Introduction to Computational Physics SS2023

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Exercise 10 from June 28, 2023
Return before noon of July 7, 2023

1 Random Numbers – Rolling Dice

- Write a simple portable random number generator, using linear congruences

$$I_{j+1} = aI_j + c \pmod{m}.$$

  It produces homogeneously distributed numbers between 0 and $m-1$; we can normalize them by $r_j = I_j/(m-1)$ to get homogeneously distributed real numbers between 0 and 1. Choose e.g. $a = 106$, $m = 6075$, $c = 1283$, from Numerical Recipe's recommendation. Experiment with other values.

- A simple way to check 'by eye' that your random number generator produces really homogeneously distributed numbers is to create two sequences with different initial values I_0 and J_0, normalize both sequences between 0 and 1 as above ($r_0 = I_0/(m-1)$, $s_0 = J_0/(m-1)$, $0 \le r_i, s_i \le 1$), and plot all pairs (r_i, s_i) in the quadrat $0 \le r \le 1$, $0 \le s \le 1$. The eye is quite sensitive to see a good distribution.

- You can see the deterministic character of this random number sequence by plotting I_{j+1} against I_j.

- Repeat the previous steps by using the routine `numpy.random.rand` in python (or `ran2` in Fortran/C).

Simulate rolling dice with your random number generator, i.e. obtain uniformly distributed random integer numbers between 0 and 6. Roll the dice ten times and sum up the result. The possible numbers lie between 10 and 60. Plot the distribution of these numbers for 10.000 experiments (of 10 dice each). The theoretical expectation for this distribution is given by the central limit theorem, which will be explained later in the lecture.

2 Probability distribution functions (Homework)

Consider a probability distribution function $p(x)$ given in the domain $[0, a\rangle$ by

$$p(x) = bx \tag{2.1}$$

Assume that $\{r_i\}$ is a random set of numbers, distributed uniformly between 0 and 1.

- Give the proper value of b as a function of a such that the probability distribution function is properly normalized.
- Use the rejection method to make a set $\{x_i\}$ that obeys Eq.(2.1) for $a = 0.5$. Hint: Use $x_i = ar_i$ $(0 < x_i < a)$. Then use another random number set $\{s_i\}$, distributed uniformly between 0 and 1; use $y_i = p(a) \cdot s_i$. We have $0 < y_i < p(a)$ with $p(a) = ab$. Accept x_i in your set $\{x_i\}$ if $y_i \leq p(x_i)$, otherwise, if $y_i > p(x_i)$, reject it (note $p(x_i) = bx_i$).
- Make a histogram of the resulting numbers and check that the histogram indeed follows Eq.(2.1), i.e. overplot Eq.(2.1). Experiment with the size of the set (the number of random numbers drawn), to find out how large you have to make it to get (by eye) a reasonable fit.

(10 points)

3 Determine the number π with random numbers RN (Homework)

Compute the number π using a rejection method with the function $f(x) = \sqrt{1 - x^2}$, for $0 \leq x \leq 1$. Hint: It is enough to use only one quadrant $x, f(x) > 0$. Vary the number of RNs widely (orders of magnitude) and plot the accuracy of the result as a function of the number of RNs. Use logarithmic variables for the plot.
(10 points)