⟨4. Relations and Partitions⟩

Relation on A and B

- Product Set ($A \times B$) ~ Catesian Product라고 하기도.

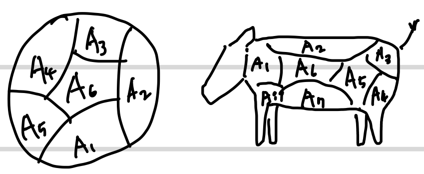
$A = \{1, 2, 3\}$ $B = \{a, b\}$

$A \times B = \{(1,a), (1,b), (2,a), \cdots (3,b)\}$

$B \times A = \{(a,1), (a,2), (a,3), \cdots (b,3)\}$

$\rightarrow A \times B \neq B \times A$

- Partition (Quotient set)



① $A_i \cap A_j = \phi$

② $A_1 \cup A_1 \cup \cdots \cup A_n = S$

$A = \{1, 2, 3\}$

$P(A) = \{\{1\}, \{2\}, \{3\}\}, \{\{1,2\}, \{3\}\}$

※(1~14) 다음 문장의 내용이 맞으면 ○표, 틀리면 ×표를 답하시오.

1 1 0 1 0 0

1. Let $U=\{1,2,3,4,5,6\}$, $A=\{2,5,6\}$ and $B=\{1,2,4\}$. Then sequence 0,1,0,0,1,1 represents f_A and 1,1,0,0,1,0 represents f_B. (f_A, f_B are characteristic functions) X

2. Let $A=\{a,b,c\}$. The expression $a^*(b\wedge c)^*bc$ is a regular expression. X

3. Let $A=\{\mathrm{ab},\mathrm{bc},\mathrm{ba}\}$. All the strings such as $abba$, $babcab$, $bcababc$, $abbcbaab$ belong to A^*. X

4. If e is an identity for a binary operation, then e is unique.

5. Let $A=\{1,2,3,4\}$ and let $R=\{(1,2),(2,2),(3,4),(4,1)\}$. Then R is antisymmetric. O

6. When the relation R on the set $A=\mathbb{Z}$ and aRb if and only if $|a-b|=2$, the relation R is symmetric and transitive.

7. The recurrence relation $f_n=f_{n-1}+2f_{n-2}-1$ is a linear homogeneous relation of degree 2.

8. Let aRb if and only if GCD$(a,b)=1$ for a,b in $A=Z^+$, then R is an equivalence relation.

9. The symmetric closure of a relation R is the smallest symmetric relation containing R.

10. Let $A=\{1,2,3,4\}$ and let $R=\{(3,3),(1,2),(2,2),(2,1)(3,4),(4,3),(1,1),(4,4)\}$. Then R is an equivalence relation.

11. Let R be the relation on $A=\{1,2,3,4\}$ that has the matrix $M_R=\begin{pmatrix}0&1&0&1\\1&0&0&0\\1&1&1&0\\1&0&1&0\end{pmatrix}$.

    Vertex 3 has in-degree 3 and out-degree 2.

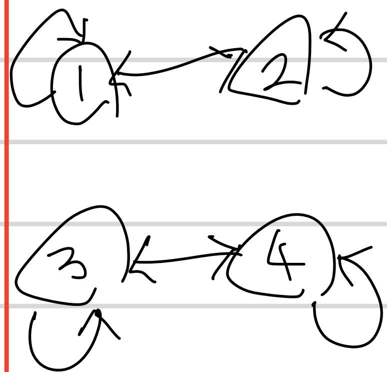
12. Let R be a relation from A to B, and let A_1 and A_2 be subsets of A. Then $R(A_1)\cap R(A_2)\subseteq R(A_1\cap A_2)$.

13. Let R be a relation on a set A where $|A|=4$ and $|R|=6$. If R is reflexive and antisymmetric then $|R\cup R^{-1}|=8$.

14. Let $A=\{1,2,3,4,5,6,7,8\}$ and R be a relation on A. If $M_R\odot M_R=M_R$ then R is transitive.

X a
X b
X c
X ab
⊘ abc
X ababc

$a^*(b\wedge c)^*bc$



$$M_R = \begin{bmatrix} 1 & 1 & 0 & 0 \\ 1 & 1 & 0 & 0 \\ 0 & 0 & 1 & 1 \\ 0 & 0 & 1 & 1 \end{bmatrix}$$

$$M_{R^2} = \begin{bmatrix} 1 & 1 & 0 & 0 \\ 1 & 1 & 0 & 0 \\ 0 & 0 & 1 & 1 \\ 0 & 0 & 1 & 1 \end{bmatrix} = R^\infty$$

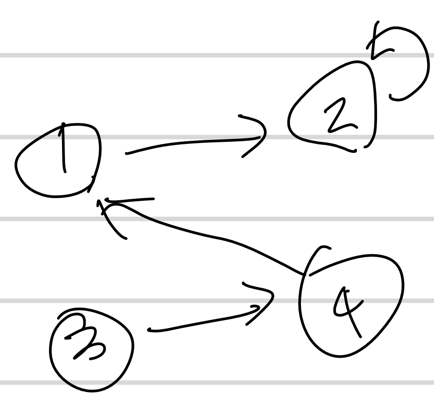
$$R = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \quad R^2 = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$A \wedge B = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 1 & 0 & 0 \end{bmatrix}$$

$A = \{0, 1\}$

$1^*(10)^*0^*$

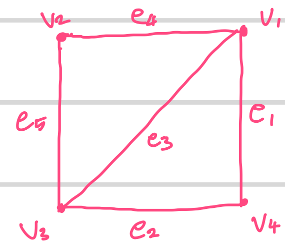
$$A \vee B = \begin{bmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{bmatrix}$$



$$\begin{bmatrix} 1 & 0 & 1 & 0 \\ 0 & 0 & 1 & 1 \\ 0 & 1 & 1 & 0 \end{bmatrix} = R$$

Ch.8 : Graph

① Graph : $G = (V, E, \gamma)$



② Vertice : V_i, V_j

③ Edge : E_k $\begin{cases} \text{directed} \\ \text{undirected} \end{cases}$

④ endpoint : $e_k = (V_i, V_j)$

⑤ degree : # of endpoint, 어떤 vertice가 몇번 사용되었는가. ⓐ degree = 2

⑥ loop : 자기 자신을 가리키는 관계 ①

⑦ isolated : degree가 0인 vertice

⑧ adjacent : 인접한 vertice

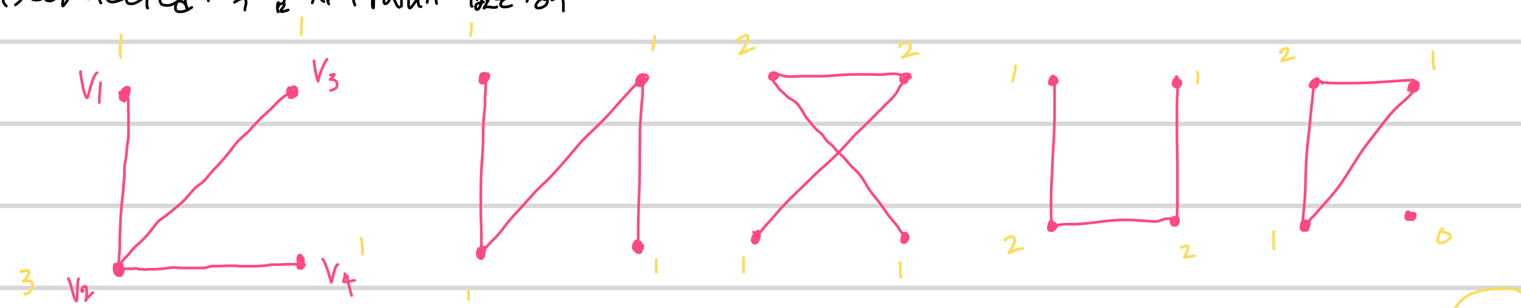
⑨ path : 경로 · sequence

⑩ Circuit : path에서 출발점과 도착점이 같은 경우.

⑪ Simple : $\begin{cases} \text{edge : 두 점사이 선분이 하나 일 때. ○ × (multiple edge)} \\ \text{Path : 두 점 사이 경로가 하나 일 때. ○ ×} \\ \text{Circuit :} \\ \text{graph : undirected, unweighted, no loop, no multiple edge graph} \end{cases}$

⑫ Connected : 두 점 사이 path가 있는 경우

⑬ disconnected : 두 점 사이 path가 없는 경우



4 + 4 + 4 + 4 + 4 = 20

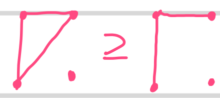
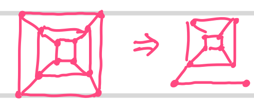
※ 점 4개일 때
선을 최대는 6개. $_6C_3 = \frac{6!}{3!3!} = 20$

③ regular graph : $\begin{cases} \text{discrete : 모든 점의 degree가 0. ○} \\ \text{Complete : 모든 점사이에 선분이 있는 그래프. ○} \\ \text{linear : 일렬로 늘어뜨릴 수 있는 그래프. 양끝의 degree가 1, 그 사이는 모두 2. ○ ○} \end{cases}$

$G = (V, E, \gamma)$

$G' = (V', E', \gamma')$

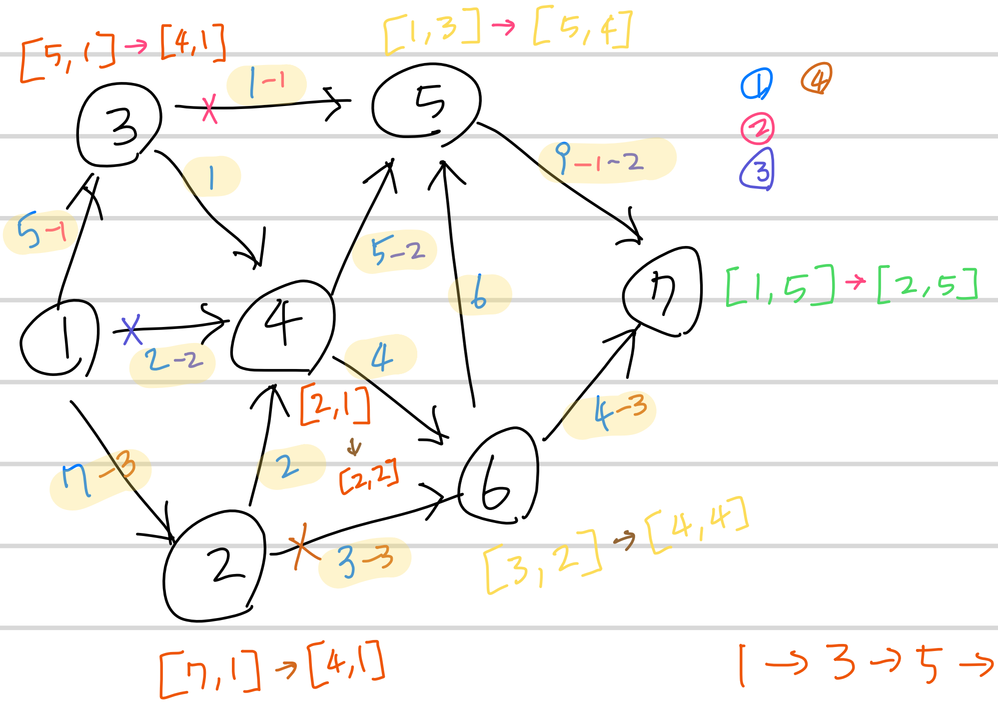
$V' \subseteq V,\ E' \subseteq E,\ \gamma' \subseteq \gamma$

$G' \subseteq G$: Subgraph  

R : E.R on V

$G'' = (V'', E'', \gamma'')$: Quotient graph

$V'' = |\mathcal{Q}| = |V/R|$



1 → 3 → 5 → 7 : 1

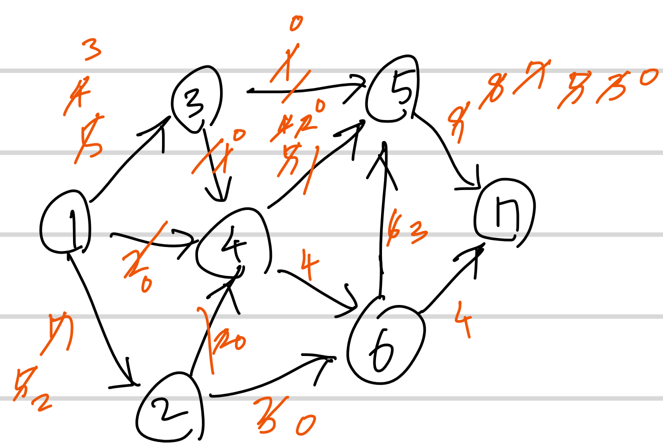
1 → 4 → 5 → 7 : 2

1 → 2 → 6 → 7 : 3

1 → 2 → 4 → 5 → 7 : 2

1 → 3 → 4 → 5 → 7 : 1

⇒ 9



1 3 5 7 : 1

1 3 4 5 7 : 1

1 4 5 7 : 2

1 2 4 5 7 : 2

1 2 6 5 7 : 3 ⇒ 9

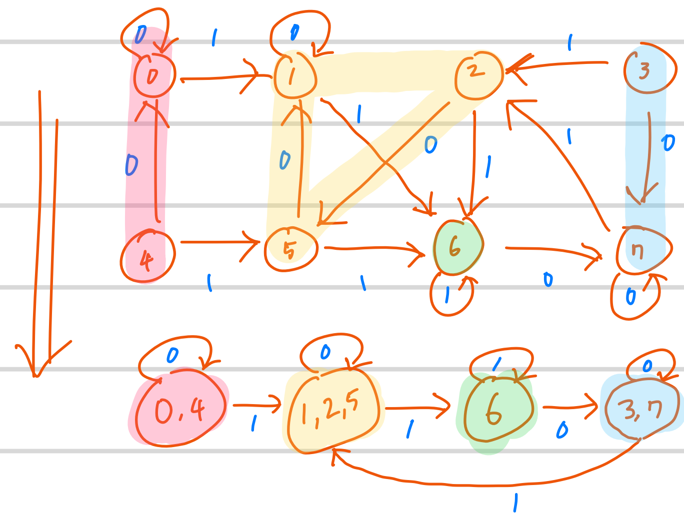
※(1~9) 다음 문장의 내용이 맞으면 ○표, 틀리면 ×표를 답하시오.

1. Let X=Y=ℤ and let formula $f(x)$ be defined by $f(x)=y$ and $y^2=x$, for $x \in X$, $y \in Y$. Then f(x) is a function. X → f(1)이 -1일수도, 1일수도.
2. If f is also everywhere defined and bijection, then f is called a one-to- one correspondence between A and B. O
3. Let f and g be functions whose domains are subsets of $\mathbb{Z}^+$, positive integers. If f is $O(g)$, then f grows faster than g dose. X
4. Let (T, v_0) be a rooted tree. The vertices of same level is called the siblings. O
5. Let (T, v_0) be a rooted tree. Then, there are no cycles in T. v_0 is the only root of T. And all vertices of T has in-degree. O
6. If (T, v_0) is a rooted tree and $v \in T$, then $T(v)$ is also a rooted tree with root v. We will say that $T(v)$ is the subtree of T beginning at v. O
7. If a graph G has exactly two vertices of odd degree, there is an Euler circuit in G. X → degree 가 odd면 Euler Circuit X.
8. The graph G is called the connected if there is a path from any vertex to any other vertex in G. O
9. The graph is called complete if each vertex of the graph has the same degree as every other vertex.

※(10~16) 다음 괄호에 알맞은 값이나 용어를 채워 넣으시오.

10. Two cycles of a set A are said to be (disjoint) if no element of A appears in both cycles.
11. The vertices of the tree that have no offspring are called the (external?) of the tree.
12. If all vertices of T, other than the leaves, have exactly 2 offsprings, we say that T is a complete (binary tree
13. Let R be a symmetric relation on a set A. R is connected and (   ).
14. If R is a symmetric, connected relation on a set A, we say that a tree T on A is a (   ) for R if T is a tree with exactly the same vertices as R and which can be obtained from R by deleting some edges of R.
15. (Weighted) graph is a graph for which each edge is labeled with a numerical value.
16. If a graph G is (Connected) and has exactly two vertices of odd degree, there is an Euler path in G.

• FSM 단순화



※ 10.5는 Skip

※(1~5) 다음 문장의 내용이 맞으면 ○표, 틀리면 ×표를 답하시오.

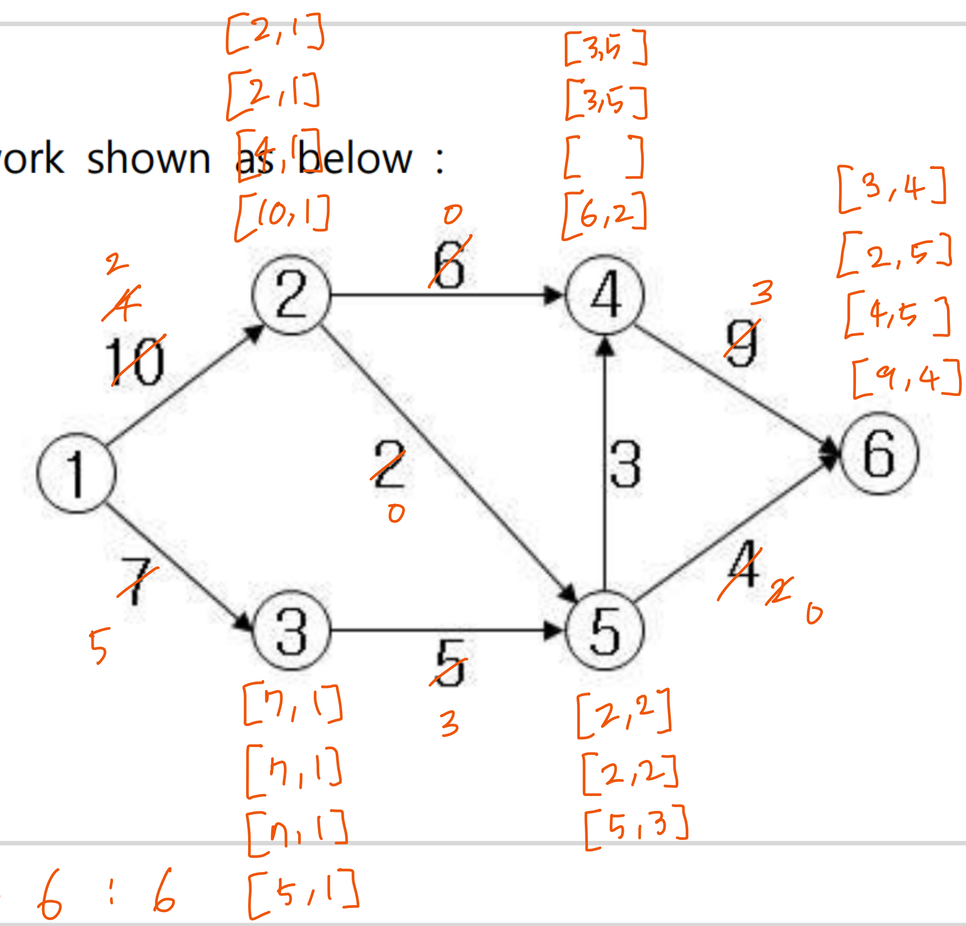
1. All the regular grammars and regular languages can be recognized by a finite state machine.
2. The Backus-Naur From (BNF) notation is for type ~~1~~ 2 grammars.
3. The syntax diagram corresponding to a grammar is unique.
4. In a Finite State Machine (FSM), every state must process every input.
5. There must be only one acceptance state in a Moore machine.

Let N be a transport network shown as below :

$L_1 = \{2, 3\}$

$L_2 = \{4, 5\}$

$L_3 = \{6\}$



1 2 4 6 : 6

1 2 5 6 : 2

1 3 5 6 : 2

1 3 5 4 6 : 3

Let N be a transport network shown as below :

1 2 4 6 : 6

1 2 5 6 : 2

1 3 5 6 : 2

1 3 5 4 6 : 3

$\Rightarrow 13.$