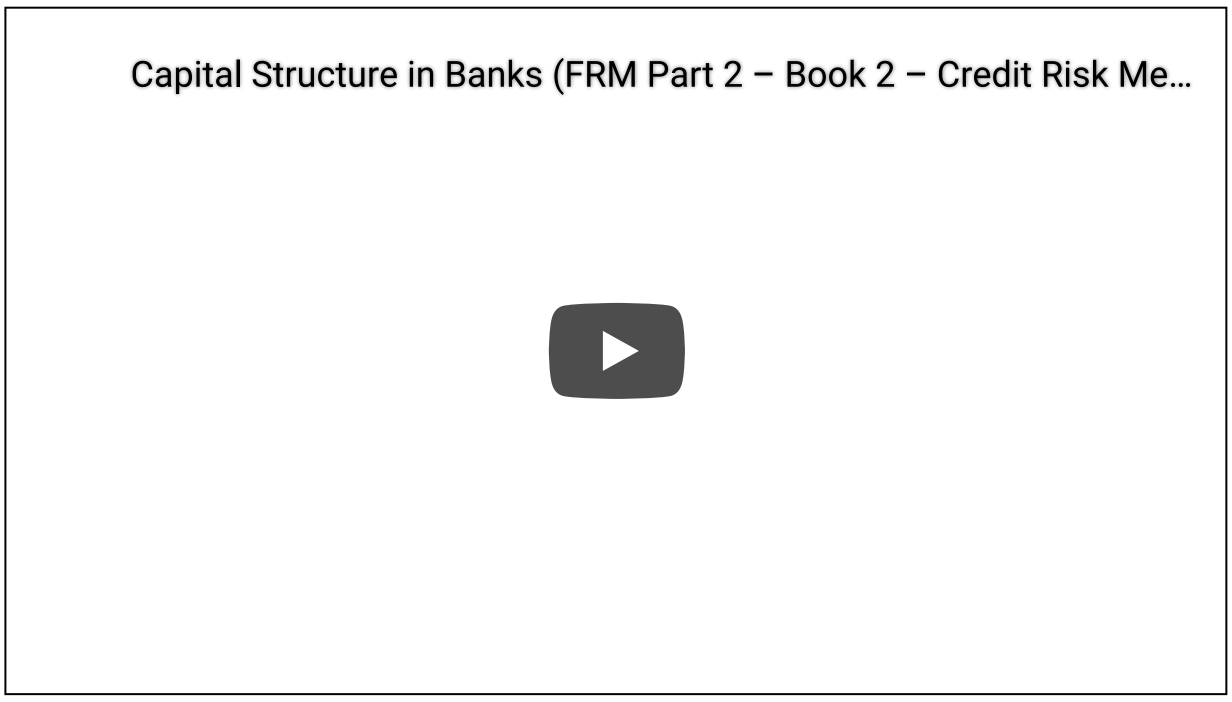
December 31, 2019 in **Credit Risk Measurement and Management**

Capital Structure in Banks



After completing this reading, you should be able to:

- Evaluate a bank’s economic capital relative to its level of credit risk.
- Identify and describe important factors used to calculate economic capital for credit risk: the probability of default, exposure, and loss rate.
- Define and calculate the expected loss (EL).
- Define and calculate unexpected loss (UL).
- Estimate the variance of default probability assuming a binomial distribution.
- Calculate UL for a portfolio and the risk contribution of each asset.
- Describe how economic capital is derived.
- Explain how the credit loss distribution is modeled.
- Describe challenges to quantifying credit risk.

Evaluating a Bank’s Economic Capital Relative to its Level of Credit Risk

The term credit risk describes the risk that arises from nonpayment or rescheduling of any promised payment. It can also arise from credit migration – events related to changes in the credit quality of the borrower. These events have the potential to cause economic loss to the bank.

The expected loss is the amount a bank can expect to lose, on average, over a predetermined period when extending credits to its customers. Unexpected loss is the volatility of credit losses around its expected loss.

Once a bank determines its expected loss, it sets aside credit reserves in preparation. However, for unexpected loss, the bank must estimate the excess capital reserves needed subject to a predetermined confidence level. This excess capital needed to match the bank's estimate of unexpected loss is known as economic capital.

To safeguard its long-term financial health, a lender must match its capital reserves with the amount of credit risk borne. Economic capital is largely determined by (I) level of confidence, and (II) level of risk. An increase in any of these two parameters causes the economic capital also to increase.

Important Factors Used to Calculate Economic Capital for Credit Risk

The Probability of Default

The probability of default (PD), describes the probability that a borrower will default on contractual payments before the end of a predetermined period. This probability is in and of itself not the greatest concern to a lender because a borrower may default but then bounce back and settle the missed payments soon afterward, including any imposed penalties. It's expressed as a percentage.

Exposure Amount

Exposure amount (EA), also known as exposure at default (EAD), is the loss exposure of a bank at the time of a loan's default, expressed as a dollar amount. It's the predicted amount of loss in the event the borrower defaults.

EAD is a dynamic amount that keeps on changing as the borrower continues to make payments.

Loss Rate

The loss rate, also known as the loss given default (LGD), is the percentage loss incurred if the borrower defaults. It can also be described as the expected loss expressed as a percentage. The loss rate is the amount that's impossible to recover after selling (salvaging) the underlying asset following a default event.

The LGD can also be expressed as:

$$LGD = 1 - Recovery \quad Rate$$

Calculation of the Expected Loss

The expected loss, EL, is the average credit loss that we would expect from an exposure or a portfolio over a given period. It's the anticipated deterioration in the value of a risky asset. In mathematical terms,

$$EL = EA \times PD \times LGD$$

Credit loss levels are not constant but rather fluctuate from year to year. The expected loss represents the anticipated average loss that can be statistically determined. The business will normally have a budget for the EL and try to bear the losses as part of the normal operating cash flows.

Exam tip: The expected loss of a portfolio is equal to the summation of expected losses of individual losses.

$$EL_P = \sum EA_i \times PD_i \times LGD_i$$

Unexpected Loss

Unexpected loss is the average total loss over and above the expected loss. It's the variation in the expected loss. It is calculated as the standard deviation from the mean at a certain confidence level.

Let UL_H denote the unexpected loss at the horizon for asset value V_H. Then,

$$UL_H \equiv \sqrt{var\left(V_H\right)}$$

You will usually apply the following formula to determine the value of the unexpected loss:

$$UL = EA \times \sqrt{PD \times \sigma_{LR}^2 + LR^2 \times \sigma_{PD}^2}$$

Where

$$\sigma_{PD}^2 = PD \times (1 - PD)$$

since default is a Bernoulli variable with a binomial distribution.

Example: Expected loss and unexpected loss

A Canadian bank recently disbursed a CAD 2 million loan of which CAD 1.6 million is currently outstanding. According to the bank's internal rating model, the beneficiary has a 1% chance of defaulting over the next year. In case that happens, the estimated loss rate is 30%. The probability of default and the loss rate have standard deviations of 6% and 20%, respectively.

Determine the expected and unexpected loss figures for the bank.

Solution

$$EL = EA \times PD \times LR$$

$EA = CAD 1,600,000$

$PD = 1\%$

$LR = 30\%$

Thus,

$$\begin{aligned} EL &= 1,600,000 \times 0.01 \times 0.3 = \text{CAD 4,800} \\ UL &= EA \times \sqrt{PD \times \sigma_{LR}^2 + LR^2 \times \sigma_{PD}^2} \\ &= 1,600,000 \times \sqrt{0.01 \times 0.2^2 + 0.3^2 \times 0.06^2} \\ &= \text{CAD 43,052} \end{aligned}$$

Calculating the Unexpected Loss of A Portfolio

Unlike expected loss, we do not compute the unexpected loss of a portfolio by summing up the unexpected loss of individual assets. And this is because the standard deviation of the sum will not be the same as the sum of standard deviation unless there is a perfect correlation.

Given a portfolio with N assets, the unexpected loss is given by:

$$UL_P = \left[\sum_i^n \sum_j^n \rho_{ij} UL_i UL_j\right]$$

For a two-asset portfolio:

$$UL_P = \sqrt{UL_i^2 + UL_j^2 + 2\rho UL_i UL_j}$$

Where:

$$UL_x = EA_x \times \sqrt{PD_x \times \sigma_{LR_x}^2 + (LR)_x^2 \times \sigma_{PD_x}^2} \qquad x = i \quad or \quad j$$

And ρ_{ij} is the correlation of default between asset i and asset j.

Thus, if $\rho_{ij} = 1$ for $i \neq j$, that's the only instance when the unexpected loss of the portfolio will be equal to the sum of the unexpected individual losses.

Due to the effects of diversification (elimination of specific risks), the risk of a portfolio is always less than the total risk of assets held separately. It follows that the unexpected loss for a portfolio is much less than the sum of unexpected individual losses.

Risk Contribution

The risk contribution of a risky asset *i* to the unexpected portfolio loss, also called the unexpected loss contribution ULC_i, is defined to be the incremental risk that the exposure of a single asset contributes to the portfolio's total risk.

The risk of a given portfolio is considerably less than the sum of the individual risk levels because each asset contributes only a portion of its unexpected loss in the portfolio. This effect is captured by the partial derivative of UL_p (portfolio unexpected loss) with respect to UL_i (unexpected loss from asset i), i.e.,

$$ULC_i = UL_i \times \frac{\delta UL_P}{\delta UL_i}$$

After differentiation, and, assuming that the portfolio consists of *n* loans, it can be shown that the risk contribution of a particular asset is given by:

$$ULC_i = \frac{UL_i \sum_{j=i}^n UL_j \rho_{ij}}{UL_P}$$

Where:

ULC_i = unexpected loss contribution of asset i

UL_i = unexpected loss from asset i

UL_j = unexpected loss from asset j

ρ_{ij} = correlation between asset i and j where i $\neq$ j

UL_P = unexpected loss from the portfolio

For a two-asset portfolio, we can calculate the risk contribution of each asset as follows:

$$RC_1 = UL_1 \times \frac{UL_1 + (\rho_{12} \times UL_2)}{UL_P}$$
$$RC_2 = UL_2 \times \frac{UL_2 + (\rho_{12} \times UL_1)}{UL_P}$$

Where

$$RC_1 + RC_2 = UL_P$$

The risk contribution is a measure of the systematic risk of an asset in the portfolio – the amount of credit risk which cannot be eliminated by placing the asset in the portfolio.

Assuming that the portfolio consists of *n* loans that have approximately the same characteristics and size (1/n), we can set ρ_{ij} = ρ = constant (for all i $\neq$ j). In this case, the unexpected loss contribution of asset *i* can be given by:

$$ULC_i = UL_i \times \sqrt{\rho}$$

Where

$$\sqrt{\rho} = \frac{\sum_{j=i}^{n} UL_j \rho_{ij}}{UL_P}$$

Example: unexpected loss contribution

Prime Bank has two outstanding loans with a correlation of 0.4. Other characteristics are as shown below:

	Asset X	**Asset Y**
EA	$\$30,000,000$	$\$12,000,000$
PD	0.5%	1.0%
LR	40%	30%
σ_{PD}	3%	4%
σ_{LR}	20%	30%

Compute EL_P, UL_P, and the risk contribution of each asset.

Solution

Step 1: Computing the EL for each asset

$$\begin{aligned} EL_X &= EA \times PD \times LR \\ &= \$30,000,000 \times 0.005 \times 0.4 \\ &= \$60,000 \end{aligned}$$

$$\begin{aligned} EL_Y &= EA \times PD \times LR \\ &= \$12,000,000 \times 0.01 \times 0.3 \\ &= \$36,000 \end{aligned}$$

Step 2: Computing unexpected loss, UL, for each asset

$$UL = EA \times \sqrt{PD \times \sigma_{LR}^2 + LR^2 \times \sigma_{PD}^2}$$

$$\begin{aligned} UL_X &= 30,000\sqrt{0.005 \times 0.2^2 + 0.4^2 \times 0.03^2} \\ &= 556,417 \end{aligned}$$

$$\begin{aligned} UL_Y &= 12,000\sqrt{0.01 \times 0.3^2 + 0.3^2 \times 0.04^2} \\ &= 387,732 \end{aligned}$$

Step 3: Computing portfolio expected loss, EL_P

$$\begin{aligned} EL_P &= \$60,000 + \$36,000 \\ &= 96,000 \end{aligned}$$

Step 4: Computing portfolio unexpected loss, UL_P

$$\begin{aligned} UL_P &= \sqrt{UL_i^2 + UL_j^2 + 2\rho UL_i UL_j} \\ &= \sqrt{556,417^2 + 387,732^2 + 2 \times 0.4 \times 556,417 \times 387,732} \\ &= 795,317 \end{aligned}$$

Note: This actually proves that $UL_P < UL_X + UL_Y$ thanks to diversification (\$795,317 < \$556,417 + \$387,732)

Step 5: Computing the risk contribution, ULC, for each asset:

$$\begin{aligned} ULC_X &= UL_X \times \frac{UL_X + (\rho_{XY} \times UL_Y)}{UL_P} \\ &= 556,417 \times \frac{556,417 + (0.4 \times 387,732)}{795,317} \\ &= 497,784 \end{aligned}$$

Note: $ULC_X < UL_X$

$$\begin{aligned} ULC_Y &= UL_Y \times \frac{UL_Y + (\rho_{XY} \times UL_X)}{UL_P} \\ &= 387,732 \times \frac{387,732 + (0.4 \times 556,417)}{795,317} \\ &= 297,532 \end{aligned}$$

Again, note that $ULC_Y < UL_Y$

We can also compute ULC_Y as \$795,317 – \$497,784 = \$297,533 since $ULC_P = ULC_X + ULC_Y$

How Economic Capital is Derived

As we saw earlier, economic capital, EC, is the amount of capital needed to match a bank's unexpected losses. The amount of EC needed is the distance between the unexpected outcome and the expected outcome, for a given level of confidence.

Let X_T be the random variable for loss and z be the percentage probability (confidence level). If we go further and let v be the minimum economic capital needed to keep the bank solvent at the time horizon t, then:

$$Pr\left[X_T \le v\right] = z$$

z can be interpreted as the desired rating for the bank, say, 99.97% for an AA rating.

Given the desired level of z, we want to determine the amount of EC such that:

$$Pr\left[X_t - EL_P \le EC\right] = z$$

Define the capital multiplier, CM, by:

$$EC = CM \times UL_P \qquad \ldots\ldots\ldots\ldots\ldots\ldots\ldots (Very \quad important)$$

Then,

$$Pr\left[\frac{X_T - EL_P}{UL_P} \leq CM\right] = z$$

How the Credit Loss Distribution is Modeled

When modeling credit risk, analysts are interested in the left tail of the chosen loss distribution. And that's because the focus is on credit loss. The normal distribution is not appropriate for this purpose for two main reasons:

I. Credit losses tend to be highly skewed
II. The maximum gain is limited to promised payment, and although extreme losses can be incurred, such heavy losses are extremely rare.

For these reasons, the favored distribution for credit risk analysis is the beta distribution. The following is its density function:

$$F(x, \alpha, \beta) = \begin{cases} \frac{\Gamma(\alpha+\beta)}{\Gamma\alpha\Gamma\beta} x^{\alpha-1}(1-x)^{\beta-1}, 0 < x < 1 \\ 0 \qquad\qquad\qquad\qquad otherwise \end{cases}$$

$Mean = \frac{\alpha}{\alpha+\beta}$

$Variance = \frac{\alpha\beta}{(\alpha+\beta)^2(\alpha+\beta+1)}$

As can be seen in the density function above, the mass of the beta distribution is distributed between 0 and 1. It follows that when modeling credit events, losses are defined between 0% and 100%. Why exactly is the beta distribution chosen?

The answer to that question is that the beta distribution is extremely flexible. Depending on the values of α and β, the distribution can be symmetrical or skewed. In fact, in the event that these two parameters are equal, then the expected loss for a portfolio equals its unexpected loss.

When modeling the very extreme losses, however, the credit loss distribution can be quite difficult to model using the beta distribution by itself. In such instances, Monte Carlo simulation techniques come in handy.

Challenges Experienced When Quantifying Credit Risk Using the Bottom-up Risk Measurement Approach

I. This approach assumes that credits are illiquid assets. As a result, credit losses are measured by their contribution to the credit portfolio without factoring the correlation among risk factors as priced in liquid markets.
II. The approach uses credit risk models that estimate expected and unexpected losses over a 1-year period. Ideally, credit risk models should estimate losses for a

much longer period.

III. The approach segregates other types of risk such as operational risk and market risk. Thus, additional resources must be made available to manage those risks as well.

Practice Questions

Question 1

Big Data Inc., a U.S. based cloud technology and computing firm, has been offered a USD 10 million term loan fully repayable in exactly two years. The bank behind the offer estimates that it will be able to recover 65% of its exposure if the borrower defaults, and the probability of that happening is 0.8%. The bank's expected loss one year from today is closest to:

A. USD 52,000

B. USD 26,000

C. USD 14,000

D. USD 28,000

The correct answer is D.

$$EL = EA \times PD \times LR$$

$EA = \text{USD 10,000,000}$

$PD = 0.8\%$

$LR = 35\%$

$$EL = 10,000,000 \times 0.008 \times 0.35 = \text{USD 28,000}$$

Maturity is irrelevant since the loan is fully repayable in two years.

A is incorrect. The loss given default is taken to be 65%. Note that $LGD = 1 - \text{Recovery rate.}$

B is incorrect. The loss given default is taken to be 65% and the final result dividend by two.

C is incorrect. The final result is incorrectly divided by 2.

Question 2

A bank has two assets outstanding, denominated in U.S. dollars. The correlation between the two assets is 0.4. Other details are as follows:

	Asset A	**Asset B**
EA	$1,600,000$	$2,000,000$
PD	1%	2%
LR	30%	40%
σ_{PD}	6%	8%
σ_{LR}	20%	25%

Calculate the unexpected loss of the portfolio as well as the risk contribution of each asset:

	Unexpected loss	Risk Cont. A	Risk Cont. B
$A.$	$118,350$	$18,350$	$100,000$
$B.$	$125,800$	$102,600$	$23,200$
$C.$	$120,000$	$98,000$	$22,000$
$D.$	$119,308$	$29,302$	$90,006$

The correct answer is D.

For a two-asset portfolio:

$$UL_P = \sqrt{UL_i^2 + UL_j^2 + 2\rho UL_i UL_j} \ldots\ldots\ldots\ldots\ldots \textit{equation} \quad 1$$

Where:

$$UL_x = EA_x \times \sqrt{PD_x \times \sigma_{LR_x}^2 + (LR)_x^2 \times \sigma_{PD_x}^2} \qquad\qquad \text{x=A or B}$$

For asset A,

$$\begin{aligned} UL_A &= 1,600,000 \times \sqrt{0.01 \times 0.2^2 + 0.3^2 \times 0.06^2} \\ &= \text{USD 43,052} \end{aligned}$$

For asset B,

$$\begin{aligned} UL_B &= 2,000,000 \times \sqrt{0.02 \times 0.25^2 + 0.4^2 \times 0.08^2} \\ &= \text{USD 95,373} \end{aligned}$$

We can now compute the unexpected loss of the portfolio using equation 1:

$$UL_P = \sqrt{43,052^2 + 95,373^2 + 2 \times 0.4 \times 43,052 \times 95,373} \qquad = \text{USD 119,308}$$

To determine the risk contributions of A and B, recall the formulas:

$$\begin{aligned} RC_1 &= UL_1 \times \frac{UL_1 + (\rho_{12} \times UL_2)}{UL_P} \\ RC_2 &= UL_2 \times \frac{UL_2 + (\rho_{12} \times UL_1)}{UL_P} \end{aligned}$$

Thus,

$$\begin{aligned} RC_A &= 43,052 \times \frac{43,052 + 0.4 \times 95,373}{119,308} \\ &= \text{USD 29,302} \end{aligned}$$

$$\begin{aligned} RC_B &= 95,373 \times \frac{95,373 + 0.4 \times 43,052}{119,308} \\ &= \text{USD 90,006} \end{aligned}$$

Note: $29,302 + 90,006 = 119,308$