

CS 724 Assignment Two: Game Playing & Propositional Logic

Dues on Feb 25, 11:59pm

Total points: 80.

1. [25] This problem exercises the basic concepts of game playing, using tic-tac-toe (noughts and crosses) as an example. We define X_n as the number of rows, columns, or diagonals with exactly n X's and no O's. Similarly, O_n is the number of rows, columns, or diagonals with just n O's. The utility function assigns +1 to any position with $X_3=1$ and -1 to any position with $O_3=1$. All other terminal positions have utility 0.

For non-terminal position, we use a linear evaluation function defined as:

$$Eval(s) = 3X_2(s) + X_1(s) - (3O_2(s) + O_1(s)).$$

- a. Show the whole game tree starting from an empty board down to depth 2 (i.e., one X and one O on the board), taking symmetry into account.
 - b. Mark on your tree the evaluations of all the positions at depth 2.
 - c. Using the minimax algorithm, mark on your tree the backed-up values for the positions at depths 1 and 0, and use those values to choose the best starting move.
2. [15] Represent each of these sentences in propositional logic.
- a) If you take Chemistry, you cannot take Physics.
 - b) You must take either Chinese or French but not both.
 - c) You must take at least two of CSci724, CSci617, and CSci764.

3. [20] For each of the following, use truth tables to show whether it is *valid*, *satisfiable*, or *unsatisfiable*.

- a. $(P \rightarrow Q) \wedge (P \rightarrow \neg Q)$
- b. $(P \rightarrow Q) \wedge (P \rightarrow R) \wedge (\neg Q \wedge \neg R) \wedge P$
- c. $(P \rightarrow Q) \vee (Q \rightarrow P)$
- d. $((P \rightarrow Q) \rightarrow (Q \rightarrow R)) \leftrightarrow (P \rightarrow R)$

4. [10]

Given: 1) $P \wedge Q \rightarrow R$

2) $\neg X \vee \neg Y \vee R$

3) $Q \wedge Y \rightarrow W$

4) Q

5) $\neg(\neg X)$

6) Y

prove: 7) $R \wedge W$

5. [10] Put the following in *clausal form (CNF)*.

- a. $(P \rightarrow Q) \wedge (X \rightarrow Y)$
- b. $(P \wedge Q \rightarrow Z) \vee (X \wedge Y)$