정보통신 수학 및 실습
Homework

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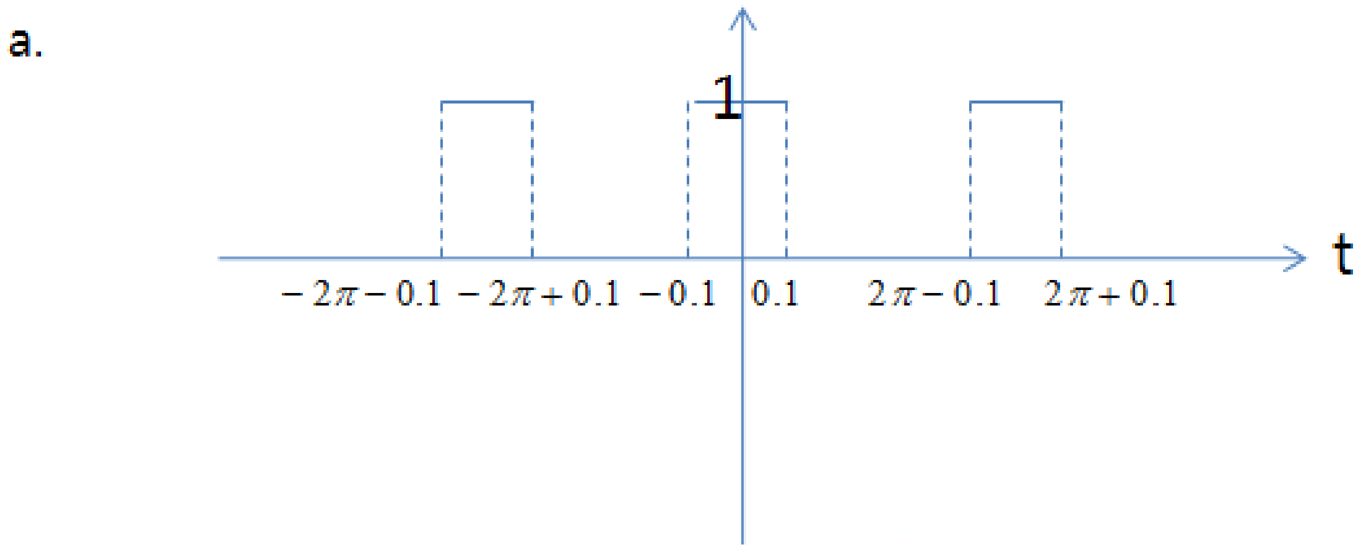
Chapter 10 Homework

1. Find the Fourier series of the following functions.



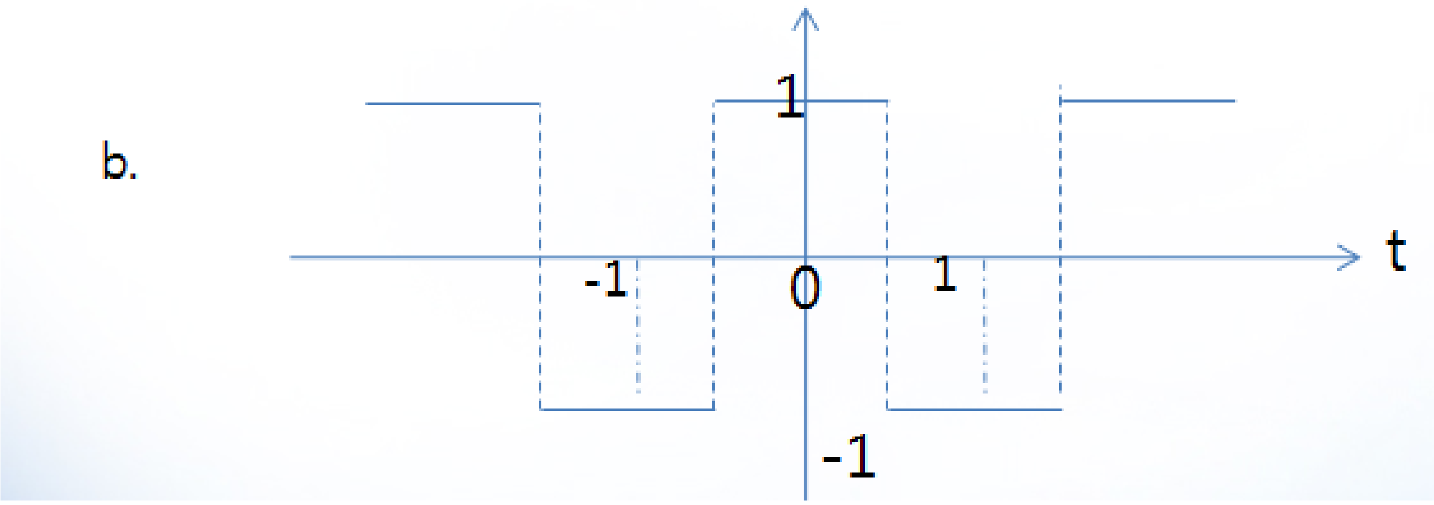
even function

$$f(t) = a_0 + b_1 \cos t + b_2 \cos 2t + \cdots$$

$$a_0 = \int_{-\pi}^{\pi} f(t)dt = 0.2$$

$$b_n = \int_{-\pi}^{\pi} f(t)\cos(nt)dt = \int_{-0.1}^{0.1} \cos(nt)dt = \left[\frac{1}{n}\sin(nt)\right]_{-0.1}^{0.1} = \frac{2}{n}\sin(0.1n)$$

$$\therefore f(t) = \sum_{k=1}^{n} 0.2 + \frac{2}{k}\sin(0.1k)$$



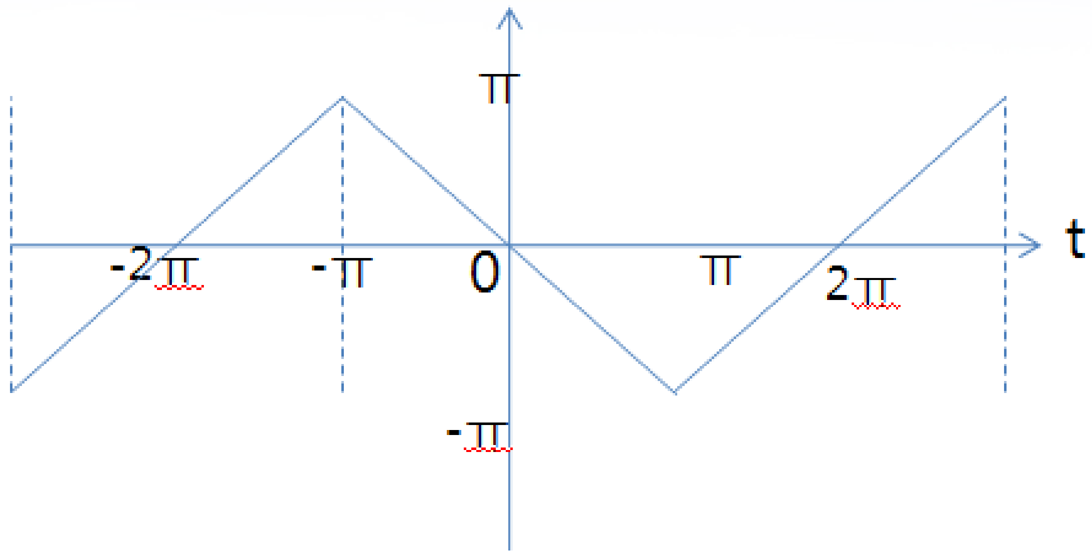
$$f(t) = a_0 + b_1 \cos \pi t + b_2 \cos 2\pi t + \cdots$$

$$a_0 = \int_{-1}^{1} f(t) = 0$$

$$b_n = \int_{-1}^{1} f(t)\cos(n\pi t)dt = 2(\int_{0}^{0.5} \cos(n\pi t)dt - \int_{0.5}^{1} \cos(n\pi t)dt) = 2(\left[\frac{1}{n\pi}\sin(n\pi t)\right]_{0}^{0.5} - \left[\frac{1}{n\pi}\sin(n\pi t)\right]_{0.5}^{1}) = \frac{4}{n\pi}$$

$$\therefore f(t) = \sum_{k=1}^{n} \frac{4}{k\pi}\cos(k\pi t)$$

c.



$$f(t) = a_0 + b_1 \sin 0.5t + b_2 \sin t + b_3 \sin 1.5t \cdots$$

$$a_0 = \int_{-2\pi}^{2\pi} f(t)dt = 0$$

$$b_n = \int_{-2\pi}^{2\pi} f(t)\sin(0.5nt)dt = 4\int_0^{\pi} f(t)\sin(0.5nt)dt = -4\int_0^{\pi} t\sin(0.5nt)dt$$

$$\int uv' = uv - \int u'v$$

$$= -4(\left[t \cdot -\frac{2}{n}\cos\frac{nt}{2}\right]_0^{\pi} + \int_0^{\pi} \frac{2}{n}\cos\frac{nt}{2}dt) = -4(\frac{-2t}{n}\cos(n\pi) + \left[\frac{4}{n^2}\cdot\sin\frac{nt}{2}\right]_0^{\pi}) = -4(\frac{-2t}{n}\cos(\frac{n\pi}{2}) + \frac{4}{n^2}\sin(\frac{n\pi}{2})$$

d.

$$f(x) = \begin{Bmatrix} 0 & if -\pi \le x < 0 \\ \cos x & if\ 0 \le x < \pi \end{Bmatrix}$$

$$f(x + 2\pi) = f(x)$$

$$f(t) = a_0 + b_1 e^{jt} + b_2 e^{2jt} + \cdots$$

$$a_0 = \int_{-\pi}^{\pi} f(t)dt = \int_0^{\pi} \cos t dt = [\sin t]_0^{\pi} = 0$$

$$b_n = \int_{-\pi}^{\pi} f(t)e^{-jnt}dt = \int_0^{\pi} \cos t e^{-jnt}dt = \left[e^{-jnt}\sin t\right]_0^{\pi} - \int_0^{\pi} -jne^{-jnt}\sin t dt = jn\int_0^{\pi} e^{-jnt}\sin t dt$$

$$= jn(\left[e^{-jnt}\cdot -\cos t\right]_0^{\pi} - \int_0^{\pi} jne^{-jnt}\cos t dt)$$

$$= jn((e^{-jn\pi} + 1) - n^2(\left[e^{-jnt} sint\right]_0^{\pi}) + \int_0^{\pi} -jne^{-jnt}\sin t dt)$$

$$b_n = jn(e^{-jn\pi} + 1) - jnb_n)$$

$$b_n = jne^{-jn\pi} + jn + n^2 b_n$$

$$\therefore b_n = \frac{jne^{-jn\pi} + jn}{1 - n^2}$$