정보통신 수학 및 실습
Homework

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날짜 : 2017년 5월 17일

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Chapter 9 Homework

1. Solve the Laplace transformations of the following functions using the tables.

a) $L[5]$

$$\int_0^\infty 5e^{-st}dt = \left[\frac{5}{-s}e^{-st}\right]_0^\infty = \frac{5}{s}$$

b) $L[5e^{-3t}]$

$$\int_0^\infty 5e^{-3t}e^{-st}dt = \int_0^\infty 5e^{-(s+3)t}dt = \frac{5}{s+3}$$

c) $L[cos(5t)]$

$$cos5t = \frac{cos5t + isin5t + cos(-5t) + isin(-5t)}{2} = \frac{e^{i5t} + e^{-i5t}}{2}$$
$$\int_0^\infty cos(5t)e^{-st}dt = \frac{1}{2}\int_0^\infty (e^{i5t} + e^{-i5t})e^{-st}dt = \frac{1}{2}\int_0^\infty (e^{(i5-s)t} + e^{(-i5-s)t})dt = \frac{1}{2}\left[\frac{e^{(5i-s)t}}{5i-s} - \frac{e^{-(s+5i)t}}{s+5i}\right]_0^\infty =$$
$$\frac{1}{2}(\frac{1}{-s+5i} - \frac{1}{s+5i}) = \frac{s}{s^2+25}$$

d) $L[5e^{-3t}cos(5t+1)]$

$$\int_0^\infty 5e^{-3t}\frac{e^{i(5t+1)} + e^{-i(5t+1)}}{2}e^{-st}dt = \frac{5}{2}\int_0^\infty e^{(5i-s-3)t+i} + e^{(-5i-s-3)t-i}dt = \frac{5}{2}\left[\frac{e^{(5i-s-3)t+i}}{5i-s-3} + \frac{e^{(-5i-s-3)t-i}}{-5i-s-3}\right]_0^\infty =$$
$$\frac{5}{2}(-\frac{e^i}{5i-s-3} - \frac{e^{-i}}{-5i-s-3})$$

e) $L[sin(5t)]$

$$sin5t = -\frac{i}{2}(cos5t + isin5t - (cos(-5t) + isin(-5t))) = -\frac{i}{2}(e^{i5t} - e^{-i5t})$$
$$\int_0^\infty sin(5t)e^{-3t}dt = \int_0^\infty -\frac{i}{2}(e^{i5t} - e^{-i5t})e^{-3t}dt$$

f) $L[t^2sin(2t)]$

$$\int_0^\infty t^2sin(2t)dt = -\frac{i}{2}\int_0^\infty t^2(e^{i2t} - e^{-i2t})dt =$$
$$(e^{i2t}t^2)' = 2ie^{i2t}t^2 + e^{i2t}2t$$
$$e^{i2t}t^2 = 2\int e^{i2t}t^2 + e^{i2t}$$
$$\therefore \int e^{i2t}t^2 = \frac{e^{i2t}(t^2-1)}{2}$$
$$t \leftarrow -t : \int e^{-i2t}t^2 = \frac{e^{-i2t}(t^2-1)}{2}$$
$$-\frac{i}{2}\left[\frac{e^{i2t}(t^2-1)}{2} - \frac{e^{-i2t}(t^2-1)}{2}\right]_0^\infty = -\frac{i}{2}$$

g) $L[\frac{d}{dt}sin(6t)]$

$$(f(x)e^{-st})' = f'(x)e^{-st} + f(x)\cdot -se^{-st}$$
$$\int \frac{d}{dt}f(x)e^{-st}dt = f(x)e^{-st} + s\int f(x)e^{-st}$$

$$\therefore L[\frac{d}{dt}sin(6t)] = sin(6t)e^{-st} + s\int sin(6t)e^{-st}dt$$

h) $L[\int_0^t ydt]$

$$F(t) = \int_0^t f(u)du$$
$$\int_0^\infty \int_0^t f(u)du \cdot e^{-st}dt = \left[\int_0^t f(u)du \cdot \frac{e^{-st}}{-s}\right]_0^\infty - \int_0^\infty f(t)\frac{e^{-st}}{-s}dt = \frac{F(s)}{s} = \frac{1}{s^3}$$
$$\because L[y] = \frac{1}{s^2}$$

i) $L[u(t-1) - u(t-2)]$

$$= \int_1^2 e^{-st}dt = \left[\frac{e^{-st}}{-s}\right]_1^2$$

j) $L[e^{-2t}u(t-2)]$

$$= \int_2^\infty e^{-(2+s)t}dt = \left[\frac{e^{-(2+s)t}}{-2-s}\right]_2^\infty$$

k) $L[6e^{-2.7t}cos(9.2t+3)]$

$$L[\cos(9.2t)] = \frac{9.2}{s^2 + 9.2^2}$$
$$L[6e^{-2.7t}cos(9.2t+3)] = \frac{9.2 \times 6}{(s+2.7)^2 + 9.2^2}$$

2. Solve the inverse Laplace transformations using the tables..

a) $L^{-1}[\frac{5}{s+1}]$

$5e^{-t}$

b) $L^{-1}[\frac{6}{s^2}]$

$6t$

c) $L^{-1}[\frac{s+2}{(s+3)(s+4)}]$

$$\frac{-1}{s+3} + \frac{2}{s+4} \longrightarrow -e^{-3t} + 2e^{-4t}$$

d) $L^{-1}[\frac{6}{-s^2+6}]$

$$-\frac{\sqrt{6}}{2}(\frac{1}{s+\sqrt{6}} - \frac{1}{s-\sqrt{6}}) \longrightarrow -\frac{\sqrt{6}}{2}(e^{-\sqrt{6}t} - e^{\sqrt{6}t})$$

e) $L^{-1}[\frac{5}{s}(1-e^{-4.5s})]$

$L[f(x)g(x)] = F(s)G(s)$
$L^{-1}\left[\frac{5}{s}\right] = 5$
$\therefore \delta(t) - \frac{5}{s+4.5}$

f) $L^{-1}[\frac{4+3j}{s+1-2j} + \frac{4-3j}{s+1+2j}]$

$2|A|e^{-\alpha t}cos(\beta t+\theta) = \frac{A}{s+\alpha-\beta j} + \frac{\bar{A}}{s+\alpha+\ betaj}$
$10e^{-t}\cos(2t)$

g) $L^{-1}[\frac{6}{4s^2+20s+24}]$

$\frac{6}{4(s+2)(s+3)} = \frac{3}{2}(\frac{1}{s+2} - \frac{1}{s+3})$
$\therefore \frac{3}{2}(e^{-2t} - e^{-3t})$

3. Solve the inverse Laplace transformations using partial fractions and the tables

a) $L^{-1}[\frac{1}{s^2-8s+25}]$

$\frac{1}{(s-4-3j)(s-4+3j)} = \frac{a+bj}{s-4-3j} + \frac{a-bj}{s-4+3j}$
$a = 0, b = -\frac{1}{6}$
$\therefore \frac{1}{3}e^{-3t}cos(4t)$

b) $L^{-1}[\frac{9s+4}{(s+3)^3}]$

$\frac{9s+27-23}{(s+3)^3} = \frac{9}{(s+3)^2} + \frac{-23}{(s+3)^3}$
$\therefore 9te^{-3t} - \frac{23}{2}t^2e^{-3t}$

4. Find the system transfer functions of the following DE's.

a) $5\frac{d^2y}{dt^2} + 6\frac{dy}{dt} + 2y = 5x(t)$

$5s^2Y(s) + 6sY(s) + 2Y(s) = 5X(s)$
$\therefore \frac{5}{5s^2+6s+2}$

b) $\frac{dy}{dt} + 8y = 3x$

$sY(s) + 8Y(s) = 3X(s)$
$\therefore \frac{3}{s+8}$