정보통신 수학 및 실습
Homework

학번 : 2016110056

학과 : 불교학부

이름 : 박승원

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Chapter 6 Homework

Show your solutions and do not use MATLAB for your homework except Problem 5.

1. Let A=[1 2 3;4 5 6], B=[1 -1;0 1], C=[-1 0;1 1;0 1], D=[-3 -2 -1;1 2 3]. Compute the followings:

a) $A–C^T$

$$\begin{bmatrix} 2 & 1 & 3 \\ 4 & 4 & 5 \end{bmatrix}$$

b) $C^T + 3D$

$$\begin{bmatrix} -10 & -5 & -3 \\ 3 & 7 & 10 \end{bmatrix}$$

c) BA

$$\begin{bmatrix} -3 & -3 & -3 \\ 4 & 5 & 6 \end{bmatrix}$$

d) CB

$$\begin{bmatrix} -1 & 1 \\ 1 & 0 \\ 0 & 1 \end{bmatrix}$$

e) B^3

$$\begin{bmatrix} 1 & -3 \\ 0 & 1 \end{bmatrix}$$

2. For each of the given matrices, find the inverse or determine that the matrix is singular.

a) [1 -1; -1 1]

ad - bc = 0이므로 singular

b) [1 2;3 4]

$$\begin{bmatrix} -2 & 1 \\ \frac{3}{2} & \frac{1}{2} \end{bmatrix}$$

c) [1 2 3;4 5 6; 7 8 9]

$$\begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \\ 7 & 8 & 9 \end{bmatrix}$$

$$A^{-1} = \frac{adj(A)}{det(A)}$$

$$det(A) = \sum_{i=1}^{n} a_{ij}C_{ij} = a_{11}C_{11} + a_{21}C_{21} + a_{31}C_{31}$$

$$C_{ij} = (-1)^{i+j}M_{ij}$$

$$det(A) = 1 \times (45 - 48) - 4 \times (18 - 24) + 7 \times (12 - 15) = -3 + 24 - 21 = 0$$

$$\therefore singular$$

d) [1 2 3; 1 3 2;0 1 1]

$$\begin{bmatrix} 1 & 2 & 3 \\ 1 & 3 & 2 \\ 0 & 1 & 1 \end{bmatrix}$$

$$det(A) = 1 \times 1 - 1 \times -1 + 0 = 2$$

$$adj(A) = \begin{bmatrix} C_{11} & C_{12} & C_{13} \\ C_{21} & C_{22} & C_{23} \\ C_{31} & C_{32} & C_{33} \end{bmatrix}^T = \begin{bmatrix} 1 & -1 & 1 \\ 1 & 1 & -1 \\ -5 & 1 & 1 \end{bmatrix}^T = \begin{bmatrix} 1 & 1 & -5 \\ -1 & 1 & 1 \\ 1 & -1 & 1 \end{bmatrix}$$

$$A^{-1} = \frac{adj(A)}{2} = \begin{bmatrix} 0.5 & 0.5 & -2.5 \\ -0.5 & 0.5 & 0.5 \\ 0.5 & -0.5 & 0.5 \end{bmatrix}$$

3. Write each of the following linear equation as a matrix equation, and solve by inverting the coefficient matrix

$$x + y + z = 2$$

$$x + 2y + z = 6$$

$$y + z = 1$$

$$\begin{bmatrix} 1 & 1 & 1 \\ 1 & 2 & 1 \\ 0 & 1 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 2 \\ 6 \\ 1 \end{bmatrix}$$

$$det(A) = 1$$

$$adj(A) = \begin{bmatrix} 1 & -1 & 1 \\ 0 & 1 & -1 \\ -1 & 0 & 1 \end{bmatrix}^T = \begin{bmatrix} 1 & 0 & -1 \\ -1 & 1 & 0 \\ 1 & -1 & 1 \end{bmatrix}$$

$$\therefore A^{-1} = \begin{bmatrix} 1 & 0 & -1 \\ -1 & 1 & 0 \\ 1 & -1 & 1 \end{bmatrix}$$

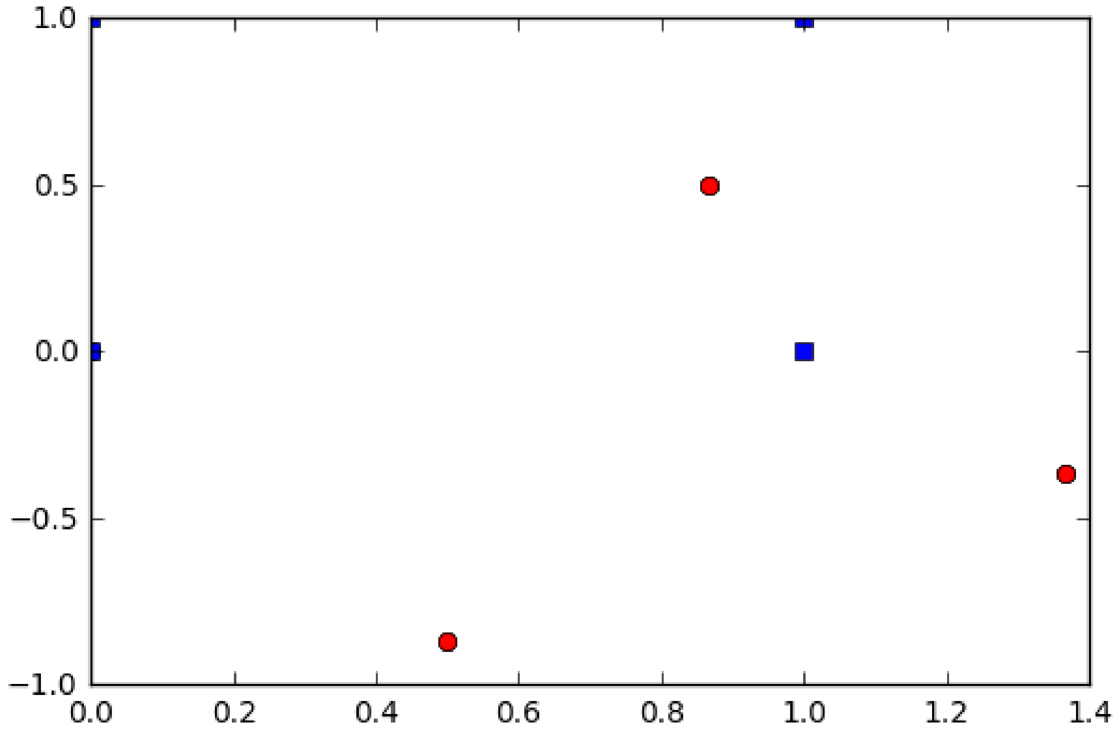
$$\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 1 & 0 & -1 \\ -1 & 1 & 0 \\ 1 & -1 & 1 \end{bmatrix} \begin{bmatrix} 2 \\ 6 \\ 1 \end{bmatrix} = \begin{bmatrix} 1 \\ 4 \\ -3 \end{bmatrix}$$

4. Find a matrix A which rotates a point by 60 degree clockwise . Draw Ax when x is on a unit square as follows:

(1.1)

$$\begin{bmatrix} \cos(-60) & -\sin(-60) \\ \sin(-60) & cos(-60) \end{bmatrix} = \begin{bmatrix} 0.5 & \frac{\sqrt{3}}{2} \\ -\frac{\sqrt{3}}{2} & 0.5 \end{bmatrix}$$

$$\begin{bmatrix} 0.5 & \frac{\sqrt{3}}{2} \\ -\frac{\sqrt{3}}{2} & 0.5 \end{bmatrix} \begin{bmatrix} 0 & 1 & 0 & 1 \\ 0 & 0 & 1 & 1 \end{bmatrix} = \begin{bmatrix} 0 & 0.5 & \frac{\sqrt{3}}{2} & \frac{1+\sqrt{3}}{2} \\ 0 & -\frac{\sqrt{3}}{2} & 0.5 & \frac{1-\sqrt{3}}{2} \end{bmatrix}$$



5. **Find the Eigen values and Eigen vectors of A = [9 14; 1 4] and draw the trace of Ax if x is a vector on the unit circle using MATLAB. Then calculate A^5 using MATLAB.**

$$Av = \lambda v$$
$$det(A - \lambda I) = 0$$

$$\left\|\begin{matrix} 9-\lambda & 14 \\ 1 & 4-\lambda \end{matrix}\right\| = 0$$

$$36 - 13\lambda + \lambda^2 - 14 = 0$$
$$(\lambda - 11)(\lambda - 2) = 0$$
$$\triangleright \lambda = 11$$
$$\begin{bmatrix} -2 & 14 \\ 1 & -7 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = 0$$
$$v = k \begin{bmatrix} 1 \\ -7 \end{bmatrix}$$
$$\triangleright \lambda = 2$$
$$v = k \begin{bmatrix} 1 \\ 2 \end{bmatrix}$$

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th = linspace(0, 2*pi, 100);
x = cos(th);
y = sin(th);
A = [9 14; 1 4]
B = A * [x;y];
t = linspace (-0.5,0.5,100) ;
plot(x,y, B (1,:) , B (2,:) , t, -7*t, t, 2*t)
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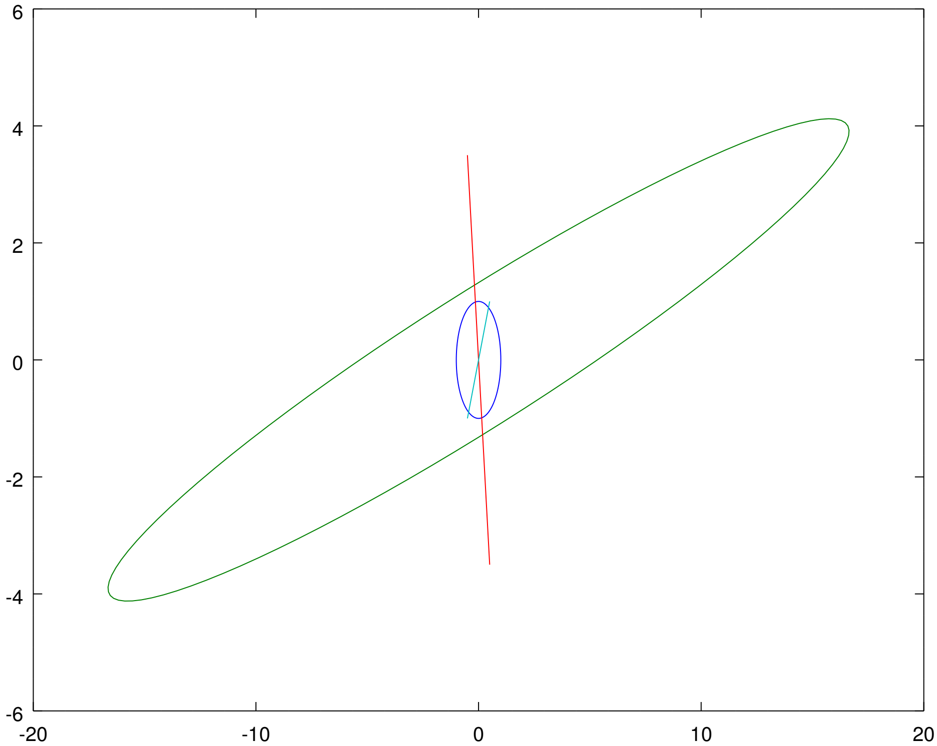

Matrix decomposition

$$V = [v_1, v_2] = \begin{bmatrix} 1 & 1 \\ -7 & 2 \end{bmatrix}$$

$$AV = \begin{bmatrix} 9 & 14 \\ 1 & 4 \end{bmatrix} \begin{bmatrix} 1 & 1 \\ -7 & 2 \end{bmatrix} = \lambda V = [11v_1, 2v_2] = \begin{bmatrix} 1 & 1 \\ -7 & 2 \end{bmatrix} \begin{bmatrix} 11 & 0 \\ 0 & 2 \end{bmatrix} = V\Lambda$$

$$\therefore A^5 = M\Lambda^5 M^{-1} = \begin{bmatrix} 1 & 1 \\ -7 & 2 \end{bmatrix} \begin{bmatrix} 11^5 & 0 \\ 0 & 2^5 \end{bmatrix} \frac{1}{9} \begin{bmatrix} 2 & -1 \\ 7 & 1 \end{bmatrix} = \begin{bmatrix} 125269 & 250474 \\ 17891 & 35814 \end{bmatrix}$$