Everything You Ever wanted To Know About Statistics

Chapter 2



Aims and Objectives

- Know what a statistical model is and why we use them.
 - The Mean
- Know what the 'fit' of a model is and why it is important.
 - The Standard Deviation
- Distinguish models for samples and populations





The Research Process

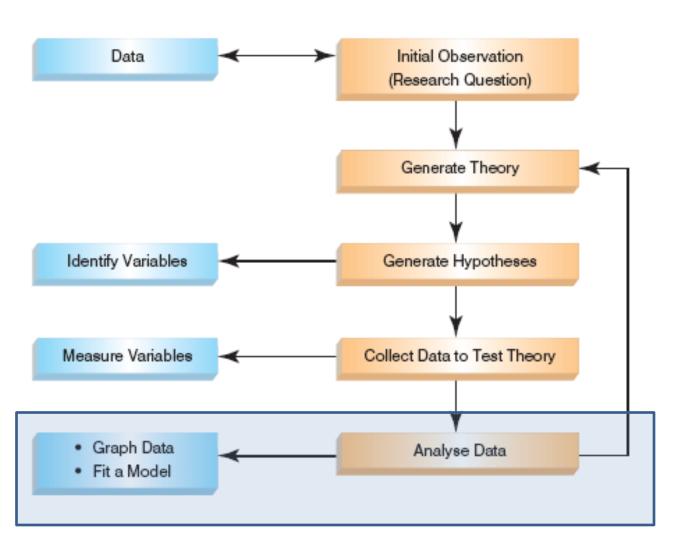
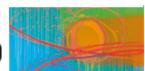


FIGURE 1.2 The research

process



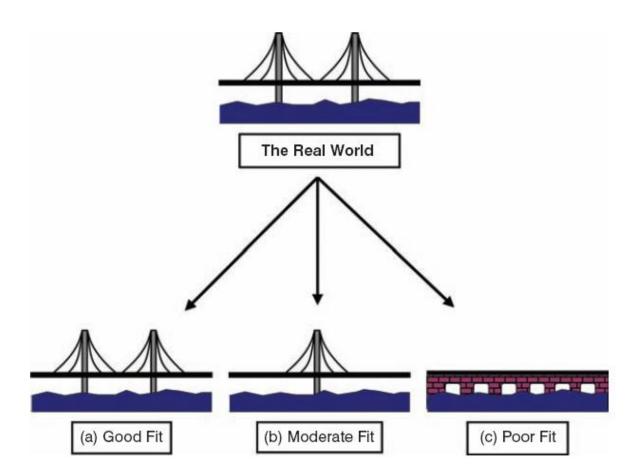
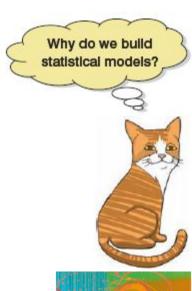


FIGURE 2.2

Fitting models to real-world data (see text for details)



ANDY FIELD

Populations and Samples

Population

The collection of units (be they people, plankton, plants, cities, suicidal authors, etc.) to which we want to generalize a set of findings or a statistical model.

Sample

 A smaller (but hopefully representative)
 collection of units from a population used to determine truths about that population



The Only Equation You Will Ever Need



A Simple Statistical Model

- In Statistics we fit models to our data (i.e. we use a statistical model to represent what is happening in the real world).
- The mean is a hypothetical value (i.e. it doesn't have to be a value that actually exists in the data set).
- As such, the mean is simple statistical model.



The Mean

- The mean is the sum of all scores divided by the number of scores.
- The mean is also the value from which the (squared) scores deviate least (it has the least error).

Mean
$$(\overline{X}) = \frac{\sum_{i=1}^{n} x_{i}}{n}$$

The Mean: Example

Collect some data:

Add them up:

$$\sum_{i=1}^{n} x_i = 1 + 3 + 4 + 3 + 2 = 13_3$$

• Divide by the number of scores, n:

$$\overline{X} = \frac{\sum_{i=1}^{n} x_i}{n} = \frac{13}{5} = 2.6$$



The mean as a model

Outcome_{lecturer1} =
$$(\overline{X})$$
+ error_{lecturer1}

$$1 = 2.6 + \text{error}_{lecturer1}$$

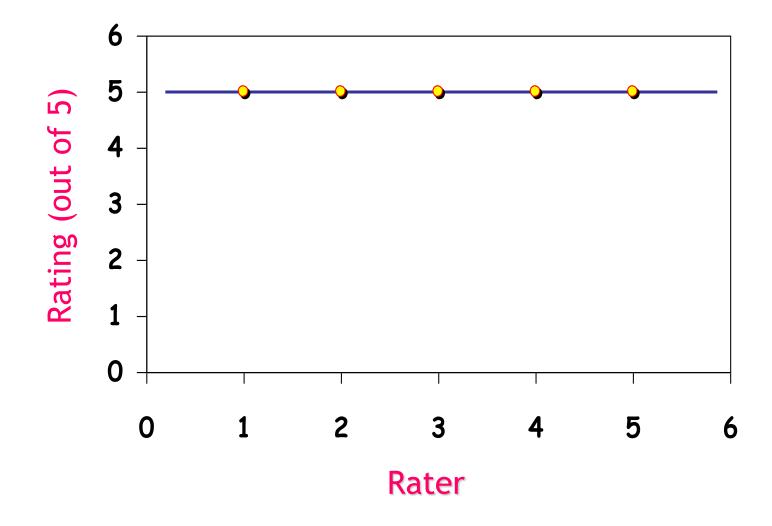
Measuring the 'Fit' of the Model

- The mean is a model of what happens in the real world: the typical score
- It is not a perfect representation of the data
- How can we assess how well the mean represents reality?





A Perfect Fit





Calculating 'Error'

- A deviation is the difference between the mean and an actual data point.
- Deviations can be calculated by taking each score and subtracting the mean from it:

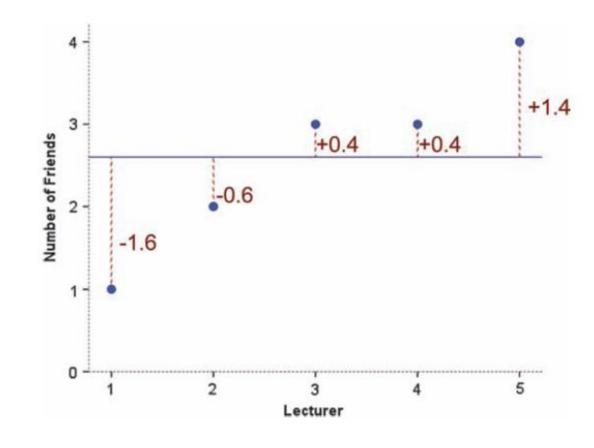
Deviation =
$$x_i - \bar{x}$$





FIGURE 2.4

Graph showing
the difference
between the
observed number
of friends that
each statistics
lecturer had, and
the mean number
of friends





Use the Total Error?

 We could just take the error between the mean and the data and add them.

Score	Mean	Deviation
1	2.6	-1.6
2	2.6 -0.6	
3	2.6 0.4	
3	2.6	0.4
4	2.6 1.4	
	Total =	0

$$\sum (X - \overline{X}) = 0$$

Sum of Squared Errors

- We could add the deviations to find out the total error.
- Deviations cancel out because some are positive and others negative.
- Therefore, we square each deviation.
- If we add these squared deviations we get the Sum of Squared Errors (SS).

Score	Mean	Deviation	Squared Deviation
1	2.6	-1.6	2.56
2	2.6	-0.6	0.36
3	2.6	0.4	0.16
3	2.6	0.4	0.16
4	2.6	1.4	1.96
		Total	5.20

$$SS = \sum (X - \overline{X})^2 = 5.20$$



Variance

- The sum of squares is a good measure of overall variability, but is dependent on the number of scores.
- We calculate the average variability by dividing by the number of scores (n).
- This value is called the variance (s^2) .

variance
$$(s^2) = \frac{SS}{N-1} = \frac{\sum (x_i - \overline{x})^2}{N-1} = \frac{5.20}{4} = 1.3$$

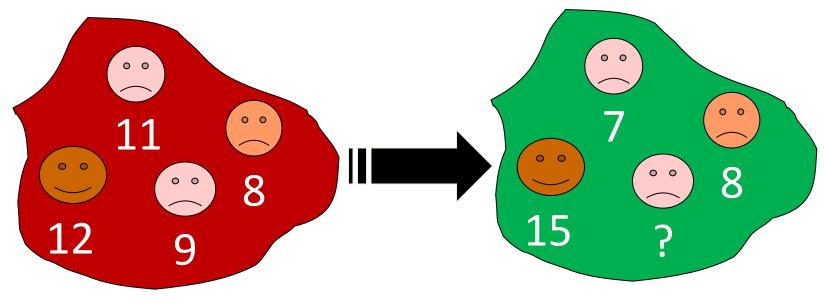


Degrees of Freedom





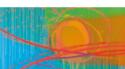
Population



$$\overline{X} = 10$$

$$\mu = 10$$

ANDY FIELD





Standard Deviation

- The variance has one problem: it is measured in units squared.
- This isn't a very meaningful metric so we take the square root value.
- This is the Standard Deviation (s).

$$S = \sqrt{\frac{\sum_{i=1}^{n} (x_i - \bar{x})^2}{n_n}} = \sqrt{\frac{5.20}{5_5}} = 1.02$$



Important Things to Remember

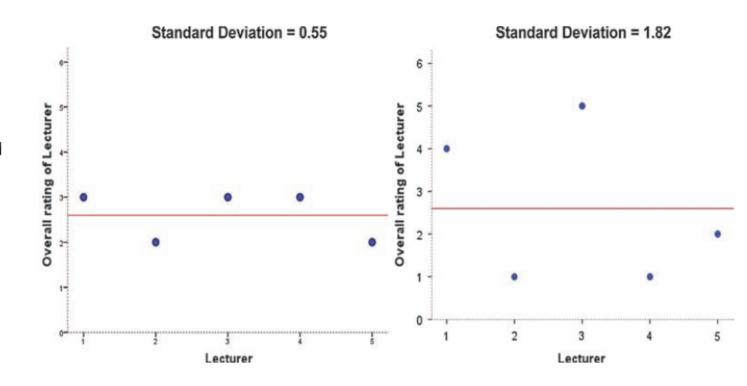
- The Sum of Squares, Variance, and Standard Deviation represent the same thing:
 - The 'Fit' of the mean to the data
 - The variability in the data
 - How well the mean represents the observed data
 - Error



Same Mean, Different SD

FIGURE 2.5

Graphs illustrating data that have the same mean but different standard deviations





The SD and the Shape of a Distribution

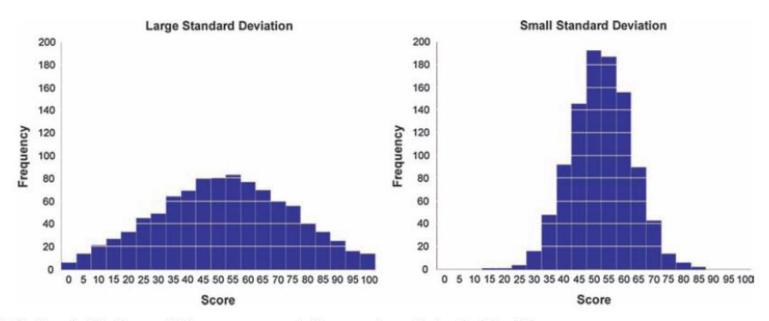


FIGURE 2.6 Two distributions with the same mean, but large and small standard deviations

Samples Vs. Populations

Sample

 Mean and SD describe only the sample from which they were calculated.

Population

 Mean and SD are intended to describe the entire population (very rare in Psychology).

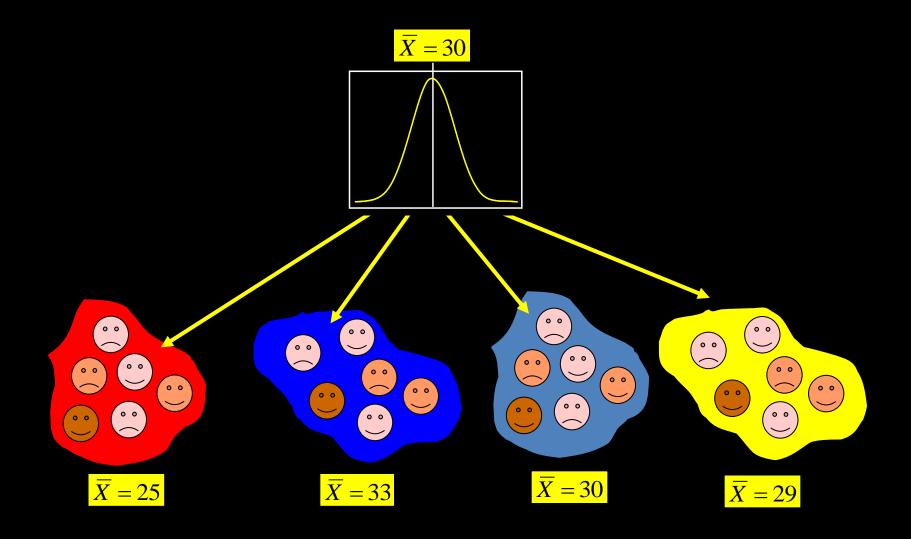
Sample to Population:

 Mean and SD are obtained from a sample, but are used to estimate the mean and SD of the population (very common in psychology).

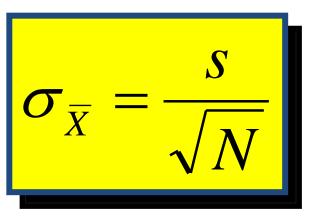




Sampling Variation



Population M = 10M = 9M = 11M = 10M = 9M = 12M = 11M = 10Mean = 10 SD = 1.22 3 Frequency 0 -11 12 13 6 7 8 9 10 14 Sample Mean



Confidence Intervals

- Domjan et al. (1998)
 - 'Conditioned' sperm release in Japanese Quail.
- True Mean
 - 15 Million sperm
- Sample Mean
 - 17 Million sperm
- Interval estimate
 - 12 to 22 million (contains true value)
 - 16 to 18 million (misses true value)
 - Cls constructed such that 95% contain the true value.

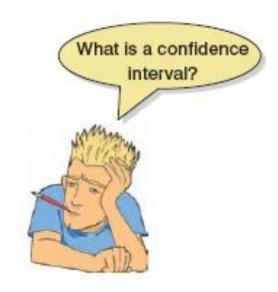
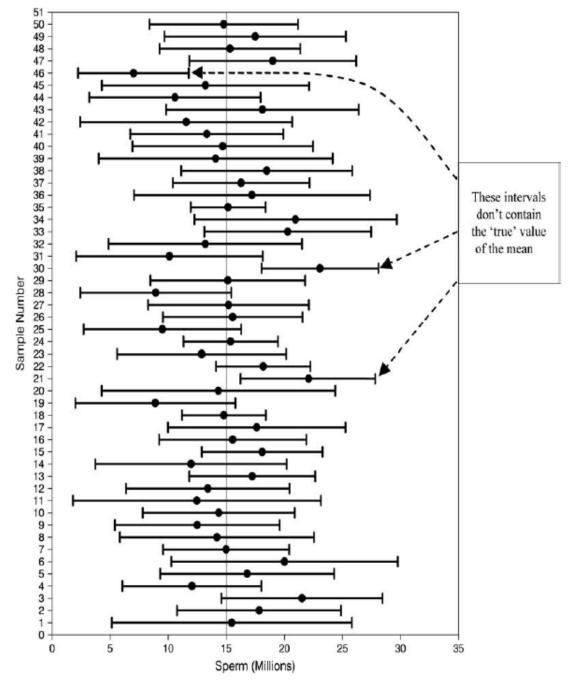
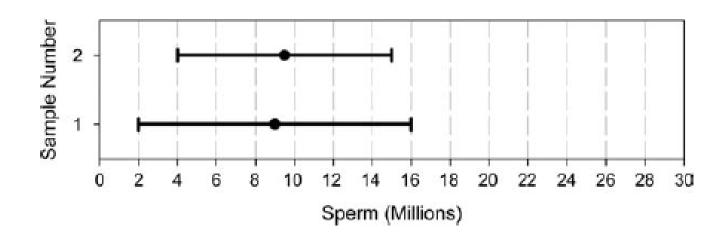
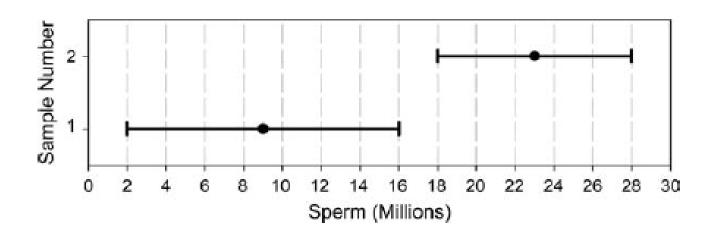


FIGURE 2.8

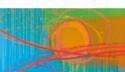
The confidence intervals of the sperm counts of Japanese quail (horizontal axis) for 50 different samples (vertical axis)











Test Statistics

- A Statistic for which the frequency of particular values is known.
- Observed values can be used to test hypotheses.

$$test \ statistic = \frac{variance \ explained \ by \ the \ model}{variance \ not \ explained \ by \ the \ model} = \frac{effect}{error}$$



One- and Two-Tailed Tests

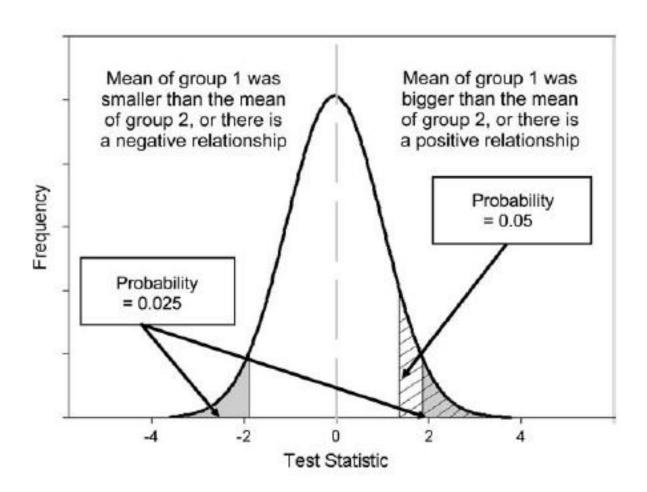
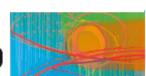


FIGURE 2.10

Diagram to show the difference between oneand two-tailed tests



Type I and Type II Errors

Type I error

- occurs when we believe that there is a genuine effect in our population, when in fact there isn't.
- The probability is the α -level (usually .05)

Type II error

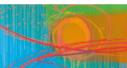
- occurs when we believe that there is no effect in the population when, in reality, there is.
- The probability is the β -level (often .2)



What does Statistical Significance Tell Us?

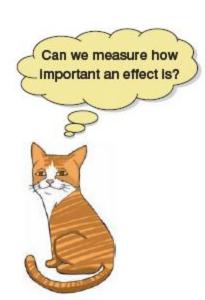
- The importance of an effect?
 - No, significance depends on sample size.
- That the null hypothesis is false?
 - No, it is always false.
- That the null hypothesis is true?
 - No, it is never true.





Effect Sizes

- An effect size is a standardized measure of the size of an effect:
 - Standardized = comparable across studies
 - Not (as) reliant on the sample size
 - Allows people to objectively evaluate size of observed effect.









Effect Size Measures

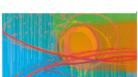
- r = .1, d = .2 (small effect):
 - the effect explains 1% of the total variance.
- r = .3, d = .5 (medium effect):
 - the effect accounts for 9% of the total variance.
- r = .5, d = .8 (large effect):
 - the effect accounts for 25% of the variance.
- Beware of these 'canned' effect sizes though:
 - The size of effect should be placed within the research context.



Effect Size Measures

- There are several effect size measures that can be used:
 - Cohen's d
 - Pearson's r
 - Glass' ∆
 - Hedges' g
 - Odds Ratio/Risk rates
- Pearson's r is a good intuitive measure
 - Oh, apart from when group sizes are different ...

Andy Field



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