
Introduction to Computational Physics SS2023

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Exercise 3 from May 3, 2023

Return before noon of May 12, 2023

1 4th Order Runge-Kutta Method (RK4)

Get acquainted with the RK4 algorithm. We provide the `rk4` subroutine from the Numerical Recipes library in `python`, `C` or `F77` on the *moodle* page of the lecture. You can find other implementations on the web. As a first test solve the simple differential equation of exponential decay,

$$\dot{x} = -rx, \quad (1.1)$$

with $r = 1$ and $x(0) = x(t_0) = 1$. The test whether your program code is working well should be that you reproduce the correct gradient of the solution in the semi-logarithmic plot. Investigate the accuracy of the integration by comparing to the analytical solution, and by varying the step size over various orders of magnitude.

2 Three-Body Problem (HOMEWORK)

Adapt your Runge-Kutta-4 integrator to the gravitational 3-body problem. Simplify the system by setting the gravitational constant to $G = 1$ and reduce the dimensionality of the problem by only considering motions in a plane. You obtain 12 coupled ordinary differential equations of first order which you can convert into the standard form $y' = f(y, t)$. Integrate the system forward in time starting from various initial configurations as given below.

- (a) In a first step, set the masses of all three bodies to $m_1 = m_2 = m_3 = 1$ and select the following initial conditions for $y(0)$:

$$\begin{aligned} (y_1, y_2) &= 0.97000436 & -0.24308753 \\ (y_3, y_4) &= -0.46620368 & -0.43236573 \\ (y_5, y_6) &= 0.0 & 0.0 \\ (y_7, y_8) &= 0.93240737 & 0.86473146 \\ (y_9, y_{10}) &= -0.97000436 & 0.24308753 \\ (y_{11}, y_{12}) &= -0.46620368 & -0.43236573 \end{aligned} \quad (2.2)$$

Here, $y_{1+4(i-1)}$ and $y_{2+4(i-1)}$ are the initial coordinates and $y_{3+4(i-1)}$ and $y_{4+4(i-1)}$ the initial velocities for the objects $i = 1, 2$ and 3 . Try to integrate with a step size h between 0.01 and 0.001 and plot the result. (10 points)

- (b) Now consider a different problem. Choose the masses of the three bodies to be $m_1 = 5, m_2 = 4$ and $m_3 = 3$, and place them at the corners of a right triangle (one angle is 90°) with edge lengths of $\ell_1 = 5, \ell_2 = 4$ and $\ell_3 = 3$, such that m_1 is opposite to the edge ℓ_1 , m_2 opposite to ℓ_2 , and m_3 opposite to ℓ_3 . Set the initial velocities to zero. This is the Meissel-Burrau problem. We recommend to place the origin of your coordinate system into the center of mass of the system.

Use the Runge-Kutta-4 integrator to follow the time evolution of the system until it dissolves. Record the points in time when two bodies have minimum separation and store this data in a file. Investigate the behavior of the system for different integration steps h , starting with $h = 0.1$. How small does h need to be in order to obtain reliable estimates for the time of the first five closest encounters. Plot for different step sizes h : (i) the trajectories of the three bodies in the orbital plane, (ii) the mutual distances of the three bodies in logarithmic scaling as well as (iii) the error of the total energy of the system in logarithmic scaling as function of time (linear). (10 points)

- (c) **Voluntary problem for further study:** Use the same initial configuration as in (b), but now add an initial velocity of $v = 0.1$ to the most massive particle ($m_1 = 5$) in the direction towards the body $m_2 = 4$. Study again the trajectories of the three bodies for the same values of h as before.

Can you find other interesting variations of the initial conditions to find “entertaining” solutions? The best may be presented in the tutorial or the lecture.