e^{-2t} + \frac{1}{2} e^{3t} + \frac{1}{6} e^{t}$$

$$= \left(\frac{1}{3}(-2) e^{-2t} + \frac{1}{2}(3) e^{3t} + \frac{1}{6} e^{t}\right]_{t=0}$$

$$= -\frac{2}{3} + \frac{3}{2} + \frac{1}{6}$$

$$\mu_1' = 1$$

$$\mu_2' = \left[\frac{d^2}{dt^2} M_0(t)\right]_{t=0}$$

$$= \left[\frac{1}{3}(-2)(-2)e^{-2t} + \frac{1}{2}(3)(3)e^{3t} + \frac{1}{6}e^{t}\right]_{t=0}$$

$$\mu_2' = \frac{4}{3} + \frac{9}{2} + \frac{1}{6}$$

$$= \frac{8+27+1}{6}$$

$$= \frac{36}{6}$$

$$\mu_2' = 6$$

$$\mu_3' = \left[\frac{d^3}{dt^3} M_0(t)\right]_{t=0}$$

$$= \left[\frac{1}{3}(-2)(-2)(-2)e^{-2t} + \frac{1}{2}(3)(3)(3)e^{3t} + \frac{1}{6}e^{t}\right]_{t=0}$$

$$= -\frac{8}{3} + \frac{27}{2} + \frac{1}{6}$$

$$\mu_3 = 11$$

$$\mu_4' = \left[\frac{d^4}{dt^4} M_o(t)\right]_{t=0}$$

$$= \left[\frac{1}{3}(-2)(-2)(-2)(-2)e^{-2t} + \frac{1}{2}(3)(3)(3)(3)e^{3t} + \frac{1}{6}e^{t}\right]_{t=0}$$

$$= \frac{16}{3} + \frac{81}{2} + \frac{1}{6}$$

$$\mu_4' = 46$$

(2) If X denotes the outcome when a fair die is thrown, find m.g.f of X about origin and first four moment about origin Also find mean and variance of X.

Sol^n :-

X	1	2	3	4	5	6
$p(x)$	1/6	1/6	1/6	1/6	1/6	1/6

(i) m.g.f. about origin is given by

$$M_o(t) = \sum p_i e^{tx_i}$$

$$= p_1 e^{tx_1} + p_2 e^{tx_2} + p_3 e^{tx_3} + p_4 e^{tx_4}$$

$$= P_1 e^{tx_1} + P_2 e^{tx_2} + P_3 e^{tx_3} + P_4 e^{tx_4} + P_5 e^{tx_5} + P_6 e^{tx_6}$$

$$M_0(t) = \frac{1}{6}\left[e^t + e^{2t} + e^{3t} + e^{4t} + e^{5t} + e^{6t}\right]$$

(ii) $\mu_r' = \left[\frac{d^r}{dt^r} M_0(t)\right]_{t=0}$

so $\mu_1' = \left(\frac{d}{dt} M_0(t)\right)_{t=0}$

$$= \left[\frac{1}{6}\left(e^t + 2e^{2t} + 3e^{3t} + 4e^{4t} + 5e^{5t} + 6e^{6t}\right)\right]_{t=0}$$

$$\mu_1' = \left[\frac{1}{6}(1 + 2 + 3 + 4 + 5 + 6)\right]$$

$$= \frac{21}{6}$$

$$\mu_1' = \frac{7}{2}$$

$$\mu_2' = \left[\frac{d^2}{dt^2} M_0(t)\right]_{t=0}$$

$$= \left[\frac{1}{6}\left(e^t + 2\cdot 2e^{2t} + 3\cdot 3e^{3t} + 4\cdot 4e^{4t} + 5\cdot 5e^{5t} + 6\cdot 6e^{6t}\right)\right]_{t=0}$$

$$\mu_2' = \frac{1}{6}(1+4+9+16+25+36)$$

$$\mu_2' = \frac{91}{6}$$

Similarly we can calculate μ_3' and μ_4'

$$\text{Mean} = \mu_1' = \frac{7}{2}$$

$$\text{Variance} = \mu_2' - (\mu_1')^2$$

$$= \frac{91}{6} - \left(\frac{7}{2}\right)^2$$

$$= \frac{91}{6} - \frac{49}{4}$$

$$\text{Variance} = \frac{35}{12}$$

Chapter 02

31/01/2023

Binomial Distribution.

1. If X follows binomial distribution with $E(X) = 2$ and $V(X) = \frac{4}{3}$. find the prob. distribution of X.

Sol^n :- Since X follows binomial

Sol^n :- Since X follows binomial distribution

$$\therefore \boxed{P(X=x) = {}^nC_x\, p^x q^{n-x}}$$

Given $E(X) = 2$

ie. mean $= 2$

$$\Rightarrow np = 2$$

Also $Var(X) = \frac{4}{3}$

$$\Rightarrow npq = \frac{4}{3}$$

$$\Rightarrow \cancel{2}q = \frac{\cancel{4}^{\,2}}{3}$$

$$\Rightarrow \boxed{q = \frac{2}{3}}$$

Also $p + q = 1$

$$\Rightarrow p + \frac{2}{3} = 1$$

$$\Rightarrow p = 1 - \frac{2}{3}$$

$$\Rightarrow \boxed{p = \frac{1}{3}}$$

Now $np = 2$

$$\Rightarrow n \times \frac{1}{3} = 2$$

$$\Rightarrow n = 2 \times 3$$

$$\Rightarrow \boxed{n = 6}$$

$$\therefore P(X=x) = {}^6C_x \left(\frac{1}{3}\right)^x \left(\frac{2}{3}\right)^{6-x}$$

where $x = 0, 1, 2, 3, 4, 5, 6$

$$P(X=0) = {}^6C_0 \left(\frac{1}{3}\right)^0 \left(\frac{2}{3}\right)^{6-0}$$

$$P(X=0) = {}^6C_0 \left(\frac{1}{3}\right)^0 \left(\frac{2}{3}\right)^6$$

$$= \frac{64}{729}$$

$$P(X=1) = {}^6C_1 \left(\frac{1}{3}\right)^1 \left(\frac{2}{3}\right)^{6-1}$$

$$= {}^6C_1 \left(\frac{1}{3}\right)^1 \left(\frac{2}{3}\right)^5$$

$$P(X=1) = \frac{64}{243}$$

$$P(X=2) = {}^6C_2 \left(\frac{1}{3}\right)^2 \left(\frac{2}{3}\right)^{6-2}$$

$$= {}^6C_2 \left(\frac{1}{3}\right)^2 \left(\frac{2}{3}\right)^4$$

$$P(X=2) = \frac{80}{243}$$

$$P(X=3) = {}^6C_3 \left(\frac{1}{3}\right)^3 \left(\frac{2}{3}\right)^{6-3}$$

$$= {}^6C_3 \left(\frac{1}{3}\right)^3 \left(\frac{2}{3}\right)^3$$

$$= {}^6C_3 \left(\frac{1}{3}\right)^3 \left(\frac{2}{3}\right)^3$$

$$P(X=3) = \frac{160}{729}$$

$$P(X=4) = {}^6C_4 \left(\frac{1}{3}\right)^4 \left(\frac{2}{3}\right)^{6-4}$$

$$= \frac{20}{243}$$

$$P(X=5) = {}^6C_5 \left(\frac{1}{3}\right)^5 \left(\frac{2}{3}\right)^{6-5}$$

$$= {}^6C_5 \left(\frac{1}{3}\right)^5 \left(\frac{2}{3}\right)^1$$

$$= \frac{4}{243}$$

$$P(X=6) = {}^6C_6 \left(\frac{1}{3}\right)^6 \left(\frac{2}{3}\right)^{6-6}$$

$$= {}^6C_6 \left(\frac{1}{3}\right)^6 \left(\frac{2}{3}\right)^0$$

$$P(X=6) = \frac{1}{729}$$

X	0	1	2	3	4	5	6
$P(X)$	$\frac{64}{729}$	$\frac{64}{243}$	$\frac{80}{243}$	$\frac{160}{729}$	$\frac{20}{243}$	$\frac{4}{243}$	$\frac{1}{729}$

② If 10% of the bolts produced by a machine are defective, determine the probability that out of 10 bolts choosen at random

(i) 1 is defective.

(ii) None is defective.

(iii) At most two bolts are defective.

Soln:- Let $p = 10\% = \frac{10}{100}$

$\Rightarrow p = 0.1$

$\therefore q = 1 - p$

$= 1 - 0.1$

$\Rightarrow q = 0.9$

and $n = 10$

$$P(X=x) = {}^{n}C_{x}\, p^{x}\, q^{n-x}$$

$$\Rightarrow P(X=x) = {}^{10}C_{x}\,(0.1)^{x}\,(0.9)^{10-x}$$

(i) prob. of one defective bolt

$$P(X=1) = {}^{10}C_{1}\,(0.1)^{1}\,(0.9)^{10-1}$$

$$= {}^{10}C_{1}\,(0.1)^{1}\,(0.9)^{9}$$

$$= 0.3874$$

(ii) prob. of zero defective bolt.

$$P(X=0) = {}^{10}C_0 (0.1)^0 (0.9)^{10-0}$$
$$= {}^{10}C_0 (0.1)^0 (0.9)^{10}$$
$$= 0.3487$$

(iii) prob. of atmost 2 defective bolts.

$$P(X \le 2) = P(X=0) + P(X=1) + P(X=2)$$

we have already calculated $P(X=0)$ and $P(X=1)$.

$\therefore$ we will calculate $P(X=2)$.

$$P(X=2) = {}^{10}C_2 (0.1)^2 (0.9)^{10-2}$$
$$= {}^{10}C_2 (0.1)^2 (0.9)^8$$
$$= 0.1937$$

$$\therefore P(X \le 2) = 0.3487 + 0.3874 + 0.1937$$
$$= 0.9298$$

② A pair of dice thrown 3 times. If getting a doublet is considered a success, find the probability of two success.

Sol^n :- Let $X \equiv$ no. of success.

Let p be the prob. of getting doublet.

$\therefore \; p = \frac{6}{36} = \frac{1}{6}$

$\therefore \; q = 1 - p$

$= 1 - \frac{1}{6}$

$q = \frac{5}{6}$

$\therefore \; P(X=x) = {}^nC_x \, p^x \, q^{n-x}$

$P(X=x) = {}^3C_x \left(\frac{1}{6}\right)^x \left(\frac{5}{6}\right)^{3-x}$

$\Rightarrow P(X=2) = {}^3C_2 \left(\frac{1}{6}\right)^2 \left(\frac{5}{6}\right)^{3-2}$

$= {}^3C_2 \left(\frac{1}{6}\right)^2 \left(\frac{5}{6}\right)^1$

$= \frac{5}{72}$