Double Exponential Smoothing

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Double Exponential Smoothing

- Concept similar to Simple Exponential smoothing
- Uses exponential weights for each observation

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- Concept similar to Simple Exponential smoothing
- Uses exponential weights for each observation
- **Also includes the trend component of the series**

$$\boxed{\hat{Y}_{t+1} = \text{Level} + \text{Trend}}$$

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Double Exponential Smoothing

$$\hat{Y}_{t+1} = \text{Level} + \text{Trend}$$

At first time step:

$$L_1 = Y_1$$

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Double Exponential Smoothing

$$\hat{Y}_{t+1} = \text{Level} + \text{Trend}$$

At first time step:

$$L_1 = Y_1$$

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Double Exponential Smoothing

$$\hat{Y}_{t+1} = \text{Level} + \text{Trend}$$

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<u>At first time step:</u>

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<u>At second time step:</u>

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At second time step:

$$L_2 = \alpha Y_2 + (1-\alpha)(\hat{Y}_1)$$

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<u>At any time step t:</u>

$$L_t = \alpha Y_t + (1-\alpha)(L_{t-1} + T_{t-1})$$

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Double Exponential Smoothing

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Holt Winters Algorithm

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- Also known as triple exponential smoothing
- Takes level, trend and seasonality into account

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- Takes level, trend and seasonality into account

$$\hat{Y}_{t+1} = \text{Level} + \text{Trend} + \text{Seasonality}$$

$$\hat{Y}_{t+1} = (\text{Level} + \text{Trend}) \times \text{Seasonality}$$

Holt Winters Algorithm

$\hat{Y}_{t+1}$ = (Level + Trend) + Seasonality

$\hat{Y}_{t+1}$ = (Level + Trend) x Seasonality

- Use when peaks and troughs are of same roughly size

- Use when peaks and troughs have significant amplitudes

Holt Winters Algorithm

$\hat{Y}_{t+1}$ = (Level + Trend) + Seasonality

$\hat{Y}_{t+1}$ = (Level + Trend) x Seasonality

$$\hat{Y}_{t+1} = (L_t + T_t) + S_t$$

$$\hat{Y}_{t+1} = (L_t + T_t) \times S_t$$

Holt Winters Algorithm

$\hat{Y}_{t+1}$ = (Level + Trend) + Seasonality

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Double Exponential Smoothing: $L_t = \alpha Y_t + (1-\alpha)(L_{t-1} + T_{t-1})$

$$L_t = \alpha(Y_t - S_{t-m}) + (1-\alpha)[L_{t-1} + T_{t-1}]$$

$$L_t = \alpha(Y_t / S_{t-m}) + (1-\alpha)[L_{t-1} + T_{t-1}]$$

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$\hat{Y}_{t+1}$ = (Level + Trend) x Seasonality

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$$L_t = \alpha(Y_t - S_{t-m}) + (1-\alpha)\,[L_{t-1} + T_{t-1}]$$

$$T_t = \beta(L_t - L_{t-1}) + (1-\beta)T_{t-1}$$

$$S_t = \gamma(Y_t - L_t) + (1-\gamma)S_{t-m}$$

$$\hat{Y}_{t+1} = (L_t + T_t) \times S_t$$

$$L_t = \alpha(Y_t / S_{t-m}) + (1-\alpha)[L_{t-1} + T_{t-1}]$$

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Holt Winters Algorithm: Initial Values

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Initial Seasonal Values: $S_{1\text{-}m} = S_i / (S_1 + S_2 + \ldots . S_m)$

Initial Value Level: $L_{m+1} = Y_{m+1} / S_1$

Initial Trend Level: $L_{m+1} - L_m$

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Notebook

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