Block-based DCT

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Assignment

- 주어진 입력 이미지를 8x8 2-D DCT 변환 및 역변환을 하시오.
- 또한 8x8 separable DCT 변환 및 역변환을 진행하시오.
- 2-D DCT와 separable DCT의 시간을 비교하시오.

Discrete Cosine Transform

- Discrete Cosine Transform(DCT)
  - Effective at transforming image data into a form that is easy to compress : energy compaction
  - Efficiently implemented in software and hardware

$$F_{x,y} = \frac{c(x)c(y)}{N/2}\sum_{i=0}^{N-1}\sum_{j=0}^{N-1} f_{i,j}\cos\left(\frac{(2i+1)\pi x}{2N}\right)\cos\left(\frac{(2j+1)\pi y}{2N}\right),\quad c(l) = \begin{cases} 1/\sqrt{2} & \text{for } l = 0 \\ 1 & \text{otherwise} \end{cases}$$

where $\begin{cases} f_{i,j} : \text{input sample at } (i, j) \\ F_{x,y} : \text{DCT coefficient at } (x, y) \end{cases}$

< Original image >

< DCT coefficient >

Discrete Cosine Transform

- Forward DCT
  - 2-D 8x8 DCT transform

$$F_{x,y} = \frac{c(x)c(y)}{N/2} \sum_{i=0}^{N-1} \sum_{j=0}^{N-1} f_{i,j} \cos\left(\frac{(2i+1)\pi x}{2N}\right) \cos\left(\frac{(2j+1)\pi y}{2N}\right)$$

$$c(l) = \begin{cases} 1/\sqrt{2} & \text{for } l = 0 \\ 1 & \text{otherwise} \end{cases}$$

46	42	32	27	27	29	98	168
44	41	32	33	29	37	140	174
37	32	29	29	31	55	162	180
38	37	34	31	33	88	173	177
34	32	28	34	47	129	164	166
32	34	31	35	81	146	151	158
46	44	59	70	105	141	137	147
83	76	94	100	112	112	123	139

< input image >

384.5	23.2	12.9	11.5	8.3	-5.0	1.7	-1.1
-30.3	32.9	28.2	20.0	4.6	24.0	1.6	-1.9
40.5	29.8	26.1	6.6	10.7	-12.5	-2.8	4.3
3.3	39.0	5.5	12.0	-1.0	9.0	2.3	-0.6
15.3	-8.2	16.9	8.5	5.0	-2.4	-3.5	3.0
-6.3	15.7	-1.8	-4.7	-2.0	4.0	2.5	-2.0
5.1	-9.5	-1.8	4.0	0.4	-3.2	-1.1	1.1
-1.7	4.0	4.7	-3.5	-3.8	5.4	-1.2	-1.4

< DCT coefficients >

Discrete Cosine Transform

❖ Inverse DCT

– 2-D 8x8 Inverse DCT transform

$$f_{i,j} = \sum_{x=0}^{N-1}\sum_{y=0}^{N-1} \frac{c(x)c(y)}{N/2} F_{x,y} \cos\left(\frac{(2i+1)\pi x}{2N}\right) \cos\left(\frac{(2j+1)\pi y}{2N}\right)$$

$$c(l) = \begin{cases} 1/\sqrt{2} & \text{for } l = 0 \\ 1 & \text{otherwise} \end{cases}$$

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< input image >

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< DCT coefficients >

Discrete Cosine Transform

❖ Result

Original image

8x8 FDCT coefficient image

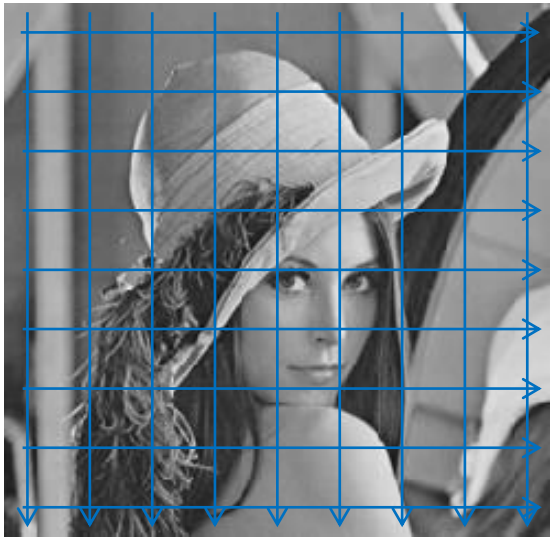
8x8 IDCT image

Discrete Cosine Transform

- Separability
  - 2-D DCT can be conducted by performing two 1-D DCTs
  - Simplifies the hardware requirements at the expense of a slight increase in the overall operations count

< Horizontal 1-D DCT >

< Vertical 1-D DCT >



< 2-D DCT >

Discrete Cosine Transform

- Separability with 8x8 DCT
  - 2-D DCT can be conducted by performing two 1-D DCTs
  - Simplifies the hardware requirements at the expense of a slight increase in the overall operations count

$$y_{kl} = \frac{c(k)}{2}\sum_{i=0}^{7}\left[\frac{c(l)}{2}\sum_{j=0}^{7} x_{ij}\cos\left(\frac{(2j+1)l\pi}{16}\right)\right]\cos\left(\frac{(2i+1)k\pi}{16}\right), \quad 0 \le k,l \le 7$$

$$z_{il} = \frac{c(l)}{2}\sum_{j=0}^{7} x_{ij}\cos\left(\frac{(2j+1)l\pi}{16}\right), \quad 0 \le i,l \le 7$$

$$\mathbf{Z} = \mathbf{TX}$$

$$\mathbf{Y} = (\mathbf{TZ}^T)^T = \mathbf{TXT}^T$$

where
- $\mathbf{T}$: 8x8 DCT transform matrix
- $\mathbf{X}$: input matrix
- $\mathbf{Y}$: coefficient matrix

END OF PRESENTATION

Q&A