CS 724 Assignment Four: Ontology and Uncertainty

Dues on April 8, 11:59pm.
Total points: 70.

1. [25] Full Joint Probability Distributions

Consider this *full joint probability distribution* involving four Boolean-valued random variables (A-D):

A	B	C	D	Prob
F	F	F	F	0.10
F	F	F	T	0.01
F	F	T	F	0.05
F	F	T	T	0.15
F	T	F	F	0.02
F	T	F	T	0.03
F	T	T	F	0.04
F	T	T	T	0.05
T	F	F	F	0.20
T	F	F	T	0.01
T	F	T	F	0.01
T	F	T	T	0.03
T	T	F	F	0.02
T	T	F	T	0.04
T	T	T	F	0.08
T	T	T	T	?

I. Compute P(A = true and B = true and C = true and D = true).
II. Compute P(A = false | B = true and C = true and D = false).
IV. Compute P(B = false).

2. [20] Representing Probability Distributions

Assume the task at hand involves 26 Boolean-valued random variables, which we'll name *A* through *Z*.

i. How big of a table (number of memory cells) would be need to explicitly represent the *full, joint probability distribution* over every possible combination of our 26 Boolean-valued random variables?

ii. How big of a table would we need if we make the *independence* assumption that each variable is independent of all other variables?

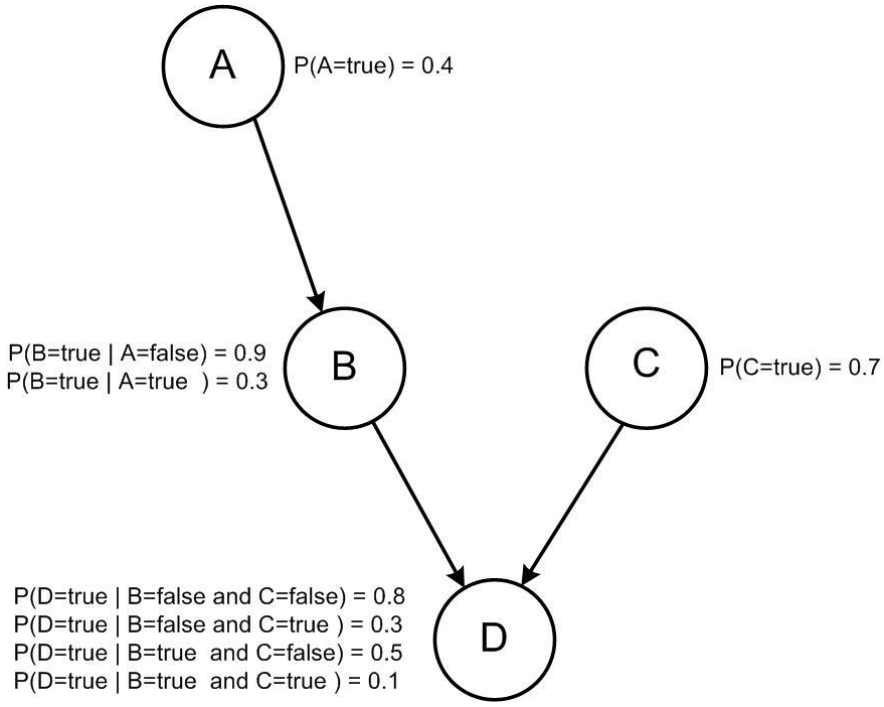
iii. This time assume that we have a Bayesian network where the following nodes have the parents listed (if a node is not listed, then it has no parents in the Bayesian network):
   - *C* has parents *A* and *B*
   - *G* has parents *B*, *C*, and *F*
   - *M* has parents *D*, *F*, *G*, and *H*
   - *Q* has parents *A*, *D*, *G*, *H*, and *N*
   - *Z* has parent *Y*

Draw this Bayesian network and next to each node report how many cells are needed to store that node's *conditional probability table* (CPT). Explain your answer (you need not explain your answer within your drawing of the Bayesian network - it is fine to place your explanation below your drawing). Finally, report the total number of cells needed to store this Bayesian network (be sure to count the memory needed to store the *parent* links - assume each link uses the same number of bytes as one cell in a probability table, ie, each parent link counts 1).

Hint: when you answer all three parts of this question, remember that if you store the value for, say, *Prob(C=true | A=true and B=true)* you do *not* need to also store the value for *Prob(C=false | A=true and B=true)* since we know that these two probabilities sum to one.

3. [25] Bayesian Networks

Consider the Bayesian network drawn below.



Show your work for the following calculations.

i. Compute *P(A = true and B = false and C = true and D = false).*
ii. Compute *P(D = true | A = false and B = true and C = false).*
iii. Compute *P(B = false).*