Question 1

I. Compute P(A = true and B = true and C = true and D = true)

I. $P(A,B,C,D) = 1 - \sum_{15}^{1} P = 1 - 0.84 = 0.16$

II. P(A = false | B = true and C = true and D = False).

$P(\neg B \mid B, C, \neg D) = \frac{P(\neg A, B, C, \neg D}{P(B, C, \neg D)} = \frac{0.04}{P(A,B,C,\neg D) + P(\neg A, B, C, \neg D)} = 1/3$

$= 0.15 + 0.05 + 0.03 + 0.16 = 0.39$

IV. P(B = false) add hidden variable give us 8 combination $P(\neg B)$

$P(A, \neg B, C, D) + P(A, \neg C, \neg D, \neg B) + P(\neg A, \neg B, \neg C, \neg D) +$

$P(\neg A, \neg B, C, D) + P(A, \neg B, \neg C, D) + P(A, \neg B, C, \neg D) +$

$P(\neg A + \neg B, C, \neg D) + P(\neg A, \neg B, \neg C, D) = 0.03 + 0.20 + 0.01$

$0.01 + 0.15 + 0.05 + 0.01 + 0.1 = 0.56$

Question 2

Virable combination Passible

i) 2^{26} or $2^{26}-1$ (full join) $= 2(26C1 + 26C2 + \dots + 26C25) =$

$= 134,217,724$ rows $* 26 = 3,489,660,824$

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ii) $26^{(2-1)} = 26$ (independent)

variable combination Passible $= (26C1 + 26C2 + \dots + 26C25) =$

$= 67,108,862$ rows $* 26 = 1,744,830,412$

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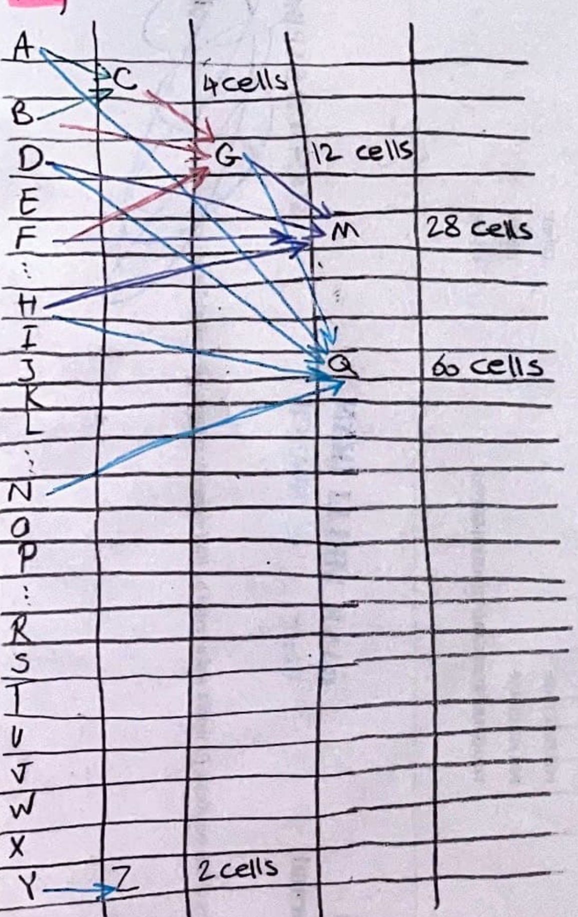
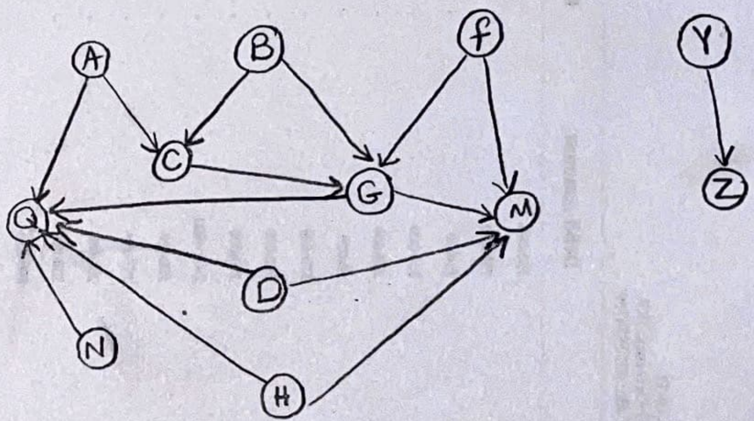
iii)





For example Q has 5 parent.
So The total number of cells required would be $1 * 2(5C1 + \dots + 5C5) = 60$

1 column is needed.

Similarly for others...

$C \to 2$ parent $\to 4$

$Q \to 5$ Parent $\to 60$

$M \to 4$ Parent $\to 28$

$Z \to 1$ parent $\to 2$

$G \to 3$ Parent $\to 12$

No Parent $2^0 = 1$

M = 1
A = 1
B = 1
F = 1
N = 1
Y = 1
D = 1

Total number of cells needed:

21 (for 21 parent link) + 4 (C) + 12 (G) + 28 (M) + 60 (Q) + 2 (Z) = 127 cells

Question 3

i) $P(A = \text{true and } B = \text{false and } C = \text{true and } D = \text{false})$

$$P(A, \sim B, C, \sim D) = P(A) \times P(\sim B \mid A) * P(C) * P(\sim D \mid \sim B, C) =$$

$$0.4 \times (1 - 0.03) \times 0.7 \times (1 - 0.3) = 0.1372$$

ii) $P(D = \text{True} \mid A = \text{false and } B = \text{True and } C = \text{false})$

$$P(D \mid \sim A, B, \sim C) = \frac{P(D, \sim A, B, \sim C)}{P(\sim A, B, \sim C)} =$$

$$P(D \mid B, \sim C) \cdot P(\sim A) \cdot P(B \mid \sim A) \cdot P(\sim C) / P(\sim A, B, \sim C) =$$

$$0.5 \times 0.6 \times 0.9 \times 0.3 / \underbrace{P(\sim A, B, \sim C)}_{X = 0.162} = 0.5$$

$$X: \text{compute } P(\sim A, B, \sim C) = \overbrace{P(\sim A, B, \sim C, D)}^{(a)} + \overbrace{P(\sim A, B, \sim C, \sim D)}^{(b)}$$

$$(a) = P(\sim A, B, \sim C, D) = P(\sim A) \cdot P(B \mid \sim A) \cdot P(\sim C) \cdot P(D \mid B, \sim C)$$

$$= 0.6 * 0.9 * 0.3 * 0.5 = 0.081$$

$$(b) = P(\sim A, B, \sim C, \sim D) = P(\sim A) \cdot P(B \mid \sim A) \cdot P(\sim C) \cdot P(\sim D \mid B, \sim C) =$$

$$= 0.6 * 0.9 * 0.3 * (1 - 0.5) = 0.81$$

$$P(\sim A, B, \sim C) = 0.162$$

$$\Rightarrow P(D \mid B, \sim C) = \frac{0.81}{0.162} = 0.5$$

iii) $P(B = \text{false}) \rightarrow P(\neg B) = 0.34$

$$P(\neg B) = (A, \neg B, C, D) + P(A, \neg B, \neg C, D) + P(A, \neg B, C, \neg D)$$
$$+ P(A, \neg B, \neg C, \neg D) + P(\neg A, \neg B, C, D) + P(\neg A, \neg B, \neg C, D) +$$
$$P(\neg A, \neg B, C, \neg D) + P(\neg A, \neg B, \neg C, \neg D) =$$

$$P(A) \cdot P(\neg B|A) \cdot P(C) \cdot P(D|\neg B, C) +$$
$$P(A) \cdot P(\neg B|A) \cdot P(\neg C) \cdot P(D|\neg B, \neg C) +$$
$$P(A) \cdot P(\neg B|A) \cdot P(C) \cdot P(\neg D|\neg B, C) +$$
$$P(A) \cdot P(\neg B|A) \cdot P(\neg C) \cdot P(\neg D|\neg B, \neg C) +$$
$$P(\neg A) \cdot P(\neg B|\neg A) \cdot P(C) \cdot P(D|\neg B, C) +$$
$$P(\neg A) \cdot P(\neg B|\neg A) \cdot P(\neg C) \cdot P(D|\neg B, \neg C) +$$
$$P(\neg A) \cdot P(\neg B|\neg A) \cdot P(C) \cdot P(\neg D|\neg B, C) +$$
$$P(\neg A) \cdot P(\neg B|\neg A) \cdot P(\neg C) \cdot P(\neg D|\neg B, \neg C) =$$

$$0.4 \times 0.7 [0.7 \times 0.3 + 0.3 \times 0.8 + 0.7 \times 0.7 + 0.3 \times 0.2] +$$

$$0.6 \times 0.1 [0.7 \times 0.3 + 0.3 \times 0.8 + 0.7 \times 0.7 + 0.3 \times 0.2] = 0.34$$