2018-2	Discrete Mathematics	Final Exams
예홍진		Test time: 60 minutes

※ (1 ~ 10) Determine whether the following statement is true or not. Write "T" for true statement and "F" for false statement. [15 points]

1. The relation O, big-O is defined as $f\,O\,g$ if and only if f and g have the same order.
2. All the complete graph K_n have an Euler path.
3. $\Theta(n^k)$ is lower than $\Theta(a^n)$ for any power n^k and any $a > 0$.
4. Let $A = \{v_1, v_2, v_3, v_4, v_5, v_6\}$ and $R = \{(v_1, v_5), (v_3, v_2), (v_3, v_4), (v_4, v_1), (v_4, v_3)\}$. Then R is a rooted tree with root v_1.
5. An undirected tree is simply the transitive closure of a rooted tree.
6. If a graph G has n vertices and $n-1$ edges, then there are only one spanning tree in G.
7. A path π in a graph G consists of a pair (V_π, E_π) of sequences. Where V_π is a sequence of vertices and E_π is a sequence of edges. Then no vertex occurs more than once in the sequence of vertices.
8. A tree is a complete n-tree if every vertex has exactly n offspring.
9. In a graph, a vertex with degree 1 is called an isolated vertex.
10. If a graph G has exactly two vertices of odd degree, there is an Euler circuit in G.
11. A maximum flow F in a network has value equal to the capacity of a maximum cut of the network.
12. The specification of the meaning of sentences is called the syntax of a language.
13. In grammar $G = (V, S, v_0, \mapsto)$, if the left-hand side of each production is a single, nonterminal symbol and the right side consists of one or more symbols. Then, we say that G is context-free grammars.
14. Let $V = \{v_0, w, a, b, c\}$, $S = \{a, b, c\}$, and let $\mapsto$ be the relation on V^* given by
    1. $v_0 \mapsto aw$. 2. $w \mapsto bbw$. 3. $w \mapsto c$.
    Then the $L(G)$ corresponds to the regular expression $a(bb)^*c$.
15. All the regular grammars and regular languages can not be recognized by a finite state machine.

※ (16 ~ 30) Complete the following statements. [15 points]
[단, 영어 단어나 철자를 정확히 모르는 경우에는 한글로 답하는 것도 인정한다.]

16. Let (T, v_0) be a rooted tree on a set A. Then T is ( ) and asymmetric and not transitive.
17. A tree with ( ) vertices has 12 edges.
18. A path in a graph G is called a(n) ( ) path if it includes every edge exactly once.
19. If R is a symmetric connected relation on A, we say that a tree T on A is a ( ) tree for R if T is a tree with exactly the same vertices as R and which can be obtained from R by deleting some edges of R.
20. For each integer $n \geq 1$, let U_n denote the graph with n vertices no edges. The graph U_n is called the ( ) graph on n vertices.
21. The ( ) of a tree is defined by the largest level number of that tree.
22. The graph is called ( ) if there is a path from any vertex to any other vertex in the graph.
23. Let a function f : A → B. If we have f(a) ≠ f(a') for the two distinct elements a and a', then we say that f is ( ).
24. If each vertex has the same degree as every other vertex in a graph, the graph is called ( ).

25. A(n) ( ) graph is a graph for which each edge is labeled with a numerical value.

26. If a graph G is ( ) and has exactly two vertices of odd degree, there is an Euler path in G.

27. Let $f: A \to B$ be an invertible function. f^{-1} is onto if and only if f is ( ).

28. A phrase structure grammar G is defined to be a 4-tuple $(V, S, v_0, \mapsto)$, where V is a finite set of symbols, S is the set of ( ) symbols, $v_0 \in V - S$ and the statement $w \mapsto w'$ is a production of G.

29. In Moore machine $M = (S, I, F, s_0, T)$, s_0 is the starting state and T is the set of ( ) states of M.

30. Let I be a set and let $L \subseteq I^*$. Then $L = L(M)$ for some Moore machine M if and only if L is a ( ) set.

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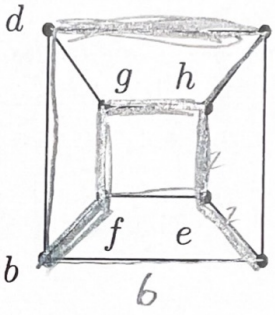
31. Order the following functions from lowest to highest Θ-class. **[3 point]**

$f_1 = 31459 + \lg(n)$ $f_2 = 3n - \lg(\lg(n)) + n^{0.5}$ $f_3 = 4n\sqrt{n} + 2\sqrt[3]{n^2} - 1.5n$

$f_4 = 1.2^n - 0.8^n + 2n^2$ $f_5 = 3n^3 - 2n^2 + 4n - 5$ $f_6 = n\lg(n^4) - \lg(n^3)$

32. Let G be the graph and the associated matrix of weights shown as below :

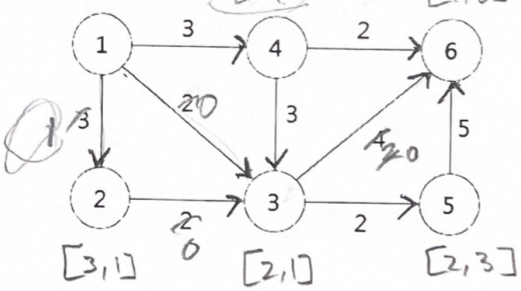


	a	b	c	d	e	f	g	h
a	0	6	3	0	2	0	0	0
b	6	0	0	5	0	7	0	0
c	3	0	0	2	0	0	0	8
d	0	5	2	0	0	0	4	0
e	2	0	0	0	0	6	0	2
f	0	7	0	0	6	0	5	0
g	0	0	0	4	0	5	0	4
h	0	0	8	0	2	0	4	0

(1) Draw the quotient graph G^R where R is defined by { {a, e}, {d, g}, {f, h} }. **[2 point]**

(2) Find a minimal spanning tree for G. **[2 point]**

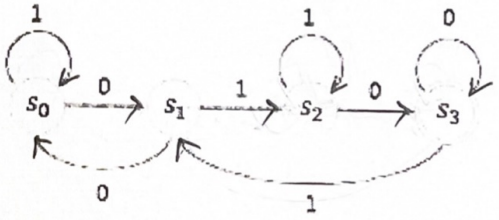
33. Use the labeling algorithm to find the maximum flow for the network.



(1) List first and second path from the source to the sink with its flow, and Write labeling of each vertices and capacity of the each edges after determined both paths. **[2 point]**

(2) Find the maximum flow for this network. **[2 point]**

34. Consider the Moore machine M given.



(1) Give the BNF representation for the Moore machine M. **[3 point]**

(2) Construct the regular expression that corresponds to $L(M)$. **[3 point]**

35. Construct a Moore machine that accepts strings having exactly two x's. (Input : x, y) **[3 point]**