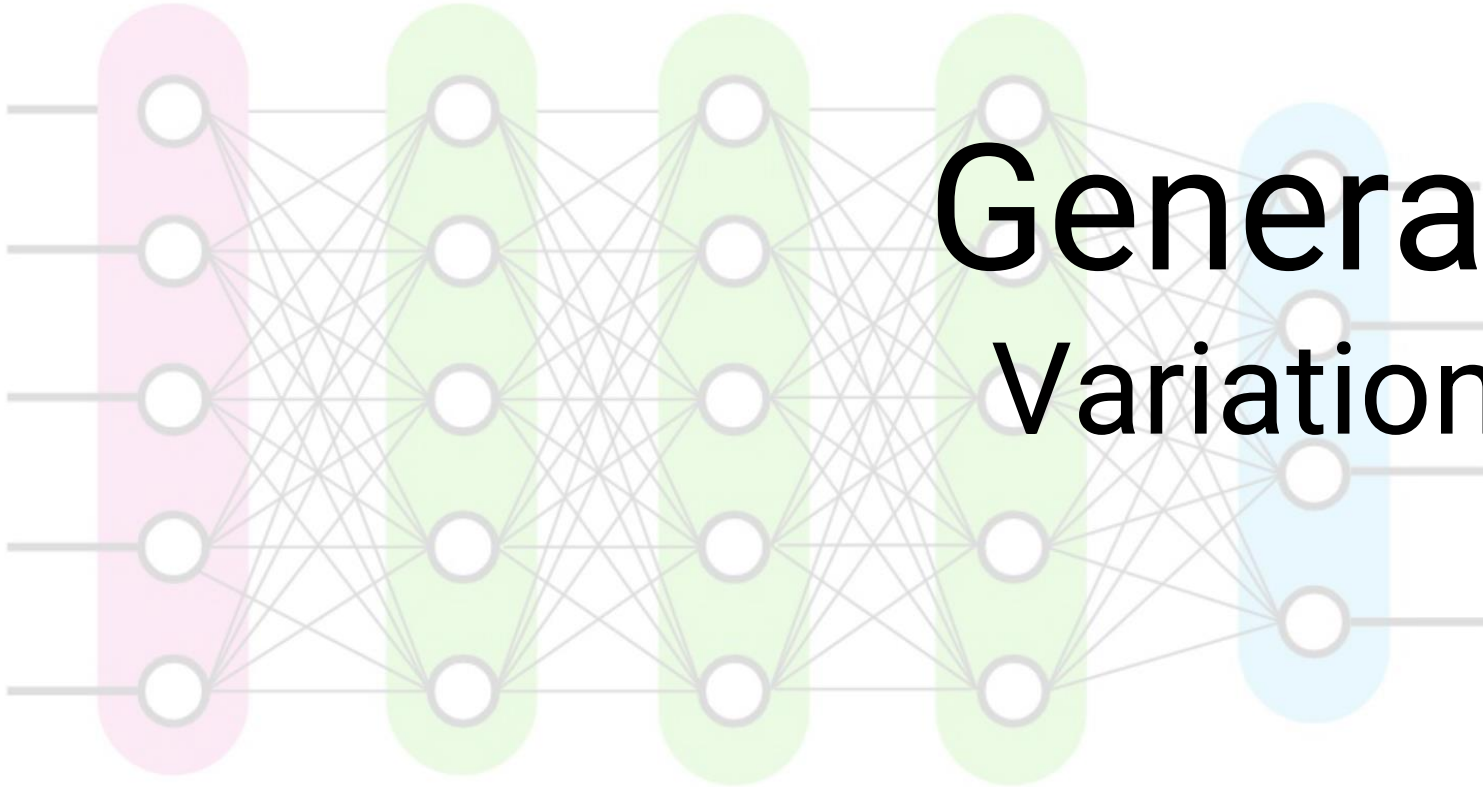


# Class core values

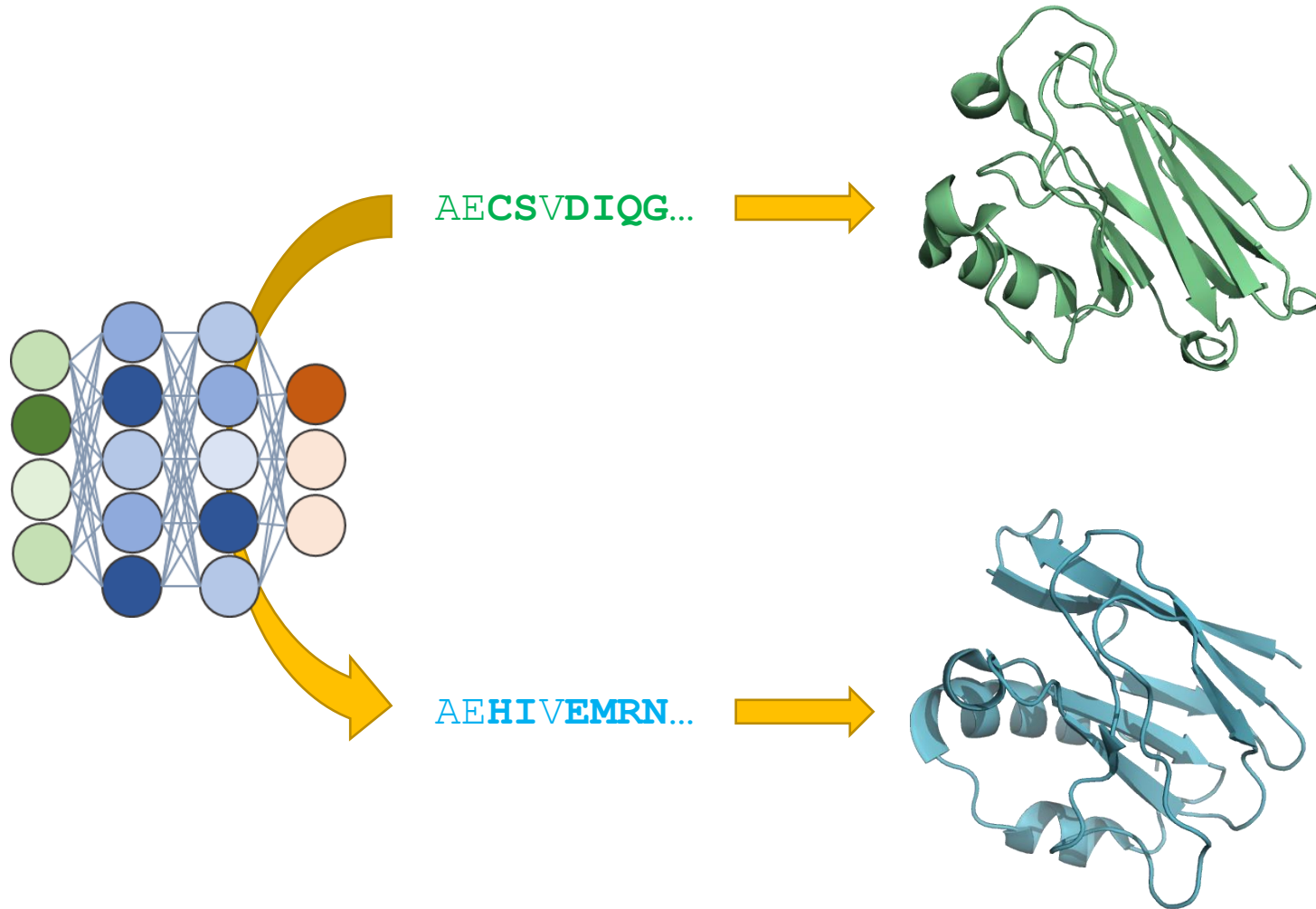
1. Be **respectful** to yourself and others
2. Be **confident** and believe in yourself
3. Always do your **best**
4. Be **cooperative**
5. Be **creative**
6. Have **fun**
7. Be **patient** with yourself while you learn
8. Don't be shy to **ask "stupid" questions**
9. Be **inclusive** and **accepting**

Week 8, Lecture 1

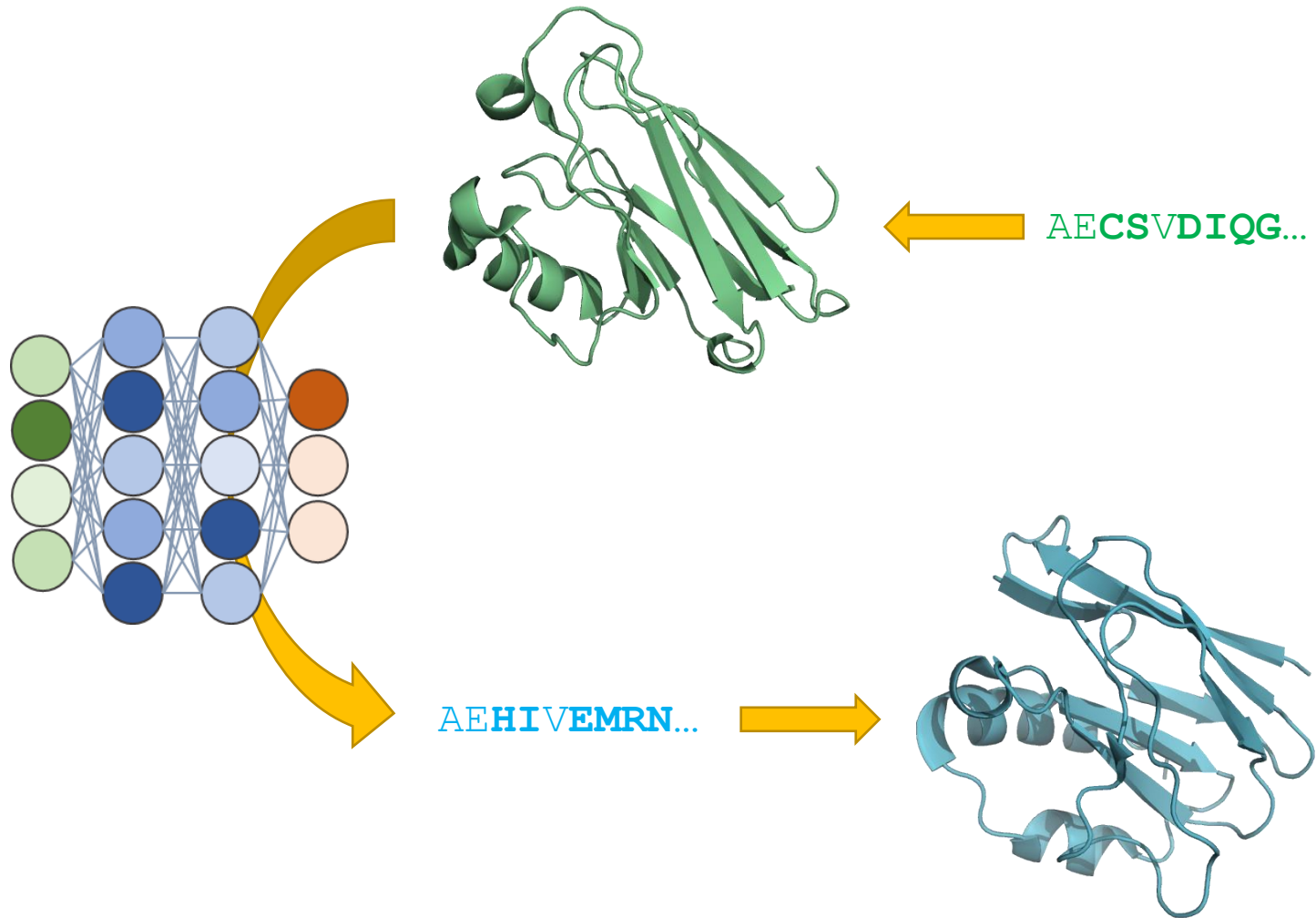
# Generative Models: Variational Autoencoders



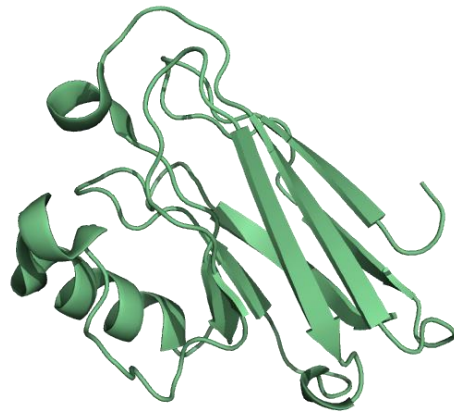
# Generating proteins



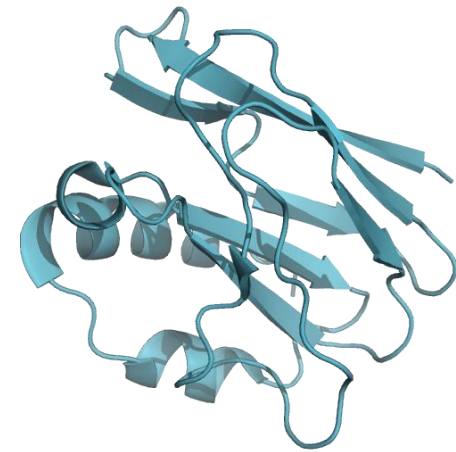
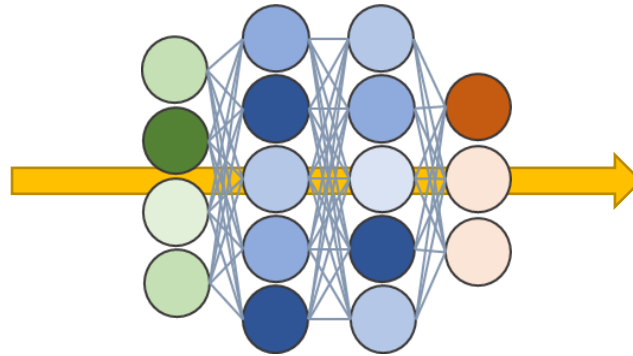
# Generating proteins



# Uses for a generative protein model?

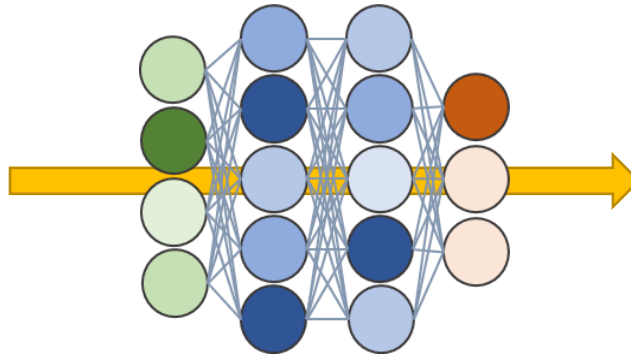


AECSVDIQG...



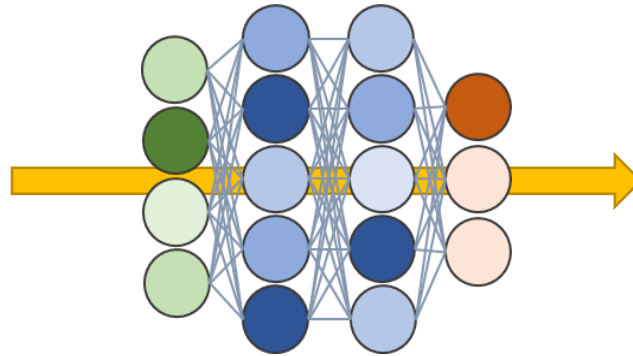
AEHIVEMRN...

# Generative models



# Generative models

The Mona Lisa is a half-length portrait painting by Italian artist Leonardo da Vinci.



Girl with a Pearl Earring is an oil painting by Dutch Golden Age painter Johannes Vermeer.



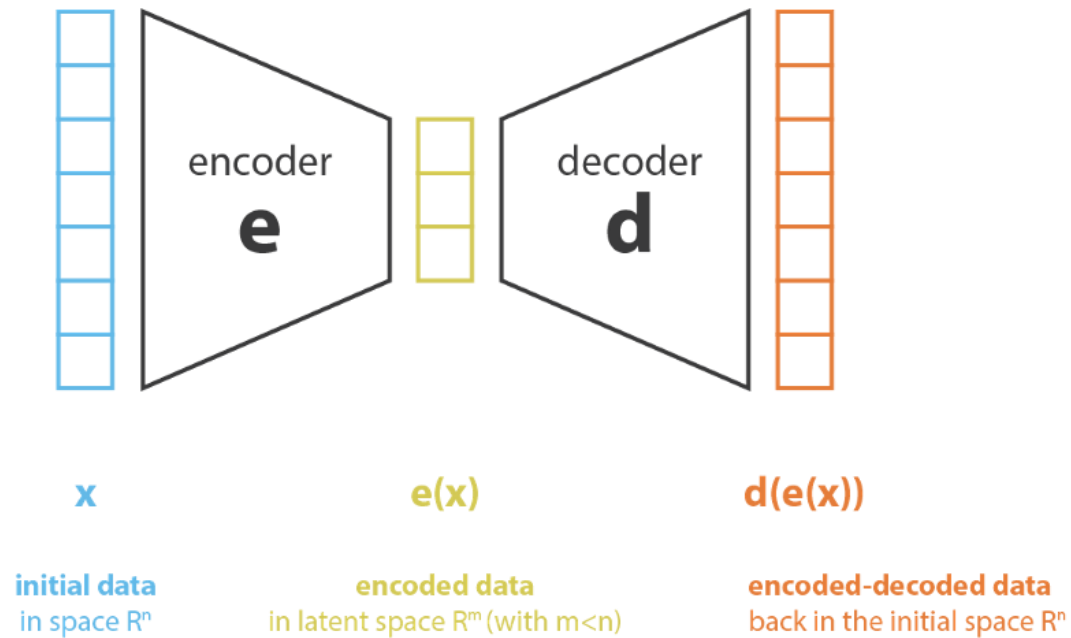
# Variational autoencoder (VAE)



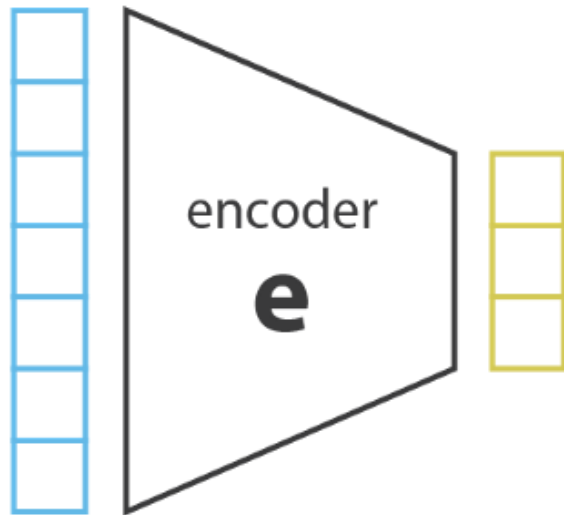
Image from [vae-celebA](https://github.com/alexmjberg/vae-celebA)



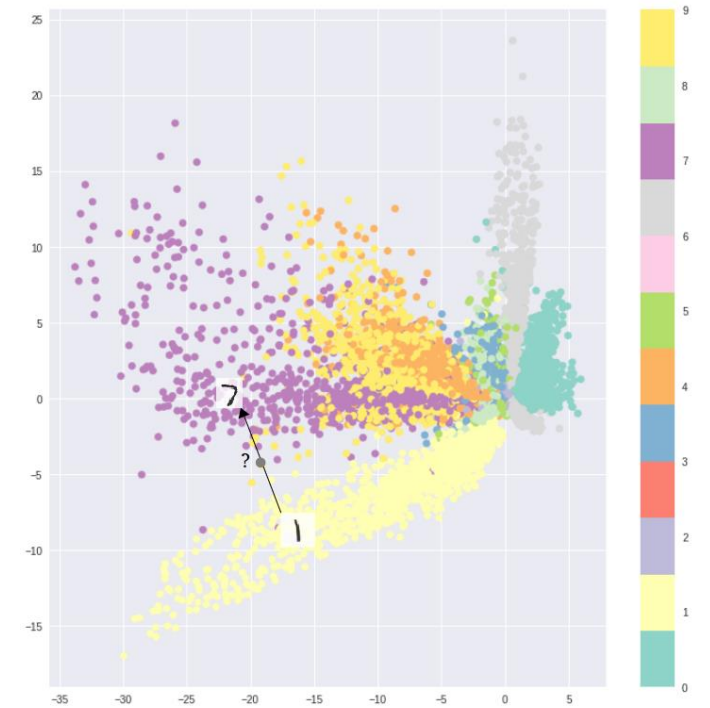
# Autoencoder



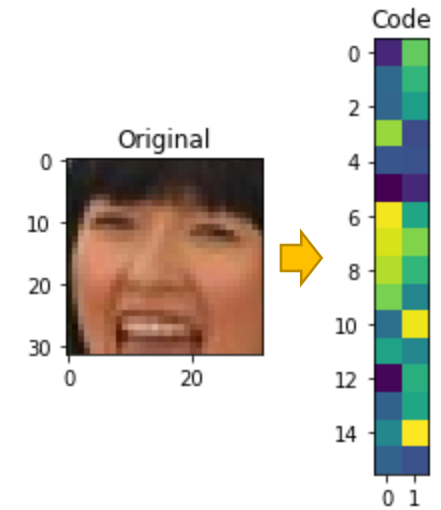
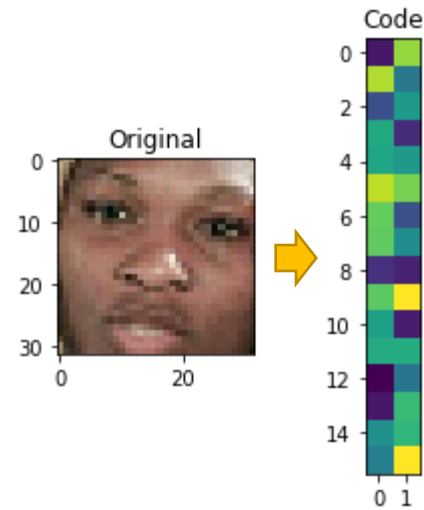
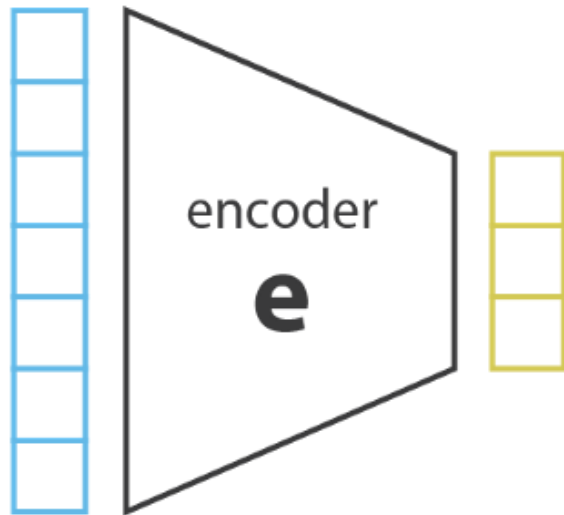
# Encoder



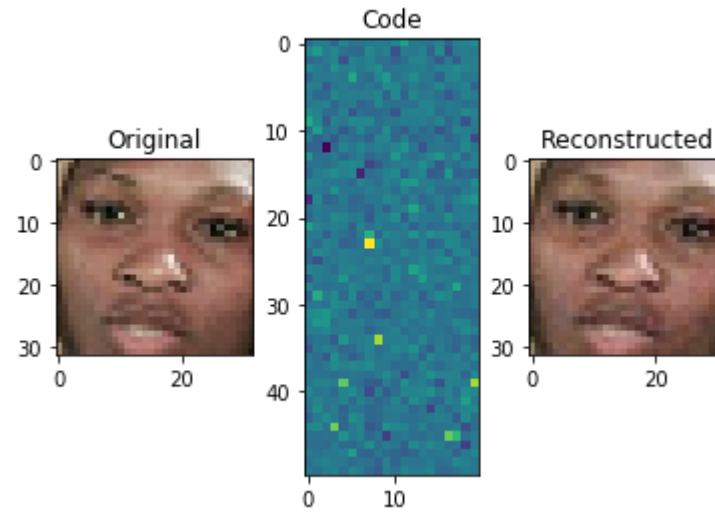
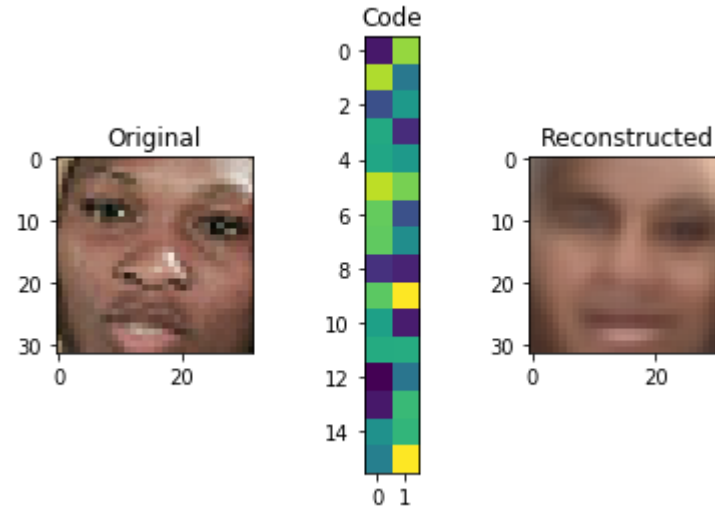
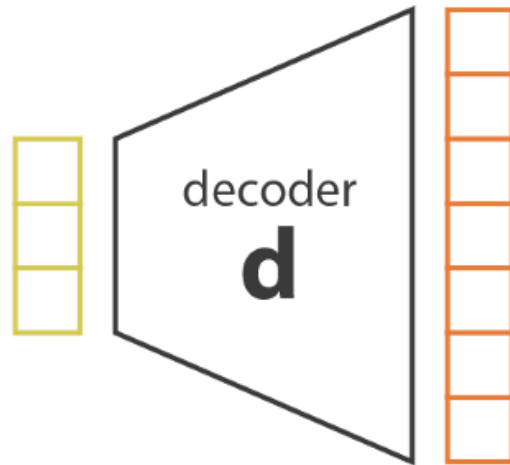
0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0  
1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1  
2 2 2 2 2 2 2 2 2 2 2 2 2 2 2 2  
3 3 3 3 3 3 3 3 3 3 3 3 3 3 3 3  
4 4 4 4 4 4 4 4 4 4 4 4 4 4 4 4  
5 5 5 5 5 5 5 5 5 5 5 5 5 5 5 5  
6 6 6 6 6 6 6 6 6 6 6 6 6 6 6 6  
7 7 7 7 7 7 7 7 7 7 7 7 7 7 7 7  
8 8 8 8 8 8 8 8 8 8 8 8 8 8 8 8  
9 9 9 9 9 9 9 9 9 9 9 9 9 9 9 9



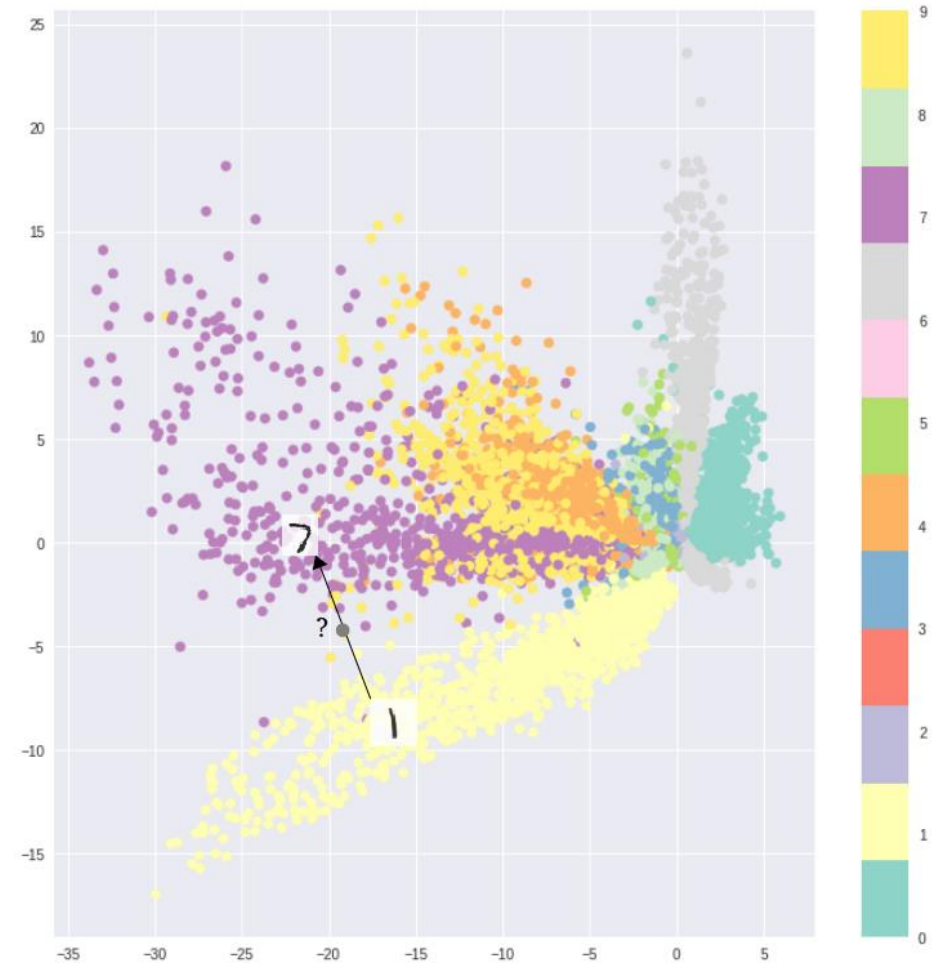
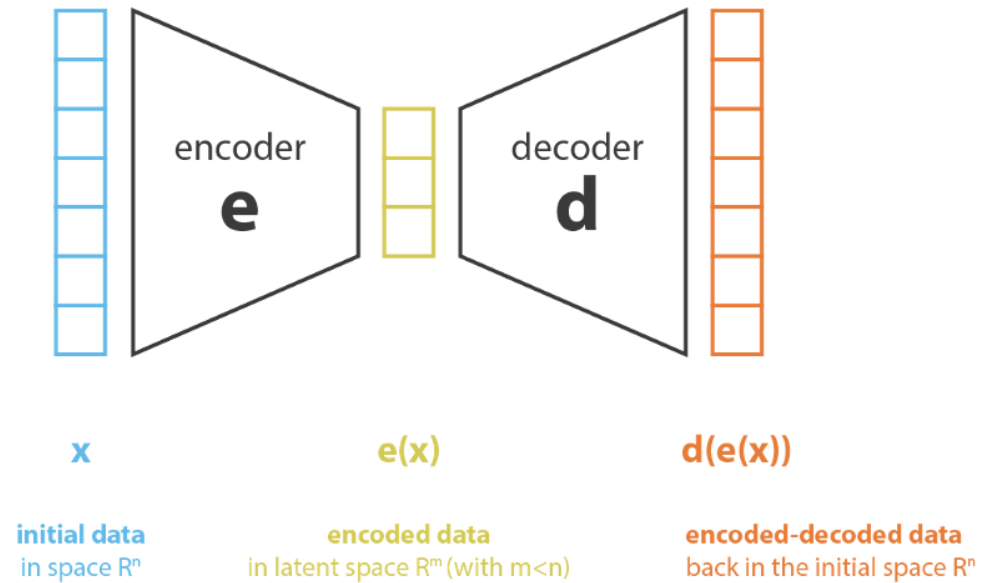
# Encoder



# Decoder



# Latent Space

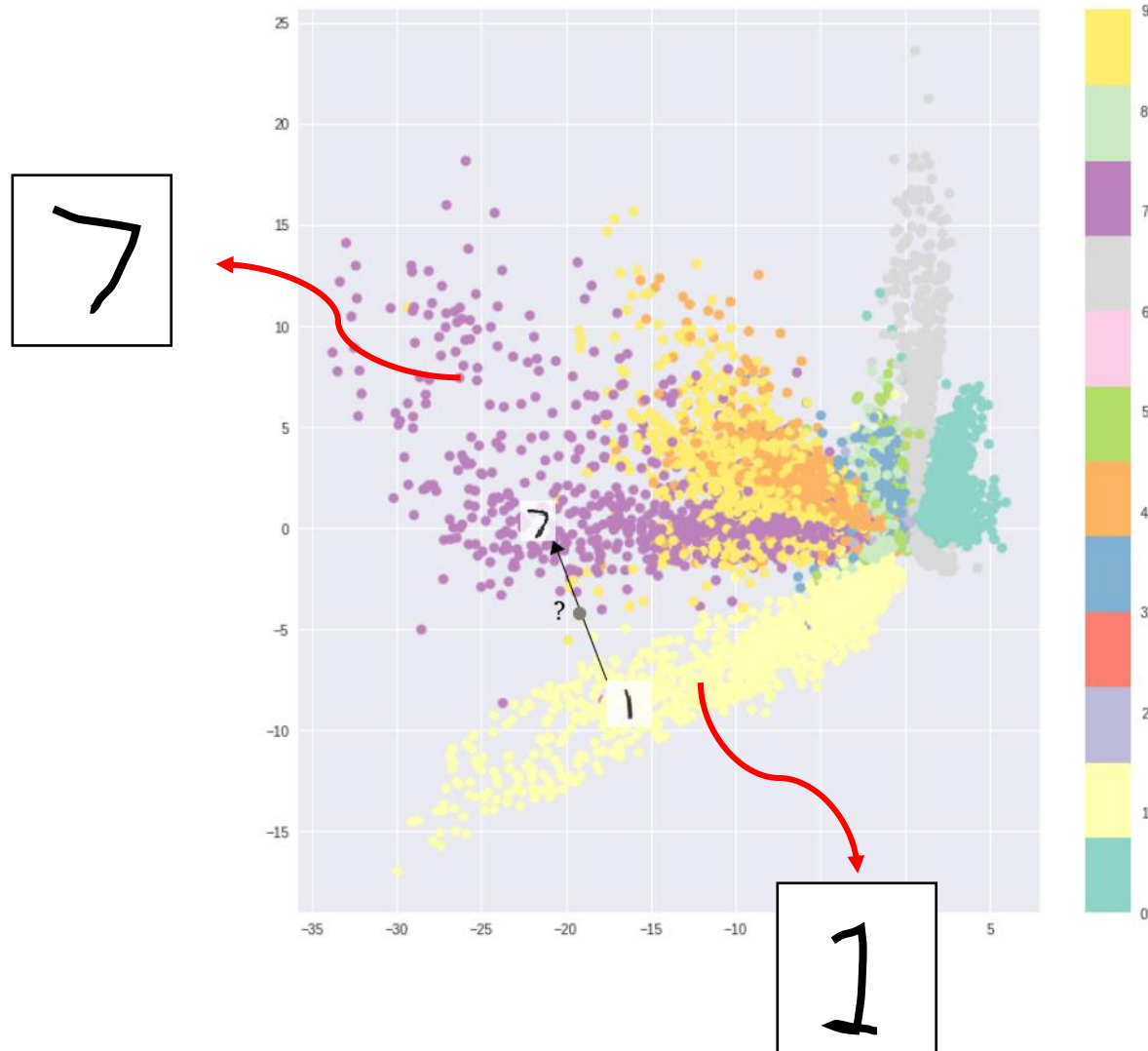


What would happen if we input embedded values from the latent space into the decoder?

Image from [TDS – Intuitively understanding VAEs](#)

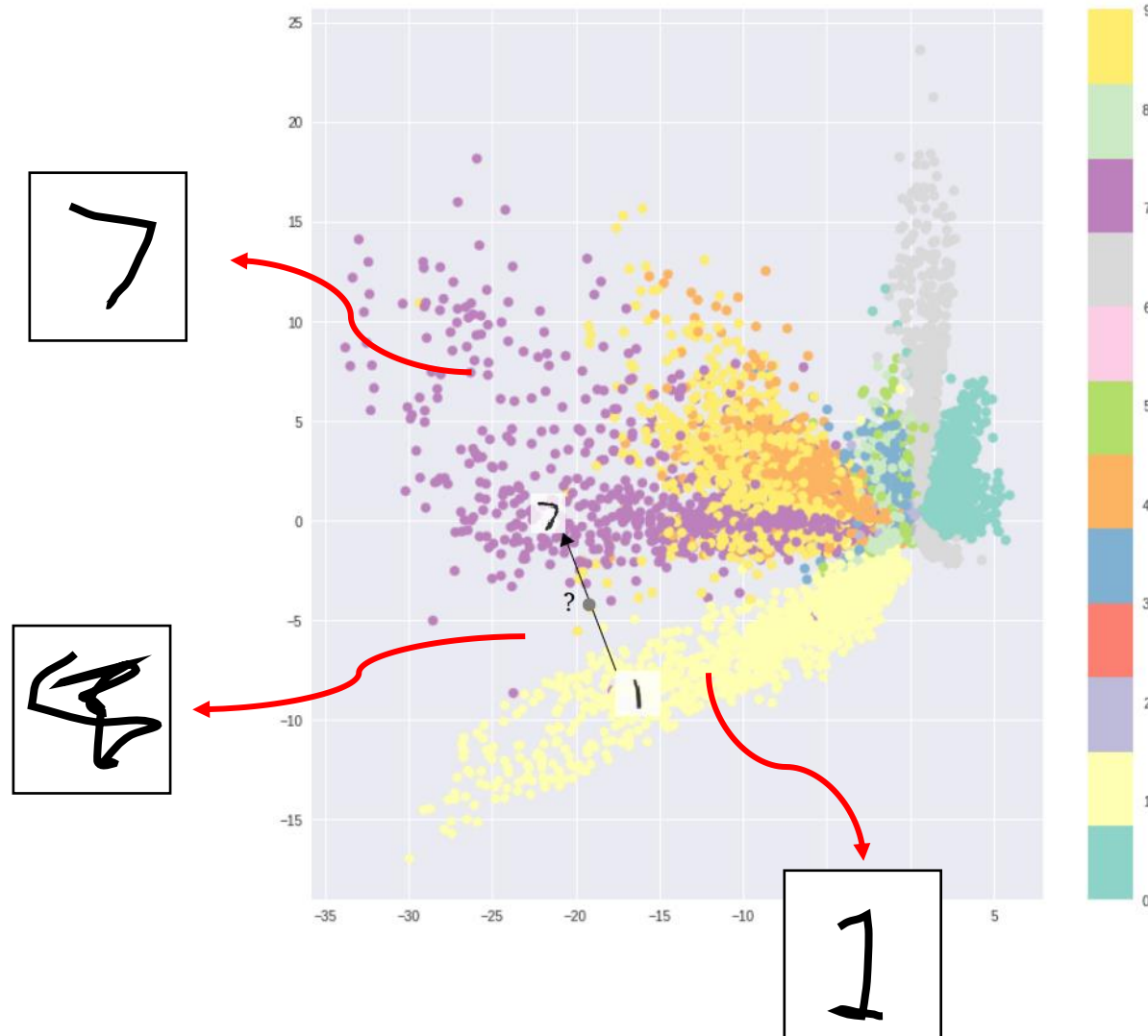


# Irregular latent space of autoencoders



What would happen if we input random values from the latent space into the decoder?

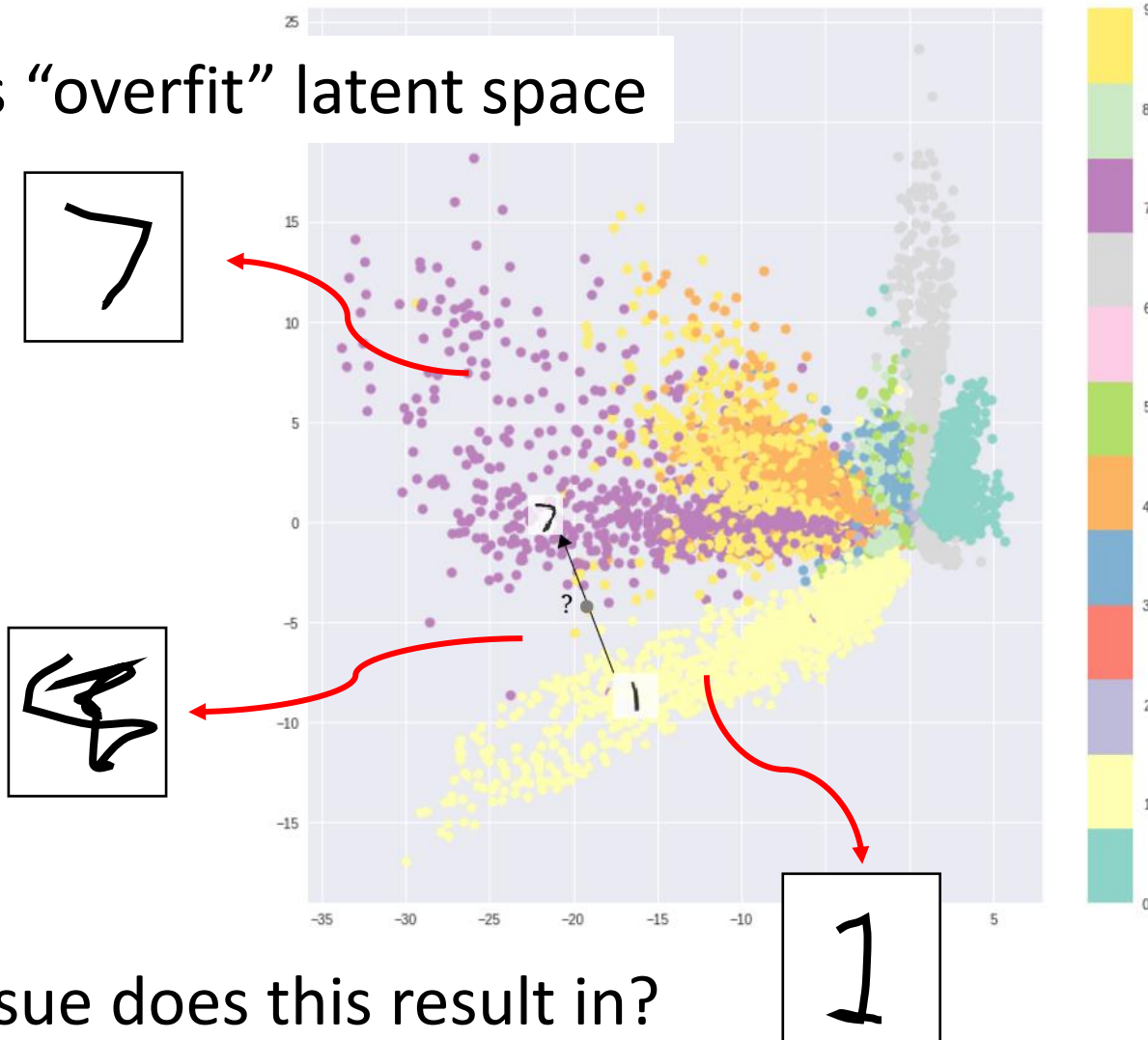
# Irregular latent space of autoencoders



Random values from the latent space decode to meaningless data

# Irregular latent space of autoencoders

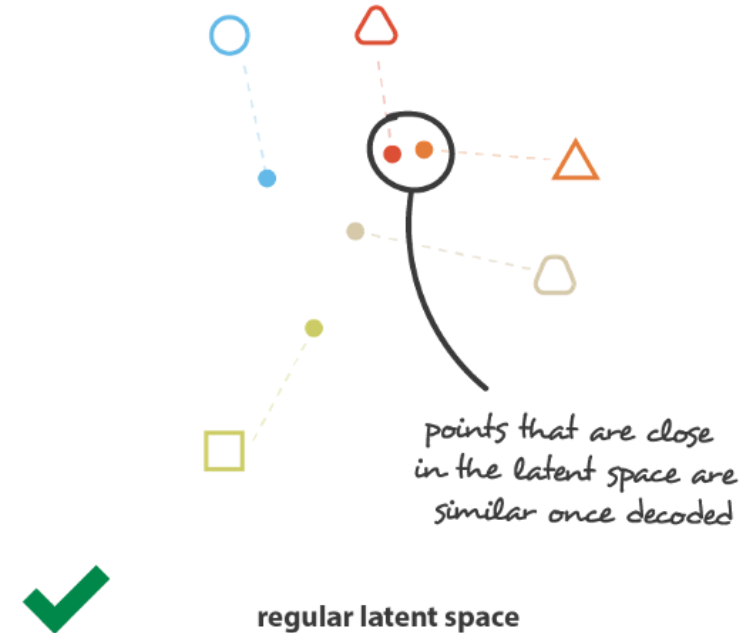
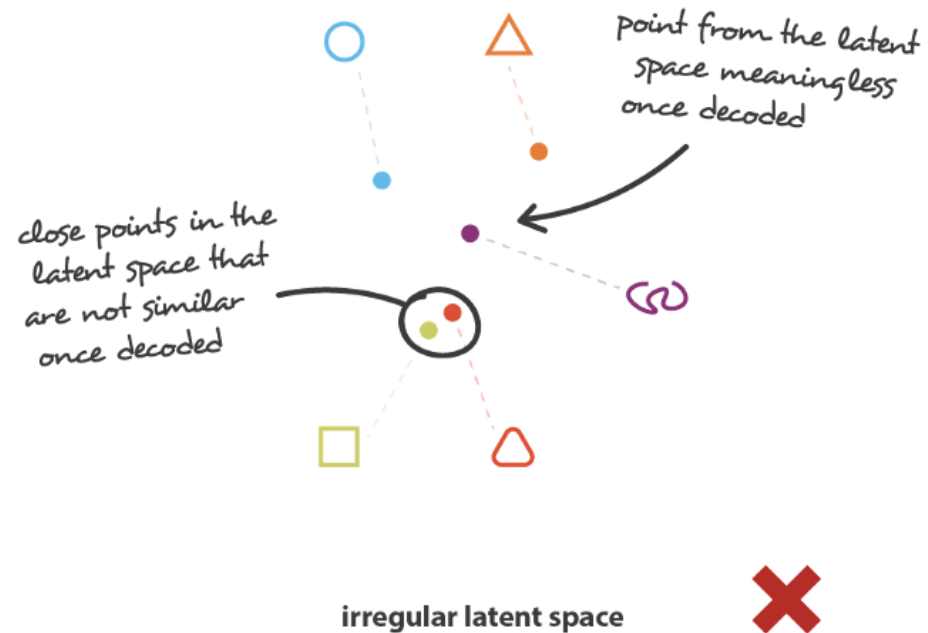
Autoencoders “overfit” latent space



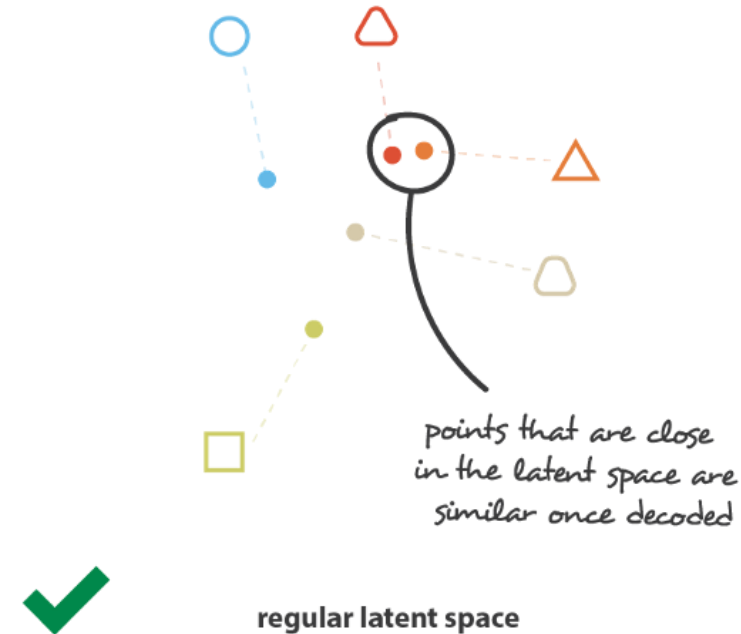
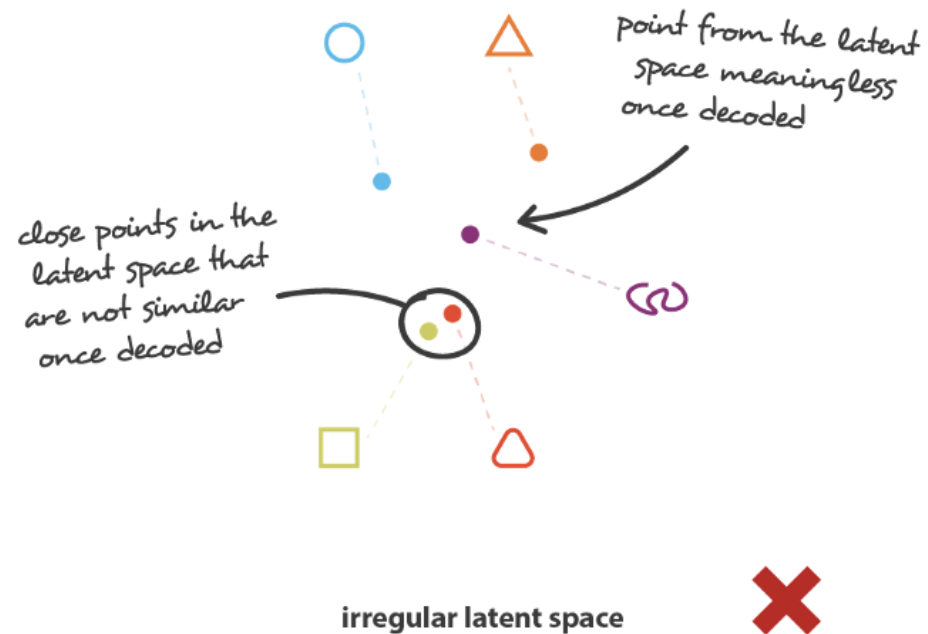
Random values from the latent space decode to meaningless data

What other issue does this result in?

# Irregular latent space of autoencoders



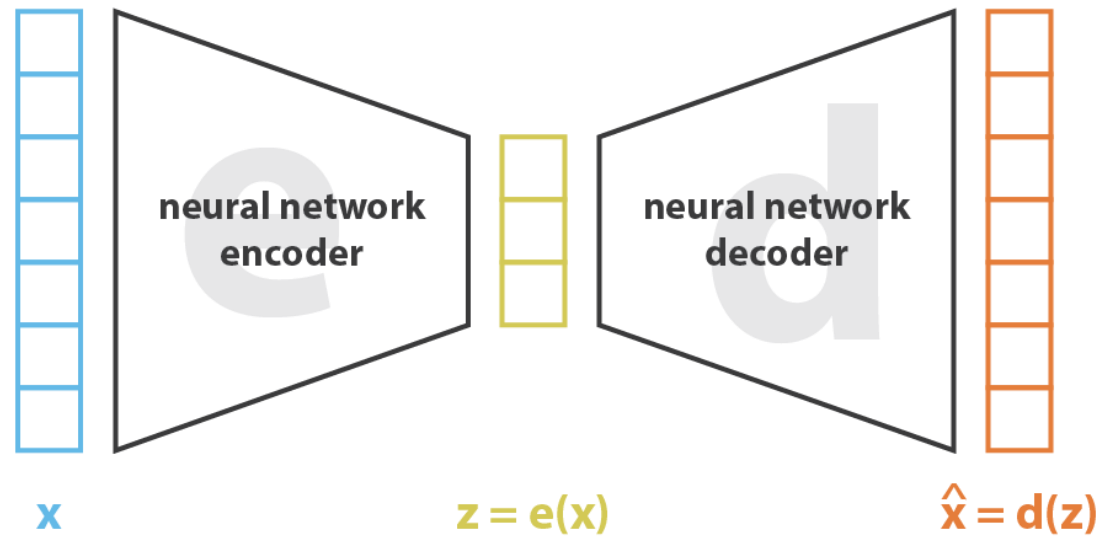
# Autoencoders cannot be used to generate new data



What needs to be changed to generate new data?



# Autoencoder loss function doesn't care about latent space

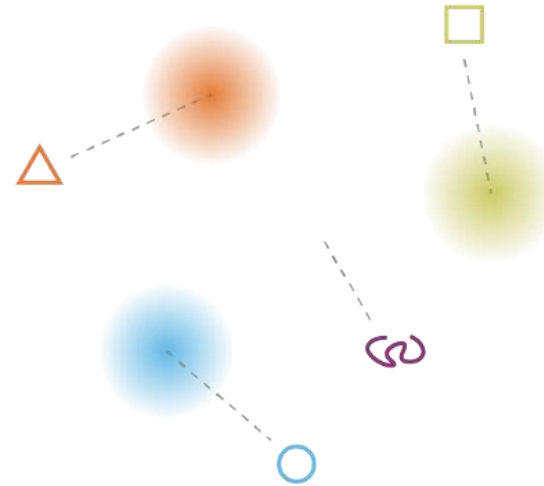
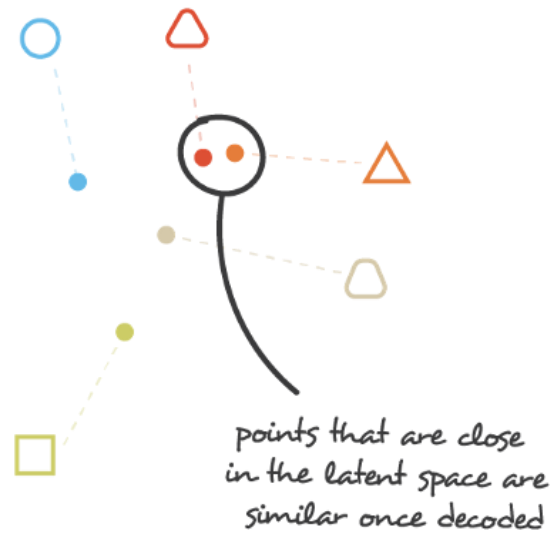


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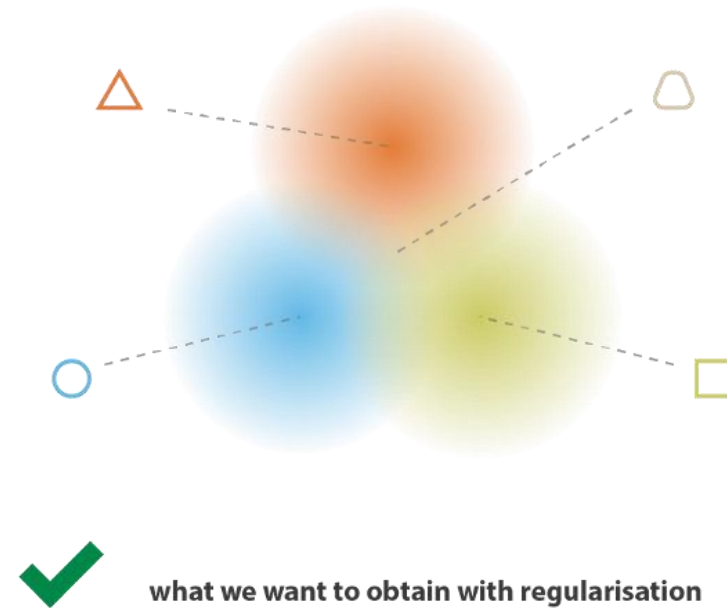
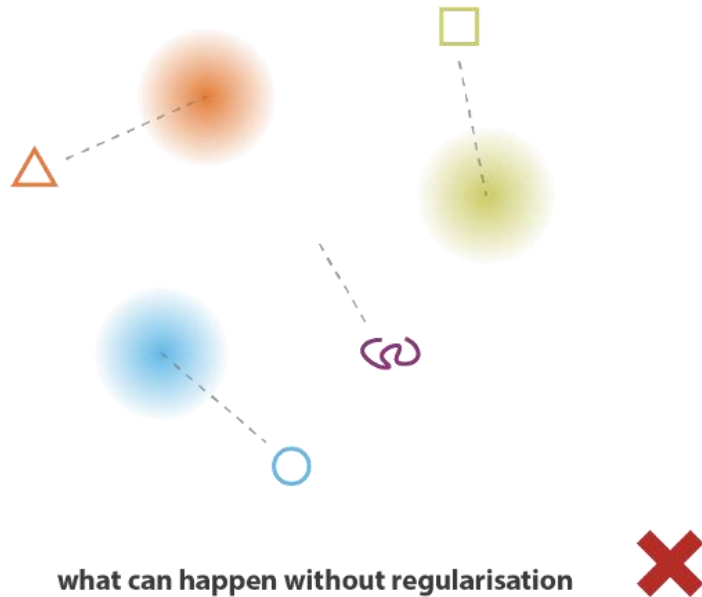
$$\text{loss} = || \textcolor{blue}{x} - \textcolor{orange}{\hat{x}} ||^2$$

Need to make some adjustments to the loss function

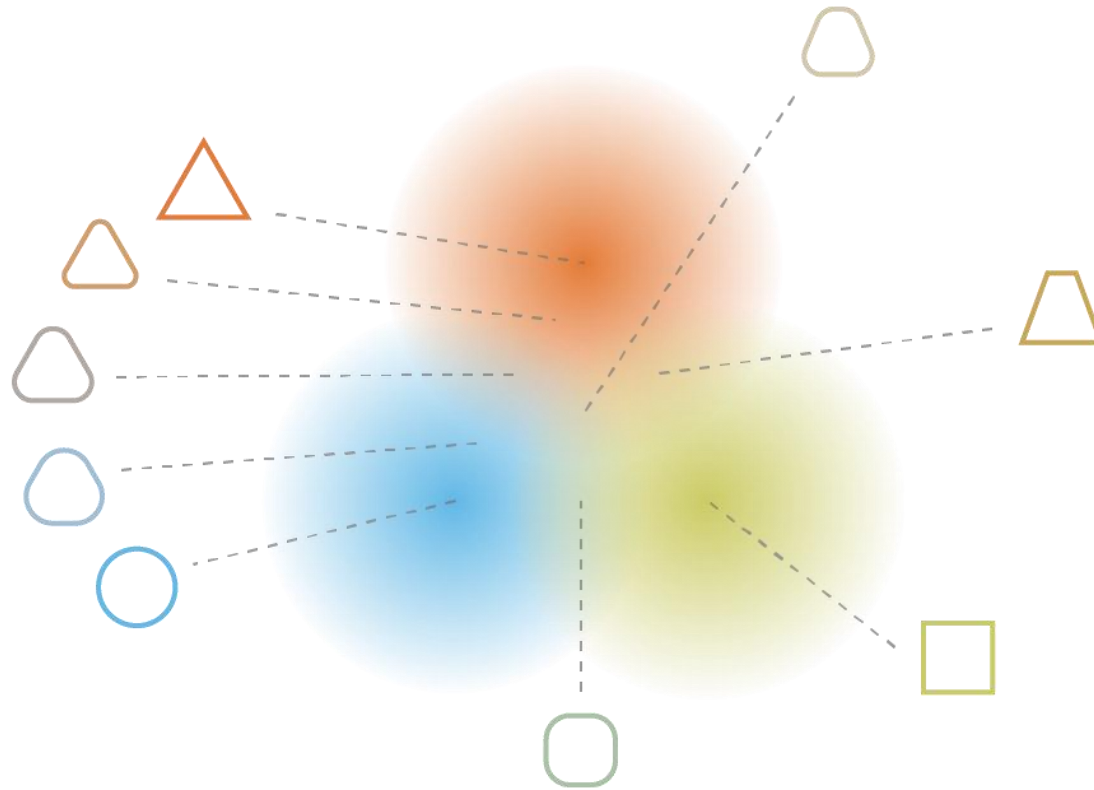
# Data is encoded as a distribution instead of single point



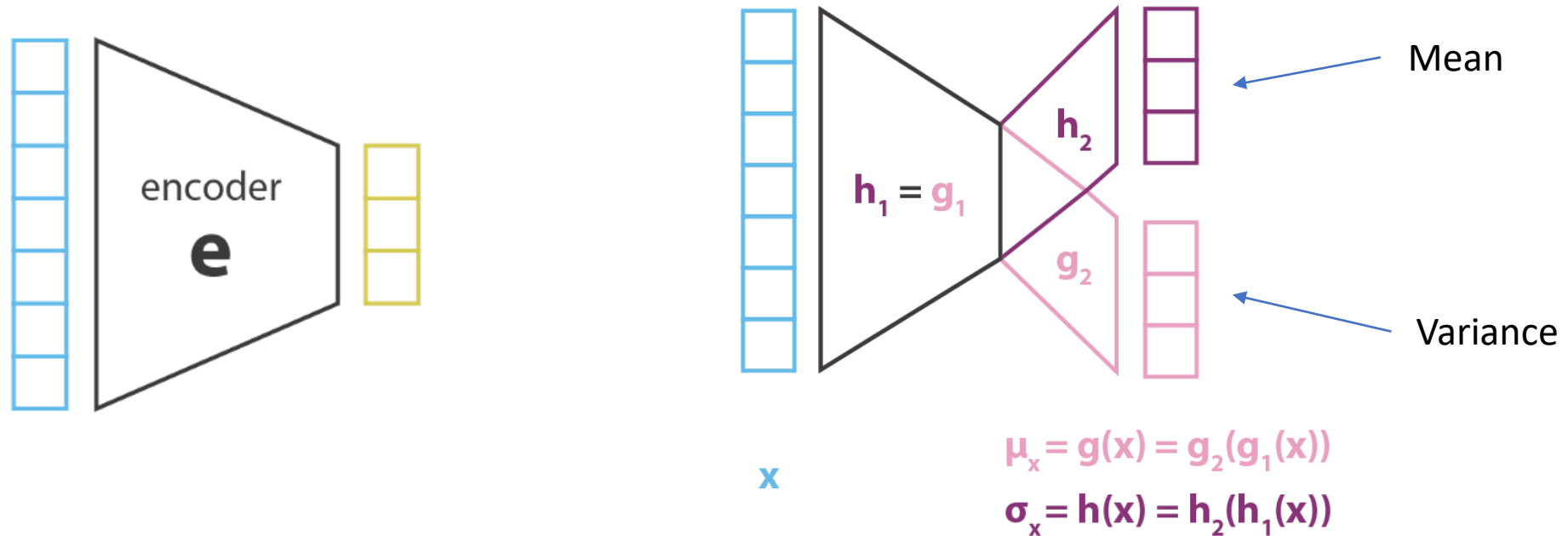
# Regularization for latent space (mean and variance)



# Regularization for latent space (mean and variance)



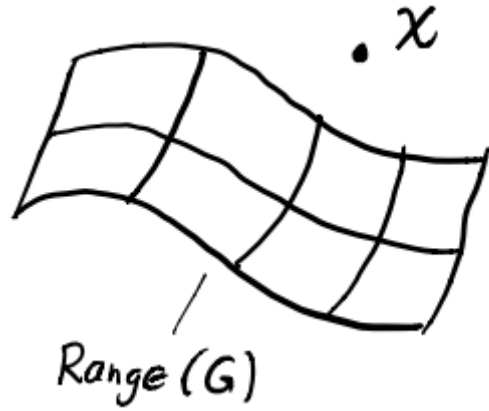
# Regularization for latent space (mean and variance)



Loss function and backpropagation?



# Variational inference

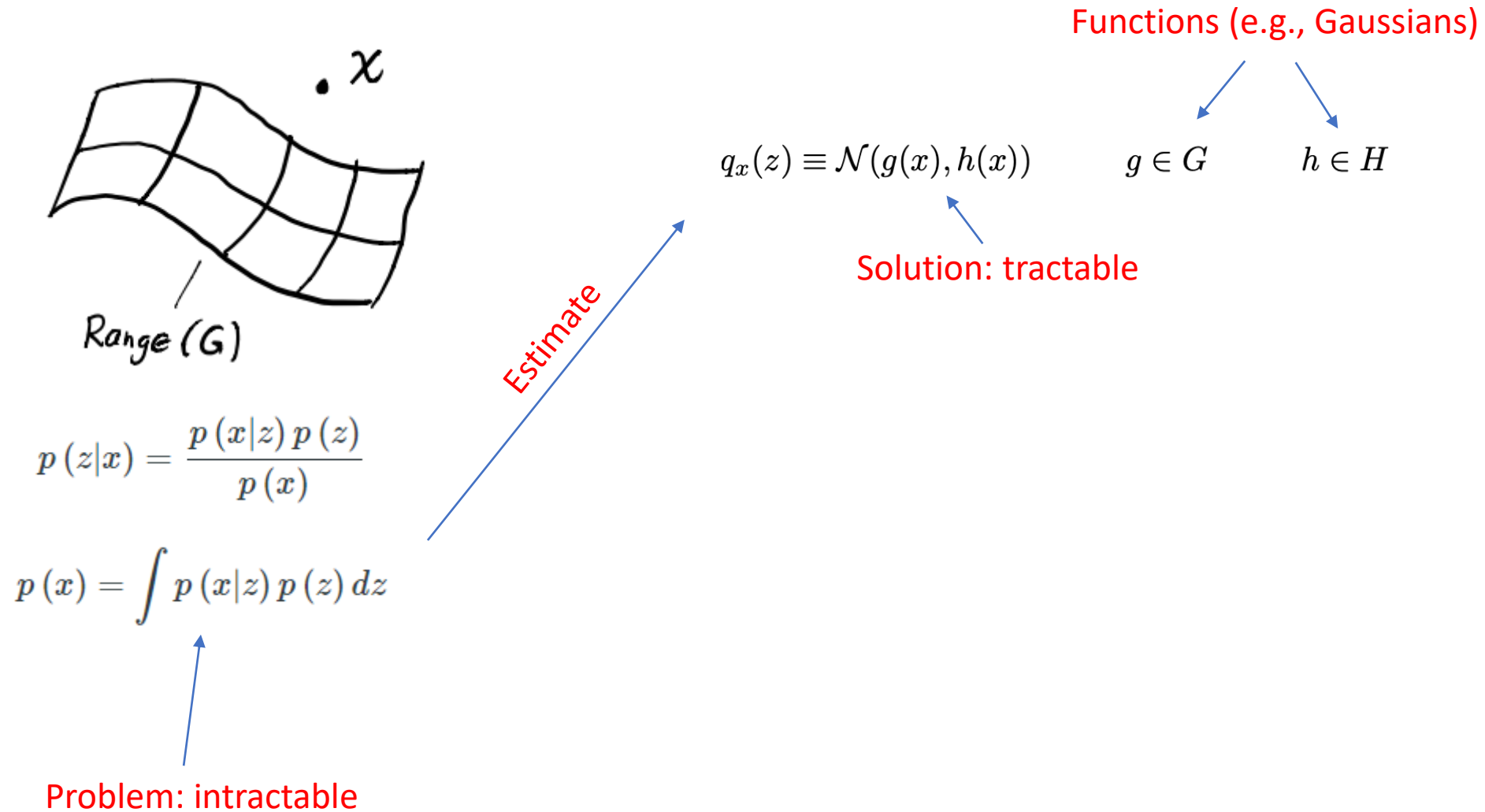


$$p(z|x) = \frac{p(x|z)p(z)}{p(x)}$$

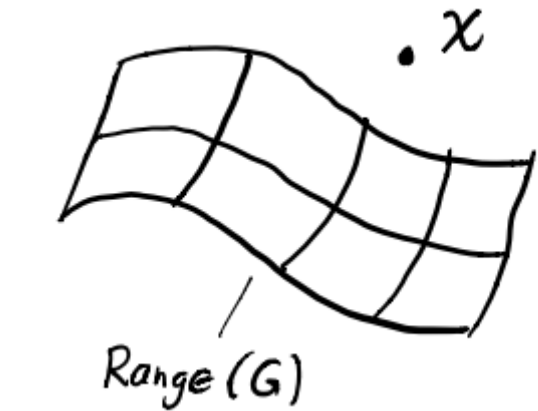
$$p(x) = \int p(x|z)p(z) dz$$

Problem: intractable

# Variational inference



# Variational inference



$$p(z|x) = \frac{p(x|z)p(z)}{p(x)}$$

$$p(x) = \int p(x|z)p(z)dz$$

Problem: intractable

Estimate

$$q_x(z) \equiv \mathcal{N}(g(x), h(x))$$

Functions (e.g., Gaussians)

$$g \in G$$

$$h \in H$$

Solution: tractable

Optimizing lower bound of probability distribution

$$(g^*, h^*) = \arg \min_{(g, h) \in G \times H} KL(q_x(z), p(z|x))$$

$$= \arg \min_{(g, h) \in G \times H} \left( \mathbb{E}_{z \sim q_x} (\log q_x(z)) - \mathbb{E}_{z \sim q_x} \left( \log \frac{p(x|z)p(z)}{p(x)} \right) \right)$$

$$= \arg \min_{(g, h) \in G \times H} (\mathbb{E}_{z \sim q_x} (\log q_x(z)) - \mathbb{E}_{z \sim q_x} (\log p(z)) - \mathbb{E}_{z \sim q_x} (\log p(x|z)) + \mathbb{E}_{z \sim q_x} (\log p(x)))$$

$$= \arg \max_{(g, h) \in G \times H} (\mathbb{E}_{z \sim q_x} (\log p(x|z)) - KL(q_x(z), p(z)))$$

$$= \arg \max_{(g, h) \in G \times H} \left( \mathbb{E}_{z \sim q_x} \left( -\frac{\|x - f(z)\|^2}{2c} \right) - KL(q_x(z), p(z)) \right)$$

# VAE loss function: reconstruction error and regularization term (KL divergence)

$$E_{q(z|x)} \log p(x|z) - KL(q(z|x) || p(z))$$

Reconstruction error

Regularization term (KL divergence)

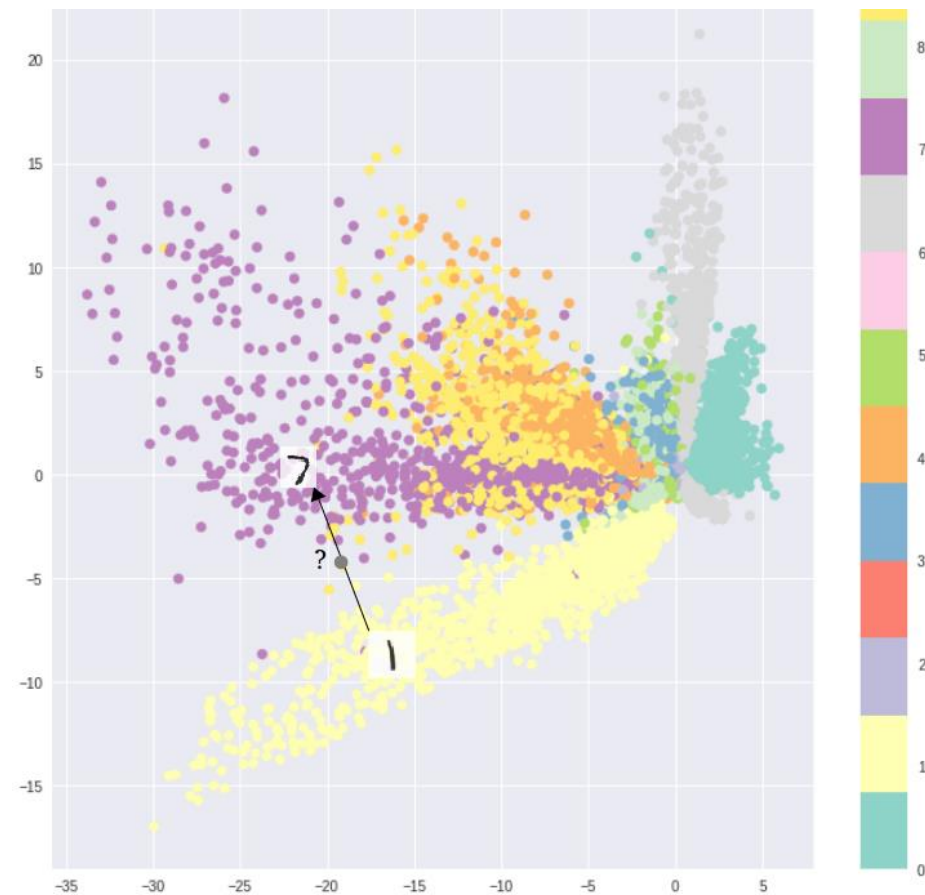
Autoencoder loss =  $\| \mathbf{x} - \hat{\mathbf{x}} \|^2$

Distance between probability distributions

What does this mean for the latent space?

# Reconstruction loss only

$$E_{q(z|x)} \log p(x|z) - KL(q(z|x) || p(z))$$

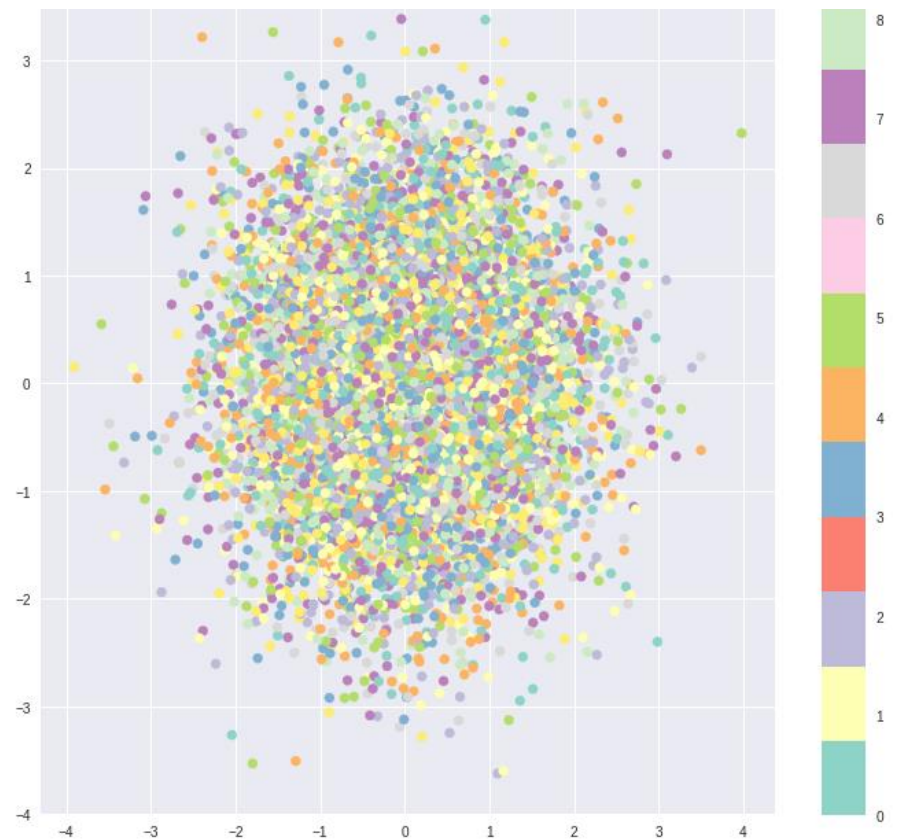


Why does it have this shape?  
What about KL only?



# KL Divergence loss only

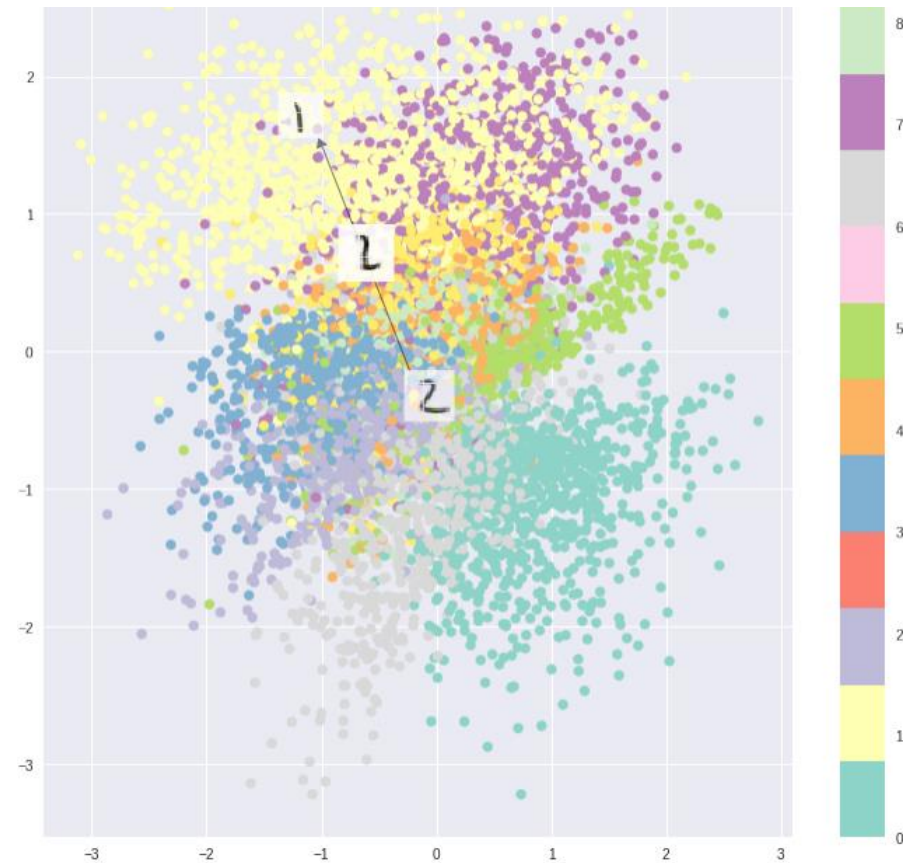
$$E_{q(z|x)} \log p(x|z) - KL(q(z|x) || p(z))$$



Why does it have this shape?  
Reconstruction + KL?

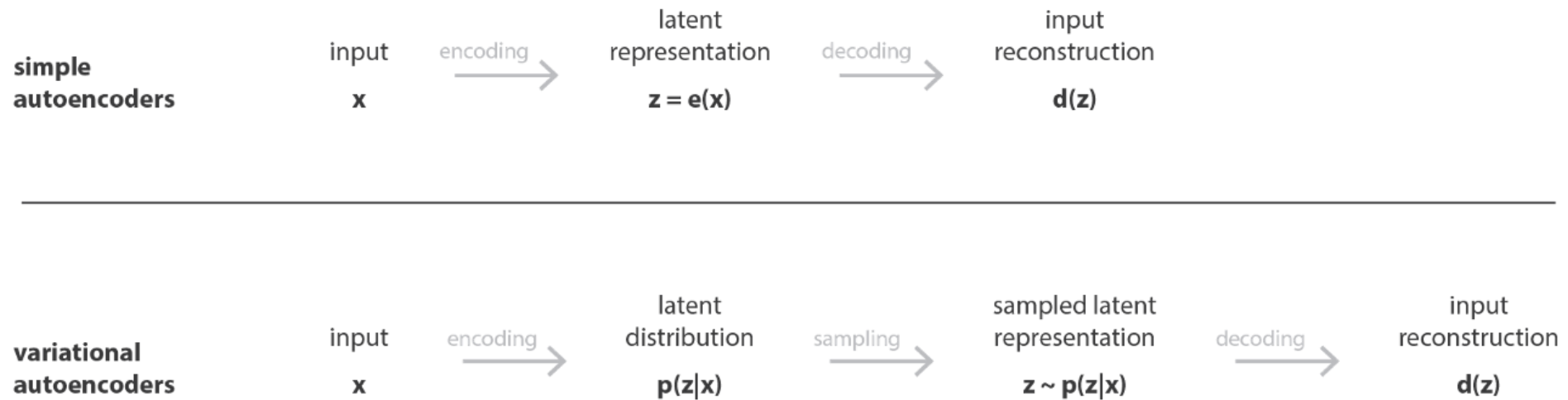
# Reconstruction and KL divergence

$$E_{q(z|x)} \log p(x|z) - KL(q(z|x) || p(z))$$



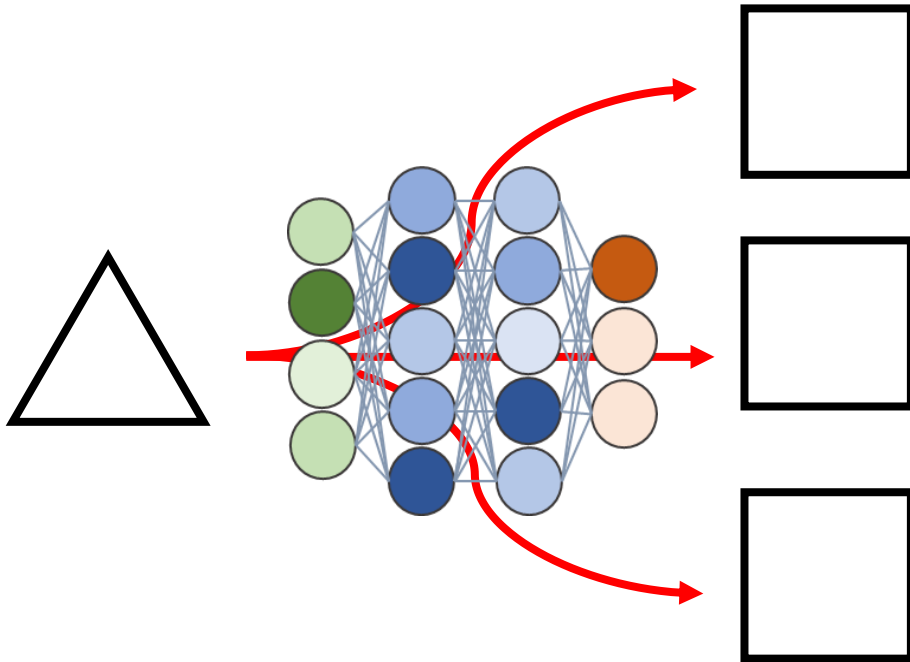
Why does it have this shape?

# VAE model steps

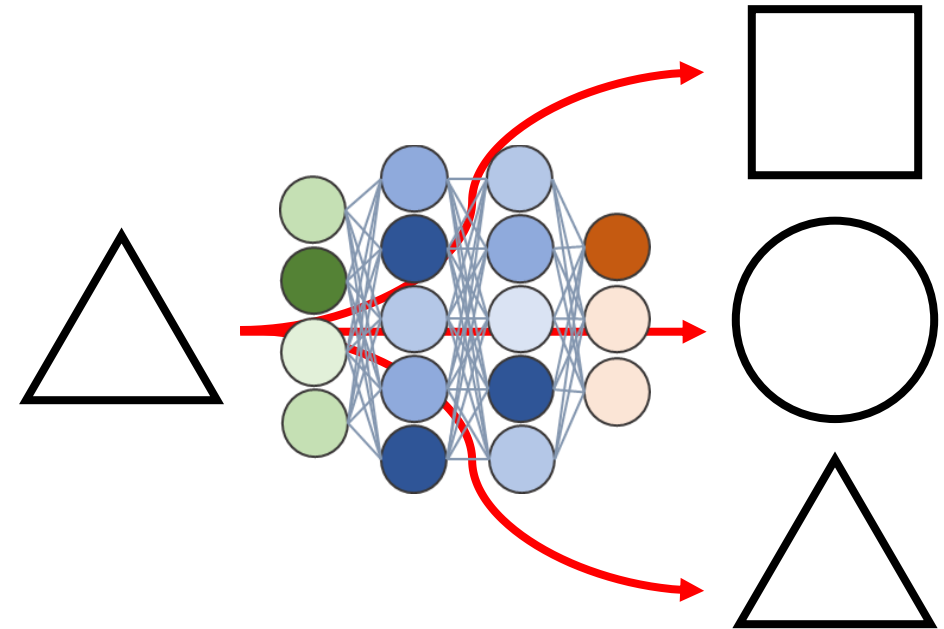


# Deterministic vs probabilistic

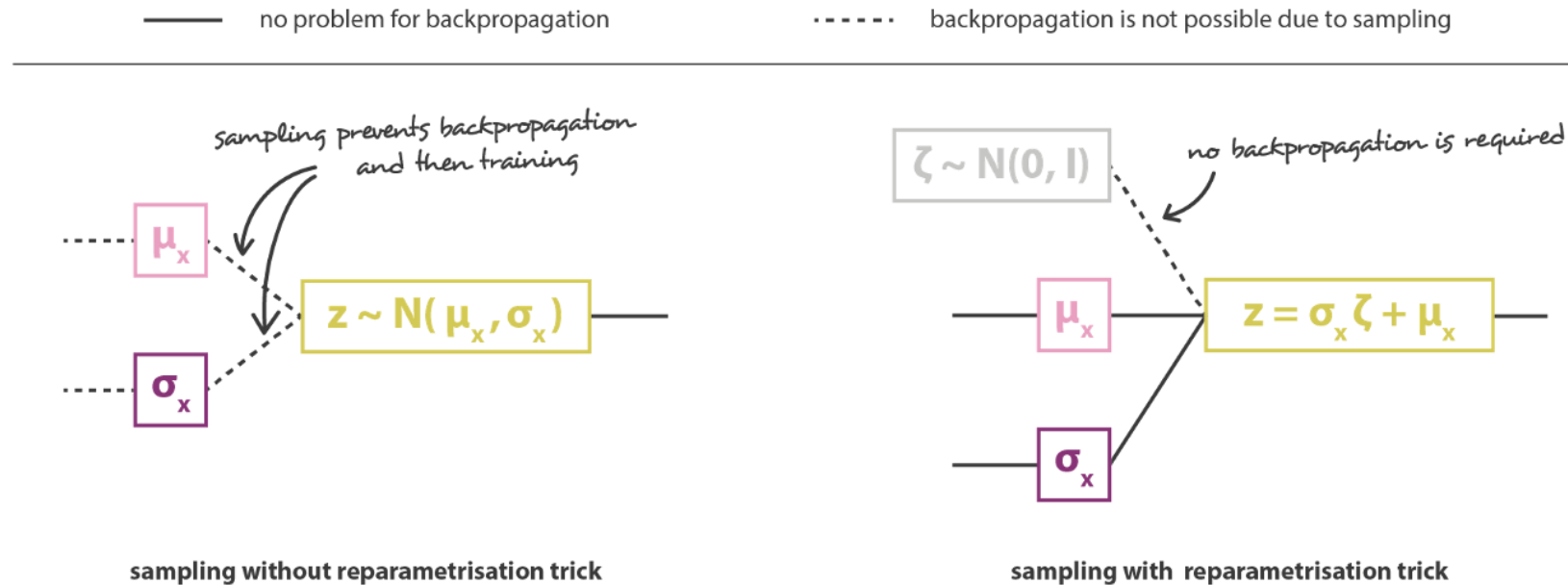
Deterministic



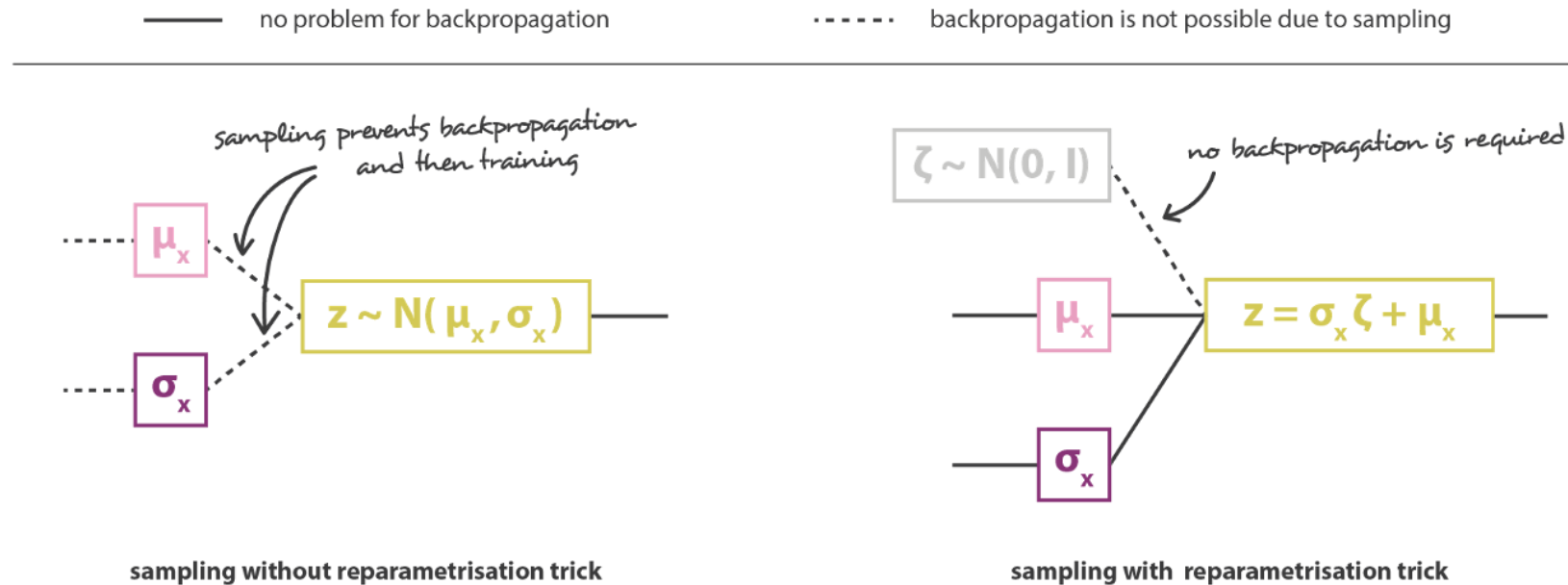
Probabilistic



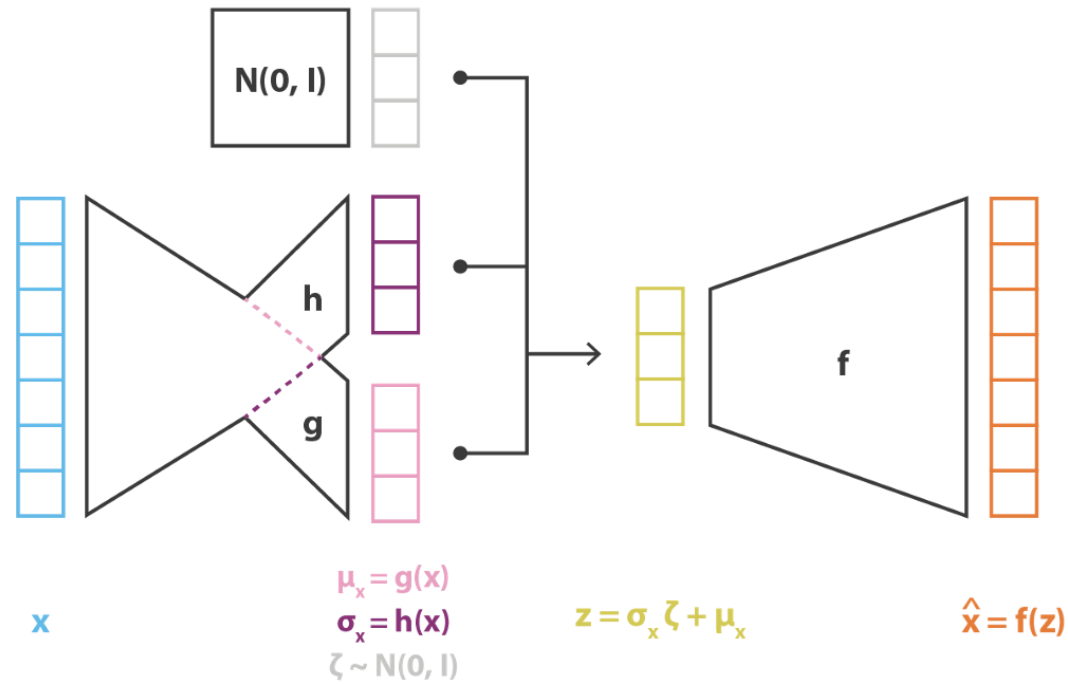
# Problem: backpropagation



# Solution: reparameterization trick



# VAE model

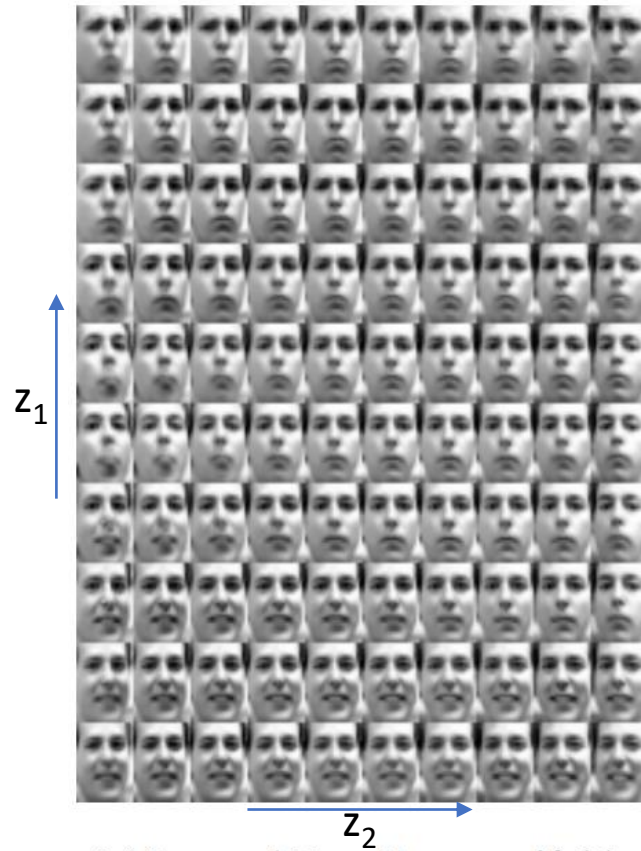


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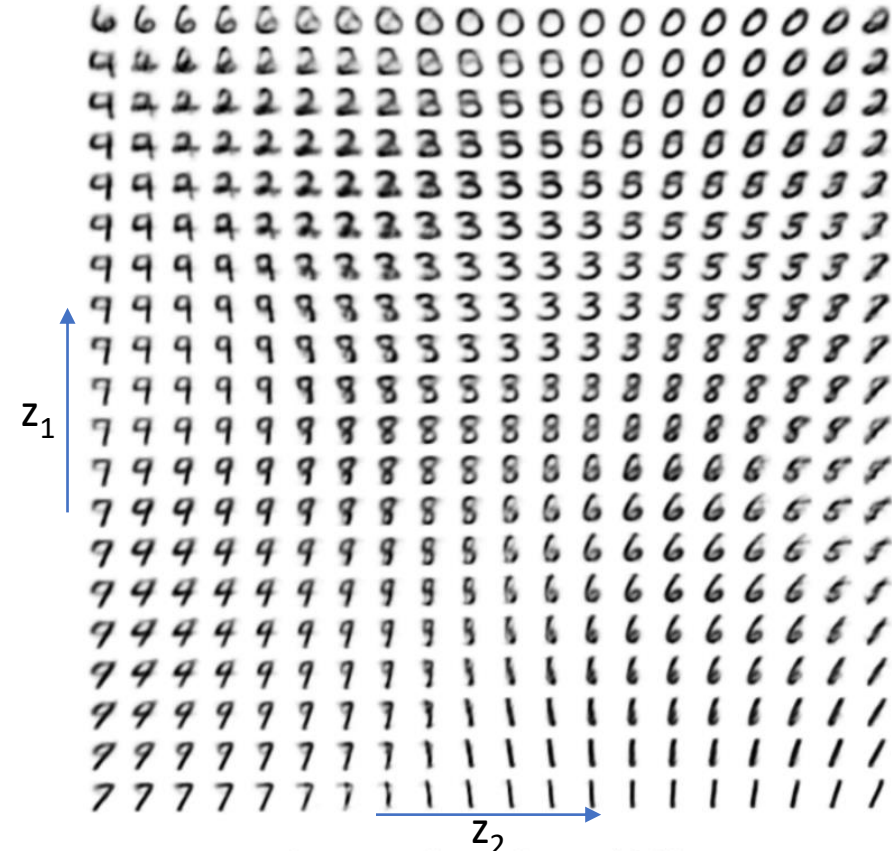
$$\text{loss} = C || x - \hat{x} ||^2 + \text{KL}[ N(\mu_x, \sigma_x), N(0, I) ]$$

# Generating data

Examples with 2-D latent space



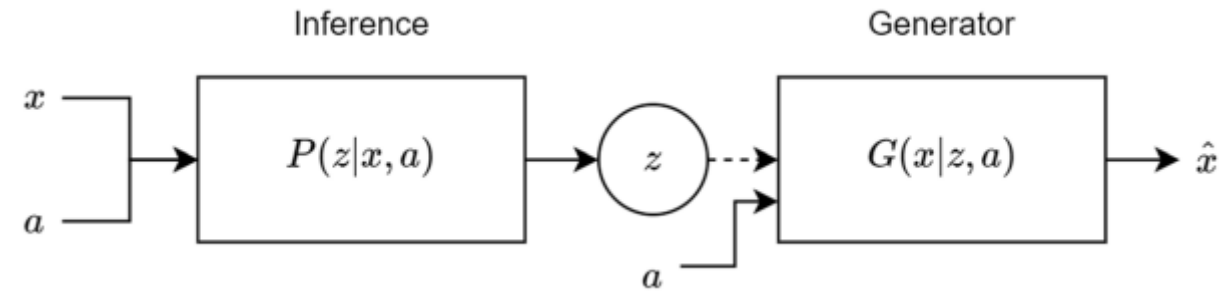
(a) Learned Frey Face manifold



(b) Learned MNIST manifold



# Generating new proteins



AEVPSGEQLFNSNCSACHIGGNNVIISHKTLRKEALEKYAMNSLEAIRYQVVNGKNAMPAFGGRLNEEEIDAITYVLGQAELD

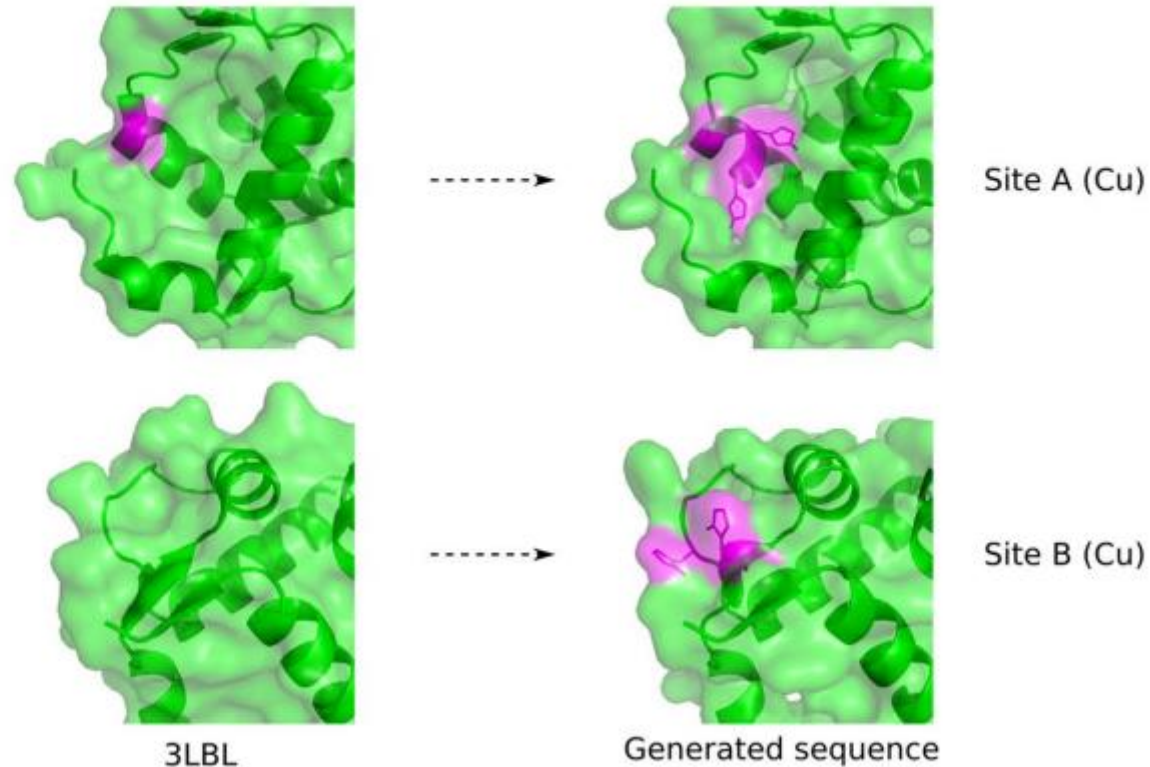


ADLEAGEQIFSANCAACHGGGNNIIMPEKTLKKDALEENGKMSVEAITYQVTNGKNAMPAFGGRLSDEDIEDVANYVLSQAEKGW  
ADLEHGAQIFSANCAACHAGGNNVIMPDKTLKKDALEKNGMNSIEAITYQVTNGKNAMPAFGGRLSDEDIEDVANYVLSQAEKGW  
ADLENGGKVFSGACAACHIGGENIVRPEKTLKKDALEEGGMDSEAITAQTNGKNAMPAFGERLVDEDIEDVAEYVL  
ADLAAGEQIFSANCAACHAGGNNVMPDKTLKKDALEKYGMSIEAITTQVTNGKNAMPAFGGRLEAEDIEDVAAYVLSQAE  
ADLEHGEQIFSANCAACHAGGNNVIMPEKTLKKDALEKYGMSVEAITTQVTNGKNAMPAFGGRLEDEQIEDVANYVLSQSEW  
ADIEHGEKIFSANCAACHAGGNNAIMRNKTLKKEALEPNGMNSIEAITYQVTNGKNAMPAFGGRLSDEDIEDVANYVLKQAEKGW  
ADLAAGEQIFSANCAACHAGGNNIIMPEKTLKKEALEKYSMSIEAITTQVTNGKNAMPAFGGRLSDEDIEDVANYVLSQAEKGW  
ADIITGEQIFSANCAACHIGGNNAIRPEKTLKPALETNGMNSVDAITTQVVPKNAMPAFGGRLEDEDIEDVANYVLSQAEK

# Generating new proteins

3LBL sequence MQIPASEQETLVRPKPLLLKLLKSVGAQKDTYTMKEVLFYLGQYIMTKRLYDEKQQHIVYCSNDLLGDLFGVPSFSVKEHARKIYTMIRNLLVVVN  
Generated sequence VQSVPSQEYSVHPGPNPKPNLPSCGVSKFHPPEKEVDLLNGYKMPKRLYDDKFFHIGKCSNDLLHDEFTPPSDEVKECRKHHTMIYRNLLVVVK

Site B Site A



# VAE takeaways

- Similar architecture to autoencoders
- Uses regularization (mean and variance) to ensure regular latent space
- Samples probability distribution from encoded inputs
- “Variational inference” used to derive loss function
- Used to generate new data that is similar but not equivalent to input data
- Output can be interpolations between inputs