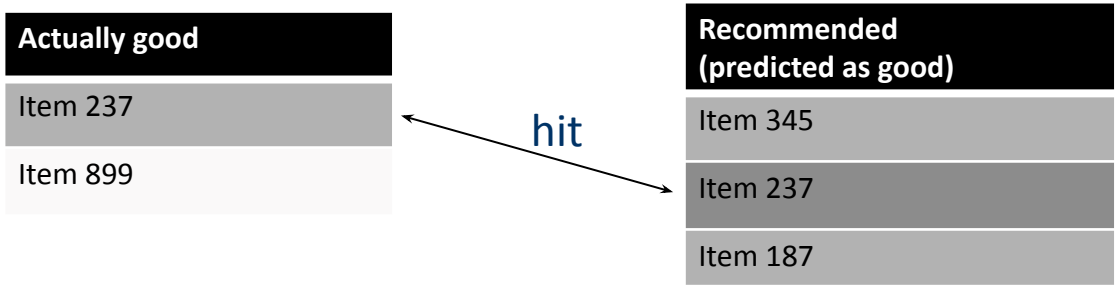
Rank Aware Metrics

Rank Aware Metrics: Rank Position Matters

For a user:



- **Rank metrics** extend recall and precision to take the positions of correct items in a ranked list into account
  - Relevant items are more useful when they appear earlier in the recommendation list
  - Particularly important in recommender systems as lower ranked items may be overlooked by users

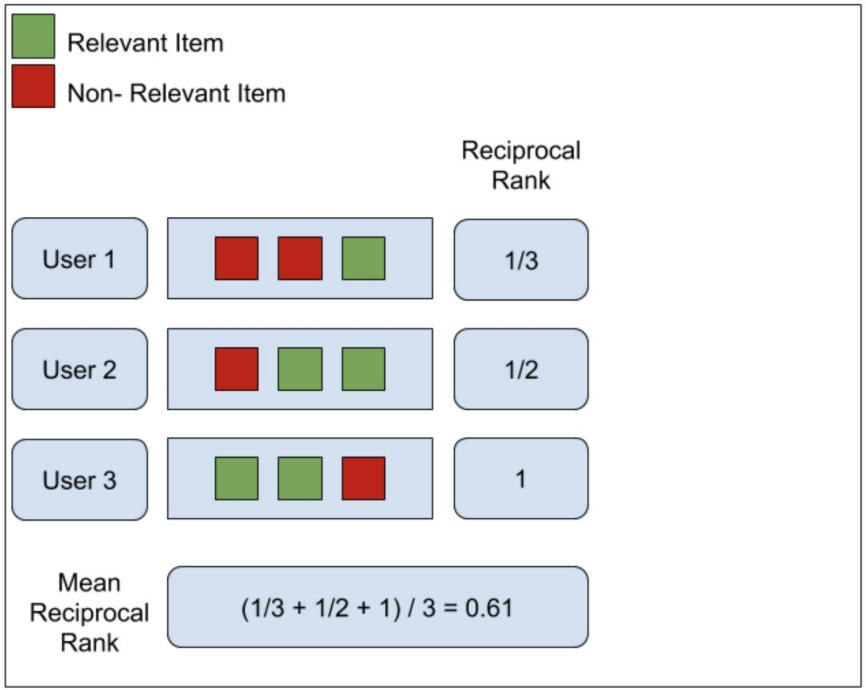
Mean Reciprocal Rank

Evaluates the list of recommendations

$$\text{MRR} = \frac{1}{Q}\sum_{i=1}^{Q}\frac{1}{\text{rank}_i}$$

- Suppose we have recommended 3 movies to a user, say A, B, C in the given order, but the user only liked movie C. As the rank of movie C is 3, the reciprocal rank will be 1/3
- For multiple recommendations across different users, the Mean Reciprocal Rank is the mean of all reciprocal ranks.
- Larger the mean reciprocal rank, better the recommendations

Mean Reciprocal Rank



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Mean Average Precision

- Average Precision (*AP*) is a ranked precision metric that places emphasis on highly ranked correct predictions (hits)
- Essentially it is the average of precision values determined after each successful prediction
- If a relevant document never gets retrieved, we assume the precision to be 0

Rank	Hit?
1	
2	X
3	X
4	X
5	

$$AP = \frac{1}{3}\left(\frac{1}{2} + \frac{2}{3} + \frac{3}{4}\right) = \frac{23}{36} \approx 0.639$$

$$AP = \frac{1}{3}\left(\frac{1}{1} + \frac{2}{4} + \frac{3}{5}\right) = \frac{21}{30} = 0.7$$

Rank	Hit?
1	X
2	
3	
4	X
5	X

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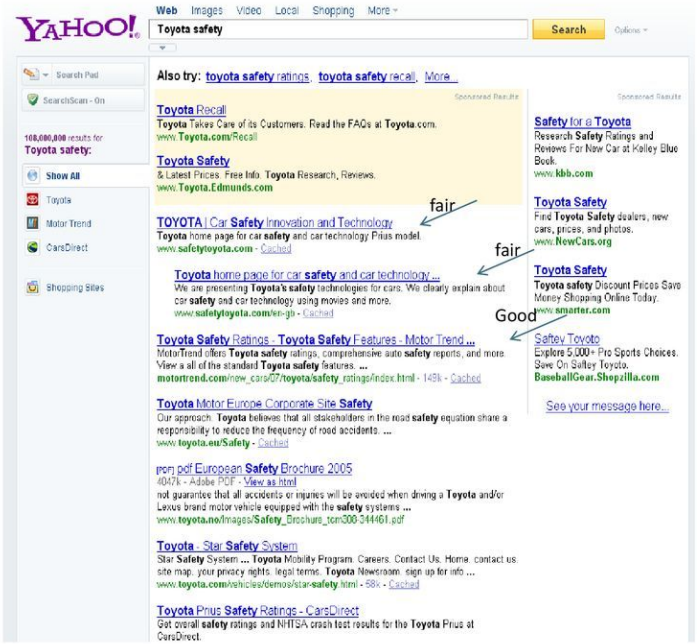
Beyond Binary Relevance



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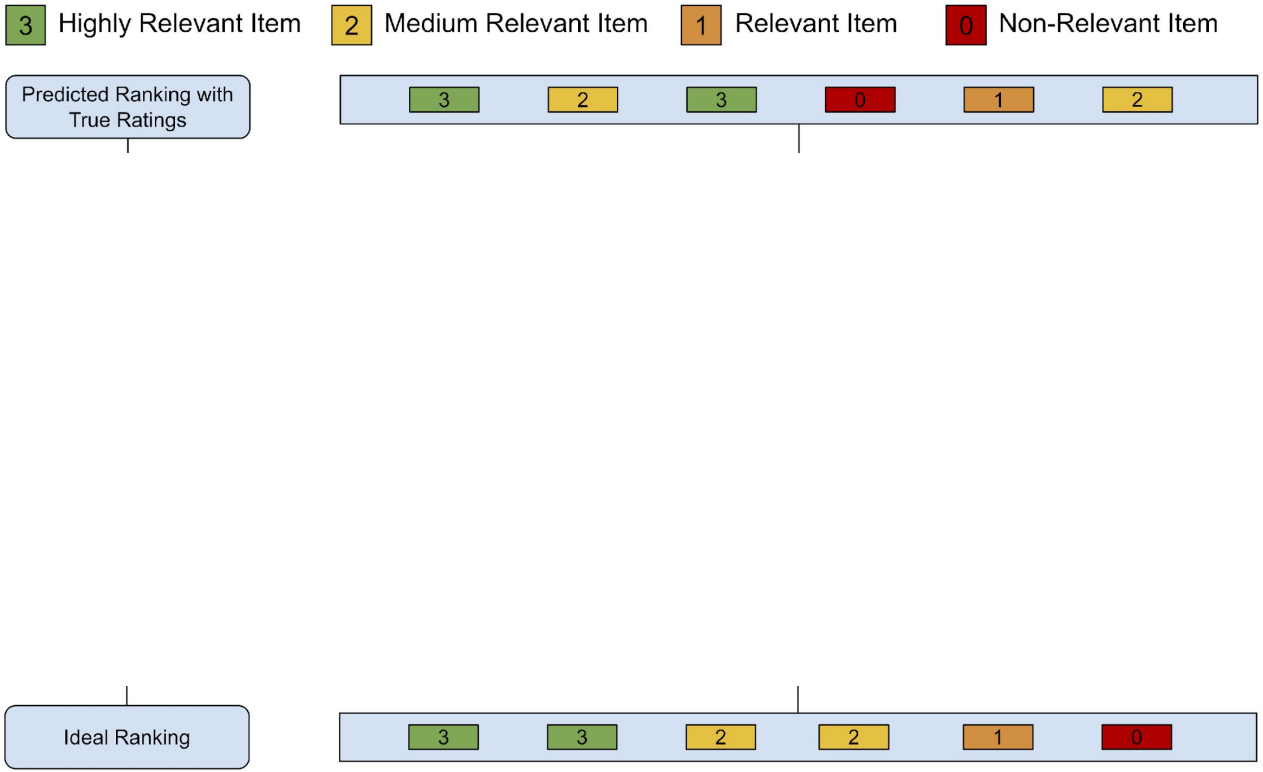
Normalised Discounted Cumulative Gain



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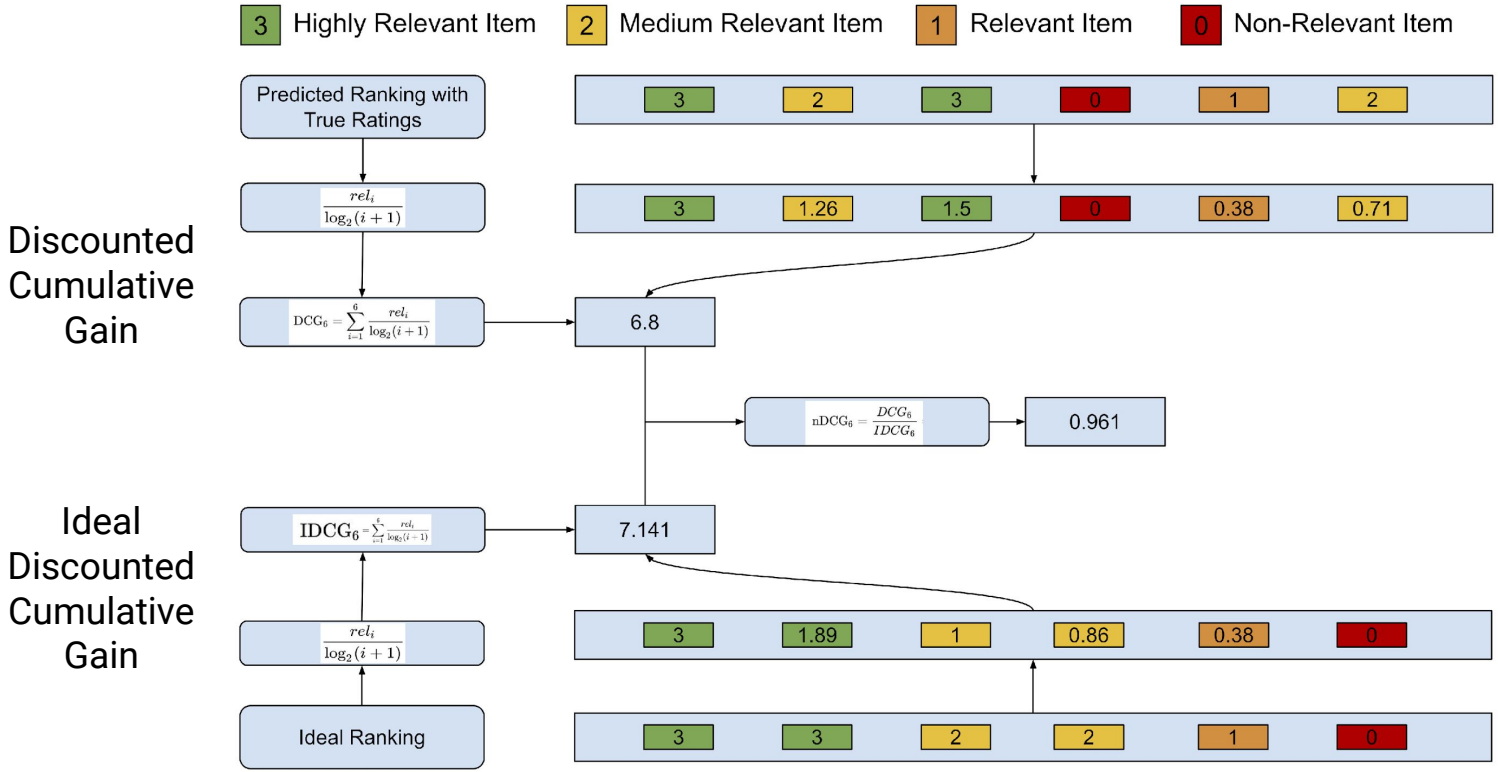

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Normalised Discounted Cumulative Gain

- Discounted cumulative gain (DCG)
  - Logarithmic reduction factor

$$DCG_p = \sum_{i=1}^{p} \frac{rel_i}{log_2(i+1)}$$

Where:
- p denotes the position up to which relevance is accumulated
- rel_i returns the relevance of recommendation at position i

- Idealized discounted cumulative gain (IDCG)
  - Assumption that items are ordered by decreasing relevance

$$IDCG_p = \sum_{i=1}^{REL_P} \frac{rel_i}{log_2(i+1)}$$

- Normalized discounted cumulative gain (nDCG)
  - Normalized to the interval [0..1]

$$NDCG_p = \frac{DCG_P}{IDCG_P}$$

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