Introduction to Computational Physics SS2023

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Exercise 2 from April 26, 2023
Return before noon of May 5, 2023

1 Numerical Simulation of the 2-Body Problem

- Write a computer program that computes the relative motion of two point-like bodies under their mutual gravitational influence. Use a) an Euler integration procedure b) a leap frog scheme; the time step remains constant. Set $G = 1$, $M_1 = M_2 = 1$. We use in the following dimensionless variables s, w, τ (for relative position, relative velocity, and time), as defined in the lecture.
- For which velocity w_0 can the two bodies rotate around each other in a circular fashion with a separation of 1?
- Now perform the numerical integration. Use, for the moment, a constant time step of $\Delta\tau = 0.01$. Choose again initial conditions of separation $s_0 = 1$ and velocity w_0 to get a circular orbit (as obtained from the previous bullet).
- Compute the eccentricity from the Laplace-Runge-Lenz (LRL) vector.
- What happens if you choose an initial velocity larger than $\sqrt{2}\,w_0$?
- Simulate more cases, like e.g. the case of an initial velocity of $w_0/\sqrt{2}$ or $w_0/4$. Experiment with decreasing the time step (and simultaneously increasing the number of time steps) and see how the results change. Try to compute the eccentricities of the orbit. Do you get a closed orbit as it should be?

2 Error Analysis (HOMEWORK)

(a) Take the Euler scheme and choose 3 different eccentricities by varying the initial velocity, and study the set-up for a wide range of different time steps. Make sure that Δt spans orders of magnitude. Integrate the 2-body problem for two orbits. Plot, in a double-logarithmic fashion, the error in the energy at the end of this orbit as a function of the time step Δt. Discuss the result, is it consistent with expectations? (10 points)

(b) Do the same as above, but now employ the leap frog integrator scheme. How does the result change? (10 points)

NOTE again, for this and all following homework exercises, which require writing a computer program, it is necessary to submit all parts of the program written by you, together with the results. It must be comprehensible for your tutor how your program generates your results.