CSE3015 Homework 2

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1. Let $F(A, B, C, D) = BD + B'D'$ is given.

- Make given function to canonical SOP form.

  The truth table for the function F is:

A	B	C	D	F	Minterm
T	T	T	T	T	m_{15}
T	T	T	F	F	m_{14}
T	T	F	T	T	m_{13}
T	T	F	F	F	m_{12}
T	F	T	T	F	m_{11}
T	F	T	F	T	m_{10}
T	F	F	T	F	m_9
T	F	F	F	T	m_8
F	T	T	T	T	m_7
F	T	T	F	F	m_6
F	T	F	T	T	m_5
F	T	F	F	F	m_4
F	F	T	T	F	m_3
F	F	T	F	T	m_2
F	F	F	T	F	m_1
F	F	F	F	T	m_0

  From the truth table above, the 1-minterm indices are $\{0, 2, 5, 7, 8, 10, 13, 15\}$. Hence, the canonical SOP form can be written as:

$$F_1(A, B, C, D) = \sum (0, 2, 5, 7, 8, 10, 13, 15) = m_0 + m_2 + m_5 + m_7 + m_8 + m_{10} + m_{13} + m_{15}$$

- With canonical SOP form, fill up the Karnaugh Map.

  AB

	00	01	11	10
00	1			1
01		1	1	
11		1	1	
10	1			1

  CD

- Using the Karnaugh Map, find the simplest(shortest) SOP form. Show each step and detail.

  The Karnaugh Map given by the previous step looks like this:

AB

	00	01	11	10
00	1			1
01		1	1	
11		1	1	
10	1			1

CD

Here, it is pretty obvious that we can group the four 1's in the middle into an implicant group as follows:

AB

	00	01	11	10
00	1			1
01		1	1	
11		1	1	
10	1			1

CD

Considering that Karnaugh Map can "wrap around" in all four directions, we can connect the four corner cells into an implicant group as well:

AB

	00	01	11	10
00	1			1
01		1	1	
11		1	1	
10	1			1

CD

Since all two implicants are not included in any other implicants, and include minterms that are not included in any other prime implicants, we can say they all are **essential prime implicants**.

The red implicant represents BD, and the green implicant represents $B'D'$. Therefore, connecting these two implicants with logical OR gives the following shortest SOP form:

$$F(A, B, C, D) = BD + B'D'$$

2. Let $G(A, B, C, D) = (A + C)(B' + C' + D')(B + C' + D)$ **is given.**

- Make given function to canonical POS form.

  The truth table for the function G is:

A	B	C	D	G	Maxterm
T	T	T	T	F	M_{15}
T	T	T	F	T	M_{14}
T	T	F	T	T	M_{13}
T	T	F	F	T	M_{12}
T	F	T	T	T	M_{11}
T	F	T	F	F	M_{10}
T	F	F	T	T	M_9
T	F	F	F	T	M_8
F	T	T	T	F	M_7
F	T	T	F	T	M_6
F	T	F	T	F	M_5
F	T	F	F	F	M_4
F	F	T	T	T	M_3
F	F	T	F	F	M_2
F	F	F	T	F	M_1
F	F	F	F	F	M_0

  From the truth table above, the 0-maxterm indices are $\{0, 1, 2, 4, 5, 7, 10, 15\}$. Hence, the canonical POS form can be written as:

$$G_1(A, B, C, D) = \prod(0, 1, 2, 4, 5, 7, 10, 15) = M_0 M_1 M_2 M_4 M_5 M_7 M_{10} M_{15}$$

- With canonical POS form, fill up the Karnaugh Map.

CD

AB	00	01	11	10
00	0	0		0
01	0	0	0	
11			0	
10				0

- Using the Karnaugh Map, find the simplest(shortest) SOP form. Show each step and detail.

  The Karnaugh Map given by the previous step looks like this:

CD

	00	01	11	10
00	0	0		0
01	0	0	0	
11			0	
10				0

AB
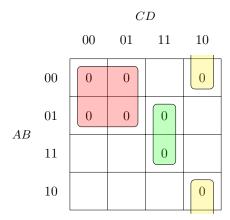
The upper-left corner has four 0-maxterms arranged like a square, so we can put them in an implicant group.

CD

	00	01	11	10
00	0	0		0
01	0	0	0	
11			0	
10				0

AB

On the third column, there are two maxterms in a line. They can put into an implicant group as well.

CD

	00	01	11	10
00	0	0		0
01	0	0	0	
11			0	
10				0

AB

Again, considering that Karnaugh Map can "wrap around" in all four directions, we can put two in the last column into their own implicant group.

$$CD$$

	00	01	11	10
00	0	0		0
01	0	0	0	
11			0	
10				0

AB

Since all two implicants are not included in any other implicants, and include maxterms that are not included in any other prime implicants, we can say they all are **essential prime implicants**.

For the red implicant, we find terms A' and C'. Taking their complement and connecting them with logical OR, we find the red implicant represents $A + C$. For the green implicant, we find terms B, C, and D. Following the same procedure, we find the green implicant represents $B' + C' + D'$. Similarly, the yellow implicant represents $B + C' + D$. Connecting all terms with logical AND, we get the shortest POS form:

$$G(A, B, C, D) = (A + C)(B' + C' + D')(B + C' + D)$$