

3. Asymptotic Notations

To measure 2. Complexities and Efficiencies of algorithm without running the code we use some mathematical tools known as Asymptotic Notations.

- We use asymptotic notations to compare the efficiencies of algorithm.
- It's a mathematical tool that estimates time based on input size without running the code.
- It focuses on how many basic operations the program perform, giving us an idea of how the algorithm behaves as input size increases.

Types of Asymptotic Notations

- Big O -> Describes that worst-case scenario or the upper bound of how the algorithm performs as the input size increases.
- Omega -> Describes the best case scenario or the lower bound.
- Theta -> Describes the average case or how the algorithm performs generally as input grows.

Big O Notation

Big-Oh defined

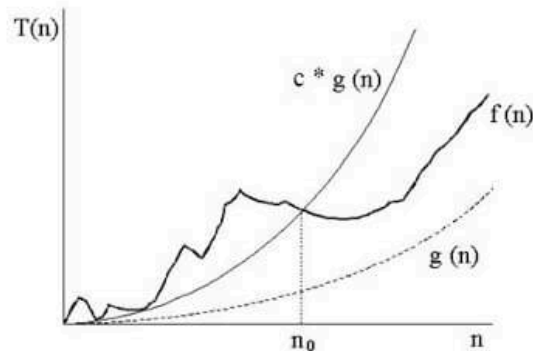
- Big-Oh is about finding an *asymptotic upper bound*.

- Formal definition of Big-Oh:

$f(N) = O(g(N))$, if there exists positive constants c , N_0 such that

$$f(N) \leq c \cdot g(N) \text{ for all } N \geq N_0.$$

- We are concerned with how f grows when N is large.
 - not concerned with small N or constant factors
- Lingo: " $f(N)$ grows no faster than $g(N)$."



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n axis -> number of input

T axis -> time of processing

$f(n)$ -> function which is working

C and n_0 -> these are the constants.

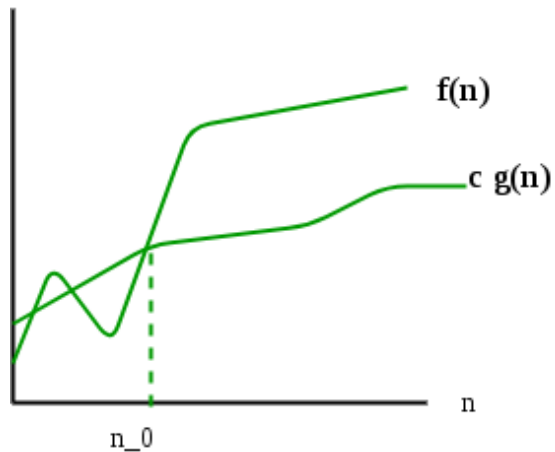
$C g(n)$ -> another function $g(n)$ which shows upper limit and C is the constant (this is actually which shows upper limit i.e. showing Big O notation in presence of C)

Explain : for $f(n)$ whatever value you put for value of n it will always be smaller than $C g(n)$ in terms of time.

Mathematically :

$$0 \leq f(n) \leq C g(n)$$

Omega Notation



$$f(n) = \Omega(g(n))$$

n axis -> number of input

T axis -> time of processing

C and n_0 -> these are the constants.

$f(n)$ -> function which is working

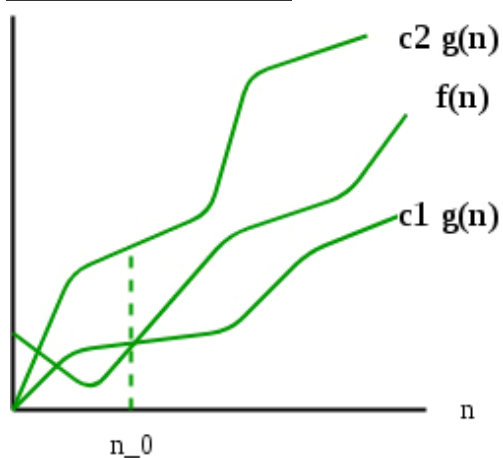
$C g(n)$ -> another function $g(n)$ which shows lower limit and C is the constant (this shows that $c g(n)$ is at the lower limit and is the best case i.e. it will not come more low than that point i.e. the best-case)

Explain : for $f(n)$ whatever value you put for value of n it will always be greater than $c g(n)$ because that is the maximum lower limit algorithm can achieve for processing.

Mathematically :

$$f(n) = \Omega(g(n))$$

Theta Notation



$$f(n) = \theta(g(n))$$

n axis -> number of input

T axis -> time of processing

C1, C2 and n_0 -> these are the constants.

f(n) -> function which is working

C1 g(n) -> another function g(n) which shows lower limit and C is the constant (this shows that c g(n) is at the lower limit and is the best case i.e. it will not come more low than that point i.e. the best-case)

C2 g(n) -> another function g(n) which shows upper limit and C is the constant (this is actually which shows upper limit i.e. showing Big O notation in presence of C, i.e. no other function can perform beyond this point as because this is the worst case)

Explain : for f(n) whatever value you put for value of n it will always in between C1g(n) and C2g(n) which shows average case algorithm will work.

Mathematically :

$$0 <= c_1 g(n) <= f(n) <= C_2 g(n)$$

Example :

Searching an element using Linear Search

3,4,5,6,7,8

Best case : when we have to search 3 in 1st attempt we can find it.

Worst case : when we have to search 8, algorithm has to traverse n times for time i.e. size of array to search the last element

Average case : when we have to search 6, algorithm will not find the element in 1st attempt and algorithm has not to traverse entire array to find the required element.

