2018-2	이산수학	1차 과제물: 1장~4장
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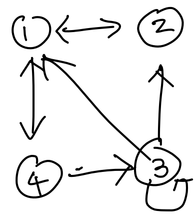
※ 문제 자체는 생략하고 문항번호와 답안만 작성하여 제출하기 바랍니다.

※(1~14) 다음 문장의 내용이 맞으면 ○표, 틀리면 ×표를 답하시오.

1. Let $U=\{1,2,3,4,5,6\}$, $A=\{2,5,6\}$ and $B=\{1,2,4\}$. Then sequence 0,1,0,0,1,1 represents f_A and 1,1,0,0,1,0 represents f_B. (f_A, f_B are characteristic functions) → X
2. Let $A=\{a,b,c\}$. The expression $a^*(b \wedge c)^* bc$ is a regular expression. → X
3. Let $A=\{\mathrm{ab},\mathrm{bc},\mathrm{ba}\}$. All the strings such as $abba$, $babcab$, $bcababc$, $abbcbaab$ belong to A^*. → X
4. If e is an identity for a binary operation, then e is unique. → O
5. Let $A=\{1,2,3,4\}$ and let $R=\{(1,2),(2,2),(3,4),(4,1)\}$. Then R is antisymmetric. → O
6. When the relation R on the set $A=\mathbb{Z}$ and aRb if and only if $|a-b|=2$, the relation R is symmetric and transitive. → X
7. The recurrence relation $f_n = f_{n-1} + 2f_{n-2} - 1$ is a linear homogeneous relation of degree 2. → X
8. Let aRb if and only if GCD$(a,b)=1$ for a,b in $A=Z^+$, then R is an equivalence relation. → X
9. The symmetric closure of a relation R is the smallest symmetric relation containing R. → O
10. Let $A=\{1,2,3,4\}$ and let $R=\{(3,3),(1,2),(2,2),(2,1)(3,4),(4,3),(1,1),(4,4)\}$. Then R is an equivalence relation. → O
11. Let R be the relation on $A=\{1,2,3,4\}$ that has the matrix $M_R = \begin{pmatrix} 0&1&0&1 \\ 1&0&0&0 \\ 1&1&1&0 \\ 1&0&1&0 \end{pmatrix}$.



Vertex 3 has in-degree 3 and out-degree 2. → X

12. Let R be a relation from A to B, and let A_1 and A_2 be subsets of A. Then $R(A_1) \cap R(A_2) \not\subseteq R(A_1 \cap A_2)$. → X
13. ? Let R be a relation on a set A where $|A|=4$ and $|R|=6$. If R is reflexive and antisymmetric then $|R \cup R^{-1}|=8$. → X
14. Let $A=\{1,2,3,4,5,6,7,8\}$ and R be a relation on A. If $M_R \odot M_R = M_R$ then R is transitive. → O

※(15~28) 다음 괄호에 알맞은 값이나 용어를 채워 넣으시오.

15. A(n) (Partition) of a nonempty set A is a collection P of nonempty subsets of A such that
   (1) Each element of A belongs to one of the sets in P.
   (2) If A_1 and A_2 are distinct elements of P, then $A_1 \cap A_2 = \varnothing$.
16. A Mathematical structure is (Closed) with respect to an operation if that operation always produces another member of the collection of objects.
17. If $\star$ is a binary operation, then $\star$ is (associative) if $(x \star y) \star z = x \star (y \star z)$.
18. If a binary operation $\star$ has an identity e, we say y is a $\star$-(inverse) of x if $x \star y = y \star x = e$.
19. A relation R on a set A is antisymmetric if whenever $a \neq b$, then $a \not R b$ (or) $a \not R b$. $b \not R a$ 오타인듯?
20. Let R be a relation on a set A. Then (R^∞) is the transitive closure of R.
21. M_R is the matrix of a relation R. Then $M_{R^{-1}}$ equals the (Symmetric) of the matrix M_R.
22. A symmetric relation R on a set A is called (Connected) if there is a path from any element of A to any other element of A.

23. Six friends discover that they have a total of $2,161 with them on a trip to the movies. One or more of them must have at least $(   ).

24. A path that begins and ends at the same vertex is called a (cycle).

25. A relation R on a set A is (anti-symmetric) if whenever aRb and bRa, then $a=b$.

26. Suppose that R is a relation on a set A. The reflexive closure of R is ($R \cup \Delta$).

27. A relation R on a set A is (asymmetric) if whenever $(a,b)\in R$, then $(b,a)\notin R$

28. A relation R on a set A is called a(n) (equivalence) if it is reflexive, symmetric, and transitive.

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29. Let $A=\begin{bmatrix}1&1&0\\0&1&1\\1&0&1\end{bmatrix}$ and $B=\begin{bmatrix}0&0&1\\1&0&0\\1&1&0\end{bmatrix}$. Compute $A\wedge B, A\vee B$ and $A\odot B$.

$M_{A\wedge B}=\begin{bmatrix}0&0&0\\0&0&0\\1&0&0\end{bmatrix}$

$M_{A\vee B}=\begin{bmatrix}1&1&1\\1&1&1\\1&1&1\end{bmatrix}$

30. Let $A=\{0,1\}$. Give the regular expression corresponding to the given regular set $\{0,1,010,110,01010,11010,\ldots\}$. $\rightarrow (0\vee 1)(10)^*$

31. What is the minimum number of members to guarantee that at least eight of them will have birthdays in the same month? $\rightarrow$ 85

32. Solve the the recurrence relation $b_n = 4b_{n-1} - 4b_{n-2}$ with initial conditions $b_1 = 2$ and $b_2 = 3$.

$b_n = \frac{5}{4}2^n - \frac{n2^n}{4}$

33. Let $A=\{x \mid x\ is\ integer, 0 \le x \le 8\}$ and $R=\{(a,b)\in A\times A \mid a\equiv b\,(\text{mod}\,3)\}$. Determine A/R. $\rightarrow \{\{0,3\},\{0,6\},\{3,6\},\{1,4\},\{1,7\},\{4,7\},\{2,5\},\{2,8\},\{5,8\}\}$

34. Let $A=\{a,b,c,d,e\}$, $R=\{(a,a),(a,b),(a,d),(b,b),(b,c),(b,e),(c,e),(d,a),(d,c),(e,b),(e,d)\}$, and $S=\{(a,a),(a,b),(a,c),(a,d),(b,b),(b,c),(c,c),(c,d),(d,a),(d,e)\}$.

(a) Find $Dom(S)$. $\rightarrow \{a,b,c,d\}$

(b) Find $(R\cap S)(\{a,b,c\})$. $\rightarrow \{(a,a),(a,b),(b,b),(b,c)\}$

(c) Compute $|S\cup\Delta|$. $\rightarrow 13$

(d) Compute $|R\cup R^{-1}|$. $\rightarrow 16$

(e) Compute $M_{(R\cap S)^2}$ $\rightarrow \begin{bmatrix}1&1&1&1\\0&1&1&0\\0&0&0&0\\1&1&0&1\end{bmatrix}$

(f) Compute W_4 as in Warshall's algorithm where $W_0 = M_{(R\cap S)}$. $\rightarrow \begin{bmatrix}1&1&1&1\\0&1&1&0\\0&0&0&0\\1&1&1&1\end{bmatrix}$

35. Let $R=\{(1,1),(1,3),(2,4),(3,1),(4,3),(4,4)\}$, $S=\{(1,2),(1,4),(2,1),(3,1),(3,3),(4,2)\}$ be the relations on $A=\{1,2,3,4\}$. Compute W_0, W_1, and W_2 using Warshall's algorithm where $W_0 = M_{S\circ R}$. $\rightarrow$ $W_0 = M_{S\circ R} = \begin{bmatrix}1&1&1&1\\0&1&0&0\\0&1&0&1\\1&1&1&0\end{bmatrix}$ $W_1 = \begin{bmatrix}1&1&1&1\\0&1&0&0\\0&1&0&1\\1&1&1&1\end{bmatrix}$ $W_2 = \begin{bmatrix}1&1&1&1\\0&1&0&0\\0&1&0&1\\1&1&1&1\end{bmatrix}$

※ 주의사항

1. 과제는 반드시 본인이 직접 손으로 풀어 제출한다.

2. 타인의 과제를 그대로 복제하면, 두 과제 모두 미제출 처리한다.

3. 과제는 제출 기한 내에 담당 조교 연구실 앞에 비치되어 있는 과제함에 제출한다.

4. 제출 기한이후에 제출하면 1점 감점하며, 일주일이 지난 후에는 미제출로 처리된다.

***** 문제를 제외하고 답안만 작성하며, 뒷면을 사용하여 가급적 한 장으로 제출하기 바랍니다. *****