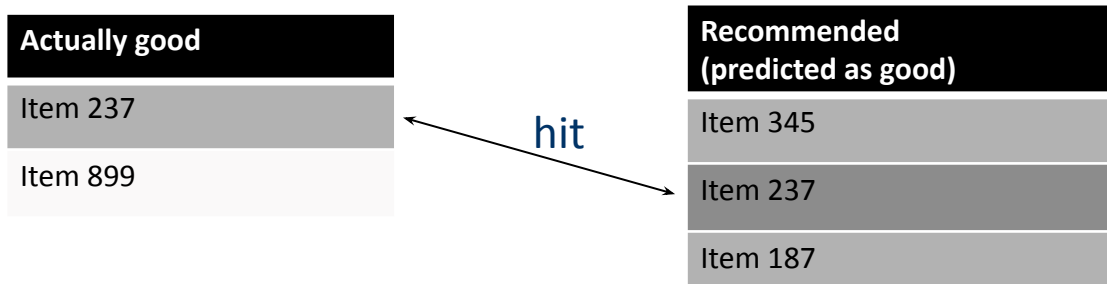


# Rank Aware Metrics

# Rank Aware Metrics: Rank Position Matters

For a user:



- **Rank metrics** extend recall and precision to take the positions of correct items in a ranked list into account
  - Relevant items are more useful when they appear earlier in the recommendation list
  - Particularly important in recommender systems as lower ranked items may be overlooked by users

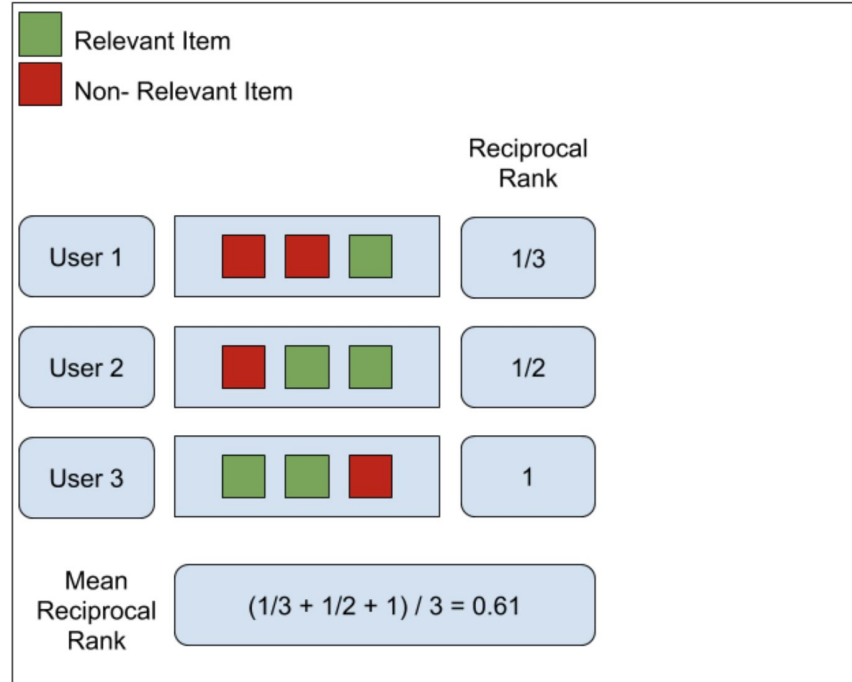
# Mean Reciprocal Rank

Evaluates the list of recommendations

$$\text{MRR} = \frac{1}{Q} \sum_{i=1}^Q \frac{1}{\text{rank}_i}$$

- Suppose we have recommended 3 movies to a user, say A, B, C in the given order, but the user only liked movie C. As the rank of movie C is 3, the reciprocal rank will be 1/3
- For multiple recommendations across different users, the Mean Reciprocal Rank is the mean of all reciprocal ranks.
- Larger the mean reciprocal rank, better the recommendations

# Mean Reciprocal Rank



# Mean Average Precision

- Average Precision (AP) is a ranked precision metric that places emphasis on highly ranked correct predictions (hits)
- Essentially it is the average of precision values determined after each successful prediction
- If a relevant document never gets retrieved, we assume the precision to be 0

Rank	Hit?
1	
2	X
3	X
4	X
5	

$$AP = \frac{1}{3} \left( \frac{1}{1} + \frac{2}{4} + \frac{3}{5} \right) = \frac{21}{30} = 0.7$$

$$AP = \frac{1}{3} \left( \frac{1}{2} + \frac{2}{3} + \frac{3}{4} \right) = \frac{23}{36} \approx 0.639$$

Rank	Hit?
1	X
2	
3	
4	X
5	X

# Beyond Binary Relevance

## Introduction to Information Retrieval

The image shows a screenshot of a Yahoo! search results page for the query "Toyota safety". The page layout includes a search bar at the top, navigation links (Web, Images, Video, Local, Shopping, More), and a sidebar on the left with links to "Search Pad", "SearchScan - On", and "168,000,000 results for Toyota safety:". The main content area displays several search results, including "Toyota Recall", "Toyota Safety", "TOYOTA | Car Safety Innovation and Technology", "Toyota home page for car safety and car technology", "Toyota Safety Ratings - Toyota Safety Features - Motor Trend", "Toyota Motor Europe Corporate Site Safety", "European Safety Brochure 2005", "Toyota - Star Safety System", and "Toyota Plus Safety Ratings - CarDirect". Handwritten annotations in blue ink are present: "fair" is written twice, once pointing to the "Toyota Safety" result and once pointing to the "Toyota home page for car safety and car technology" result. "Good" is written once, pointing to the "Toyota Safety Ratings - Toyota Safety Features - Motor Trend" result. The page also features sponsored results on the right side, including "Safety for a Toyota", "Toyota Safety", and "Toyota Safety".

# Normalised Discounted Cumulative Gain

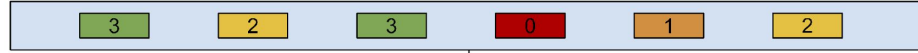
3 Highly Relevant Item

2 Medium Relevant Item

1 Relevant Item

0 Non-Relevant Item

Predicted Ranking with  
True Ratings



Ideal Ranking



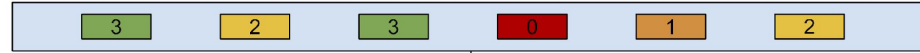
# Normalised Discounted Cumulative Gain

3 Highly Relevant Item    2 Medium Relevant Item    1 Relevant Item    0 Non-Relevant Item

Discounted  
Gain

Predicted Ranking with  
True Ratings

$$\frac{rel_i}{\log_2(i+1)}$$



Ideal  
Discounted  
Gain

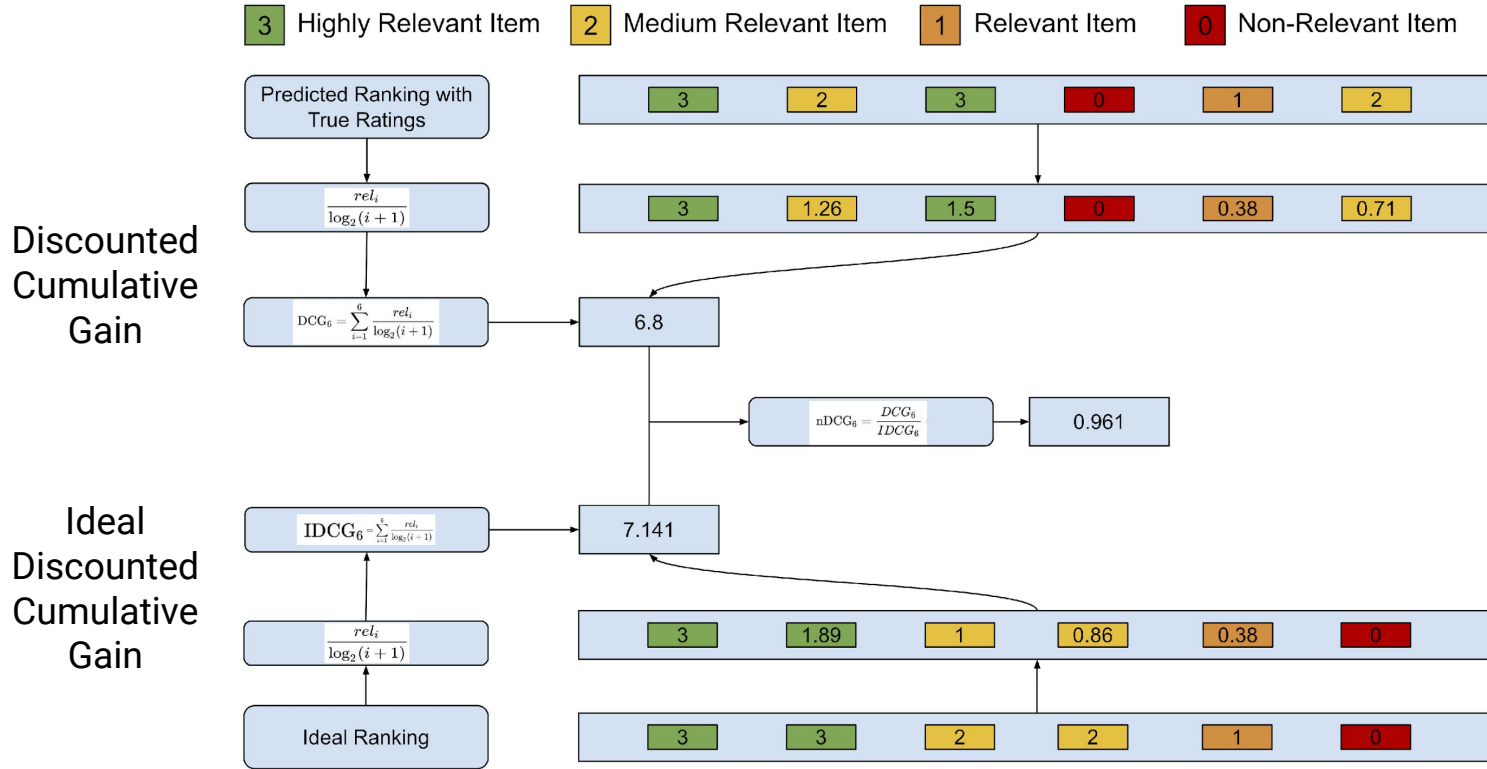
$$\frac{rel_i}{\log_2(i+1)}$$

Ideal Ranking





# Normalised Discounted Cumulative Gain



# Normalised Discounted Cumulative Gain

- Discounted cumulative gain (DCG)
  - Logarithmic reduction factor

$$DCG_p = \sum_{i=1}^p \frac{rel_i}{\log_2(i+1)}$$

Where:

- $p$  denotes the position up to which relevance is accumulated
- $rel_i$  returns the relevance of recommendation at position  $i$

- Idealized discounted cumulative gain (IDCG)
  - Assumption that items are ordered by decreasing relevance

$$IDCG_p = \sum_{i=1}^{REL_p} \frac{rel_i}{\log_2(i+1)}$$

- Normalized discounted cumulative gain (nDCG)
  - Normalized to the interval [0..1]

$$NDCG_p = \frac{DCG_p}{IDCG_p}$$