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Assignment 1

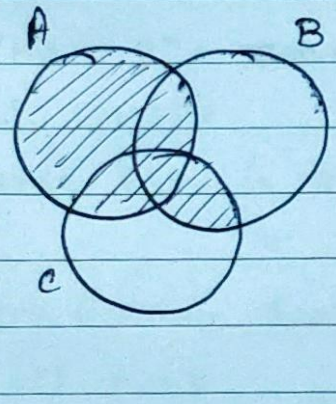
1) let U = {John, Mary, Dave, Lucy, Peter, Larry}
A = {John, mary, Dave} and B = {John, Larry, Lucy}
Find $A \cap B$, $A \cup B$, $A - B$, $B - A$, $\bar{A}$ and $\bar{B}$

→ let U = {John, Marry, Dave, Lucy, Peter, Larry}
A = {John, Marry, Dave}
B = {John, Larry, Lacy

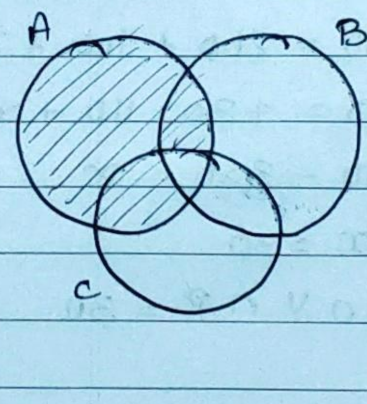
$A \cap B$ = {john}
$A - B$ = {Marry, Dave}
$B - A$ = {Larry, Lucy}
$\bar{A}$ = {Lucy, Peter, Larry}
$\bar{B}$ = {Marry, Dave, Peter}

2) let A, B, C be Three sets. for each of the following sets, draw a venn diagram and shade the area representing the given set.

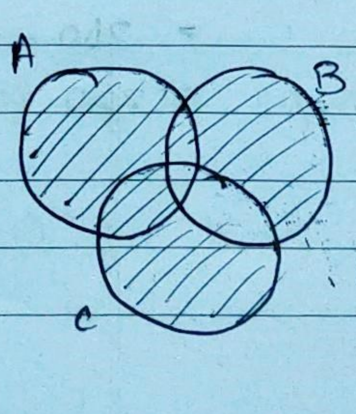
i) $A \cup (B \cap C)$



ii) $A - (B \cap C)$



iii $A \cup (B \cap C)$



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3) There are 350 farmers in a large region, 260 farm beetroot, 100 farm yarns, 70 farm radish, 40 farm beetroot and radish, 40 farm yarns and radish, and 30 farm beetroot and yarns. let B, Y and R denote the set of farms that farm beetroot, yarns and radish respectively. Determine the no. of farmers that farm beetroot, yarns and radish. Also find ~~the~~ total number of farmers they farm only one. Draw venn diagram.

-> Total farmers -> 350

Beetroot (B) -> 260

Yarn (Y) -> 100

Radish (R) -> 70

$B \cap R \rightarrow 40$

$Y \cap R \rightarrow 40$

$B \cap Y \rightarrow 30$

$B \cap Y \cap R = ?$

Let x be the number of people who farm

$$|B \cup Y \cup R| = |B| + |Y| + |R| - |B \cap R| - |Y \cap R| - |B \cap Y| + |B \cap Y \cap R|$$

$$350 = 260 + 100 + 70 - 40 - 40 + 30 + x$$

$$350 = 430 - 110 + x$$

$$350 - 320 = x$$

$$x = 30$$

$$B \cap Y \cap R = 30$$

$$|B - Y - R| = |B| - |B \cap R| - |B \cap Y| + |B \cap Y \cap R|$$

$$= 260 - 40 - 30 + 30$$

$$= 220$$

$|Y-R-B| = |Y| - |Y \cap R| - |B \cap Y| + |B \cap Y \cap R|$

$= 100 - 40 - 30 + 30$

$= 60$

$|R-B-Y| = |R| - |Y \cap R| - |B \cap R| + |B \cap Y \cap R|$

$= 70 - 40 - 40 + 30$

$= 20$

4) Using mathematical induction. Prove that $n! > 2n$ where n is a positive integer greater than or equal to 4

→ let $P(n)$ be defined by $n! \geq 2^n$

S1 → Show that $P(4)$ is true

Let $n = 4$

$n! = 2n$

$LHS = 4! = 24$

$RHS = 2^n = 24 = 16$

$\therefore 24 > 16$

hence $P(4)$ is true

S2 → Assume that $P(k)$ is true

let $n = k$

$k! = 2k$ (i)

Show that $P(k+1)$ is true

Multiplying $(k+1)$ both sides of the eqn (i)

$k!(k+1) > 2^k (k+1)$

The left side is equal to $(k+1)'$ for $k > 4$

$\therefore (k+1) > 2$

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Multiply both sides by 2^k to obtain

$2^k (k+1) > 2 \cdot 2^k$

$\therefore 2^k (k+1) > 2^{k+1}$

$\therefore$ we proved $(k+1)! \; 2^k (k+1)$ and $2^k (k+1) > 2^{k+1}$

$\therefore (k+1)! \; 2^{k+1}$

$P(k)$ is true

hence $p(k+1)$ is also true

5) Show that $1^2 + 3^2 + 5^2 + \dots (2n-1)^2 = \frac{n(2n-1)(2n+1)}{3}$ by mathematical induction

$\rightarrow$ let $P(n) = 1^2 + 3^2 + 5^2 + \dots + (2n-1)^2 = \frac{n(2n-1)(2n+1)}{3}$

$S1 \rightarrow$ Show that $P(n)$ is true

let $n = 1$

$L.H.S = (2 \times 1 - 1)^2 = 1$

$RHS = \frac{n(2n-1)(2n+1)}{3} = \frac{1(2 \times 1 - 1)(2 \times 1 + 1)}{3} = \frac{3}{3} = 1$

$LHS = RHS$

$\therefore P(n)$ is true for $n = 1$

$S2 \rightarrow$ Assume it is true for $n = k$

$P(n)$ is true for $n = k$

i.e. $1^2 + 3^2 + 5^2 + \dots + (2k-1)^2 = \frac{k(2k-1)(2k+1)}{3}$ (i)

to prove $P(n)$ is true for $n = k+1$

$1^2 + 3^2 + 5^2 + \dots (2(k+1)-1)^2 = \frac{k+1(2(k+1)-1)(2(k+1)+1)}{3}$

$1^2 + \dots + (2k-1)^2 + (2k+1)^2 = \frac{(k+1)(2k+1)(2k+3)}{3}$ (ii)

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From eq (1)

$$1^2 + 3^2 + 5^2 + \dots + (2k-1)^2 = \frac{k(2k-1)(2k+1)}{3}$$

Adding $(2k+1)^2$ on both sides

$$1^2 + 3^2 + 5^2 + \dots + (2k-1)^2 + (2k+1)^2 = k(2k-1)(2k+1)/3 + (2k+1)^2$$

$$= \frac{k(2k-1)(2k+1) + 3(2k+1)^2}{3}$$

$$= (2k+1)\left(\frac{k(2k-1) + 3(2k+1)}{3}\right)$$

$$= (2k+1)\left(\frac{2k^2 - k + 6k + 3}{3}\right)$$

$$= (2k+1)\left(\frac{2k^2 + 5k + 3}{3}\right)$$

$$= (2k+1)\left(\frac{2k^2 + 2k + 3k + 3}{3}\right)$$

$$= (2k+1)\left(\frac{2k(k+1) + 3(k+1)}{3}\right)$$

$$= (2k+1)\left(\frac{(2k+3)(k+1)}{3}\right)$$

$$= \frac{(2k+1)(2k+3)(k+1)}{3}$$

$$\therefore 1^2 + 3^2 + 5^2 \dots (2k-1)^2 + (2k+1)^2 = \frac{(k+1)(2k+1)(2k+3)}{3}$$

is true $\therefore$ $P(k+1)$ is true when $P(k)$ is true

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6) Prove that $n^3 + 2n$ is divisible by 3 for all $n > 1$

→ let $P(n)$ be true

S1 → let $n = 1$

$n^5 + 2n \rightarrow 1^3 + 2 + 1 = 3$

∴ 3 is divisible by 3

hence $P(n)$ is true for $n = 1$

S2 → Assume that $P(k)$ is true

$n = k$

$k^3 + 2k$ is divisible by 3 which is equivalent to $k^3 + 2k = 3A$

where A is a positive integer

to prove $P(n)$ is true for $n = k+1$

$(k+1)^3 + 2(k+1)$

$= (k+1)^3 + (2k+1)$

$= k^3 + 3k^2 + 5k + 3$

$= (k^3 + 2k) + (3k^2 + 3k + 3)$

$= 3A + 3(k^2 + k + 1)$

$= 3(A + k^2 + k + 1)$

Hence $(k+1)^3 + 2(k+1)$ is also divisible by 3 and therefore $P(k+1)$ is also true

7) ~~$(12+56)_{10} \rightarrow ()_2 \rightarrow ()_8$~~

3) $(12456)_{10} \rightarrow (\ )_2 \rightarrow (\ )_8$

→

2	12456	0
2	6228	0
2	3114	0
2	1557	1
2	778	0
2	389	1
2	194	0
2	97	1
2	48	0
2	24	0
2	12	0
2	6	0
2	3	1
	1	

$(12456)_{10} \rightarrow (11000010101000)_2$

→

8	12456
8	1557
8	194
8	24
8	3

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ii) $(1011011)_2 = ( )_8 = ( )_{16}$

$\underline{001}\,\underline{110}\,\underline{011}$

1 6 3

$\Rightarrow (163)_8$

$\underline{0101}\,\underline{1011}$

6 B

$(6B)_{16}$

iii) $(BDF.78)_{16} \rightarrow ( )_8 \rightarrow ( )_{10}$

BDF.78

$\overline{101}\ \overline{111}\ \overline{011}\ \overline{111}$

5737

$78 \times 8 = 6.24$

$24 \times 8 = 1.92$

$92 \times 8 = 7.36$

$36 \times 8 = 2.88$

$88 \times 8 = 7.04$

$04 \times 8 = 0.32$

$32 \times 8 = 2.56$

$56 \times 8 = 4.48$

$48 \times 8 = 3.84$

$84 \times 8 = 6.72$

$B \times 16^2 + D \times 16^1 + F \times 16^0 . 7 \times 16^{-1} + 8 \times 16^{-2}$

$\Rightarrow 11 \times 16^2 + 13 \times 16^1 + 15 \times 16^0 . 7 \times 16^{-1} + 8 \times 16^{-2}$

$\Rightarrow 2816 + 208 + 15.468$

$(3039.468)_{10}$

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iv) $(7045)_8 \rightarrow ()_2 \rightarrow ()_{10} \rightarrow ()_{16}$

$(111000100101)_2$

$(7045)_8 \rightarrow ()_{10}$

$= 7\times8^3 + 0\times8^2 + 4\times8^1 + 5\times8^0$

$= (3621)_{10}$

$(7045)_8 \rightarrow ()_{16}$

$\underline{1110}\ \underline{0010}\ \underline{0101}$

E 2 5

$(E25)_{16}$

8) Use Elucidian algorithm to find gcd(1000, 5040

let a = 5040

b = 1000

a mod b = R

5040 mod 1000 = 40

a = 1000, b = 40

1000 mod 40 = 24

a = 40, b = 25

40 mod 25 = 15

a = 25, b = 15

25 mod 15 = 10

$a = 15, \quad b = 10$

$15 \bmod 10 = 5$

$a = 10, \quad b = 5$

$10 \bmod 5 = 0$

$gcd = 5$

b) $gcd(9888, 6060)$

$a = 9888, \quad b = 6060$

~~a a mod b~~

$a \bmod b$

$9888 \bmod 6060 = 3828$

$a = 6060,$ ~~[illegible]~~ $3828 = 2232$

$6060 \bmod 3828 = 2232$

$a = 3828 \quad b = 2232$

$2838 \bmod 2232 = 1596$

$a = 2232, \quad b = 1592$

$2232 \bmod 1592 = 640$

$a = 1592 \quad b = 640$

$1592 \bmod 640$

$a = 640, \quad b = 312$

$640 \bmod 312 = 16$

$a = 312, \quad b = 16$

$312 \bmod 16 = 8$

$a = 16, \quad b = 8$

$16 \bmod 8 \rightarrow 0$

9) Find the power set of the following sets

i) $\{a, b\}$

→ Power Set = $\{ \phi, \{a\}, \{b\}, \{a, b\} \}$

ii) $\{x, y, 3, 10\}$

Power Set = $\{ \phi, \{x\}, \{y\}, \{3\}, \{x, y\}, \{x, 3\}, \{y, 3\},$
$\{x, y, 3\} \}$