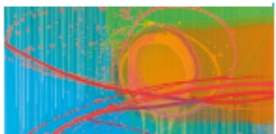
Analysis of Covariance, ANCOVA (GLM2)

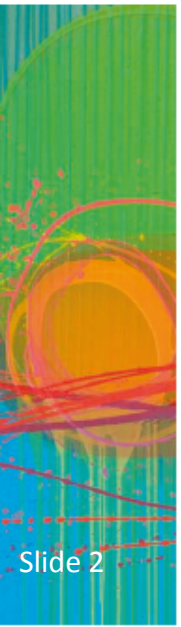
Lecture 10

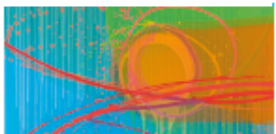
ANDY FIELD



Aims

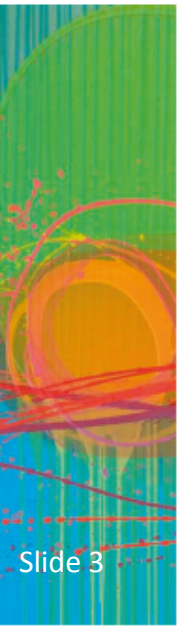
- When and Why do we use ANCOVA?
- Partitioning Variance
- Carrying out on PASW/SPSS
- Interpretation
  - Main Effects
  - Covariates

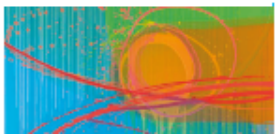




When And Why

- To test for differences between group means when we know that an extraneous variable affects the outcome variable.
- Used to control known extraneous variables.

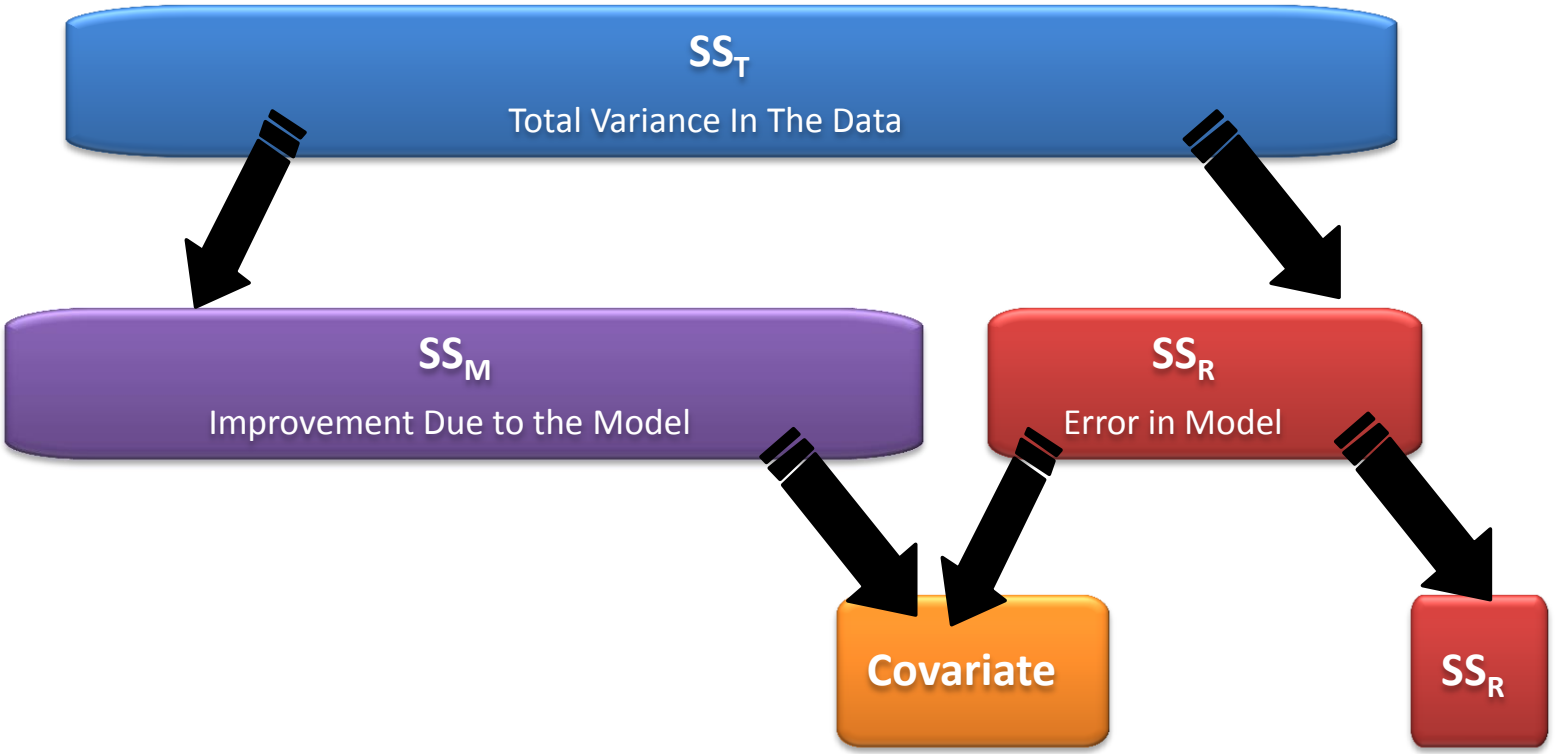




Advantages of ANCOVA

- Reduces Error Variance
  - By explaining some of the unexplained variance (SS_R) the error variance in the model can be reduced.
- Greater Experimental Control:
  - By controlling known extraneous variables, we gain greater insight into the effect of the predictor variable(s).

Variance

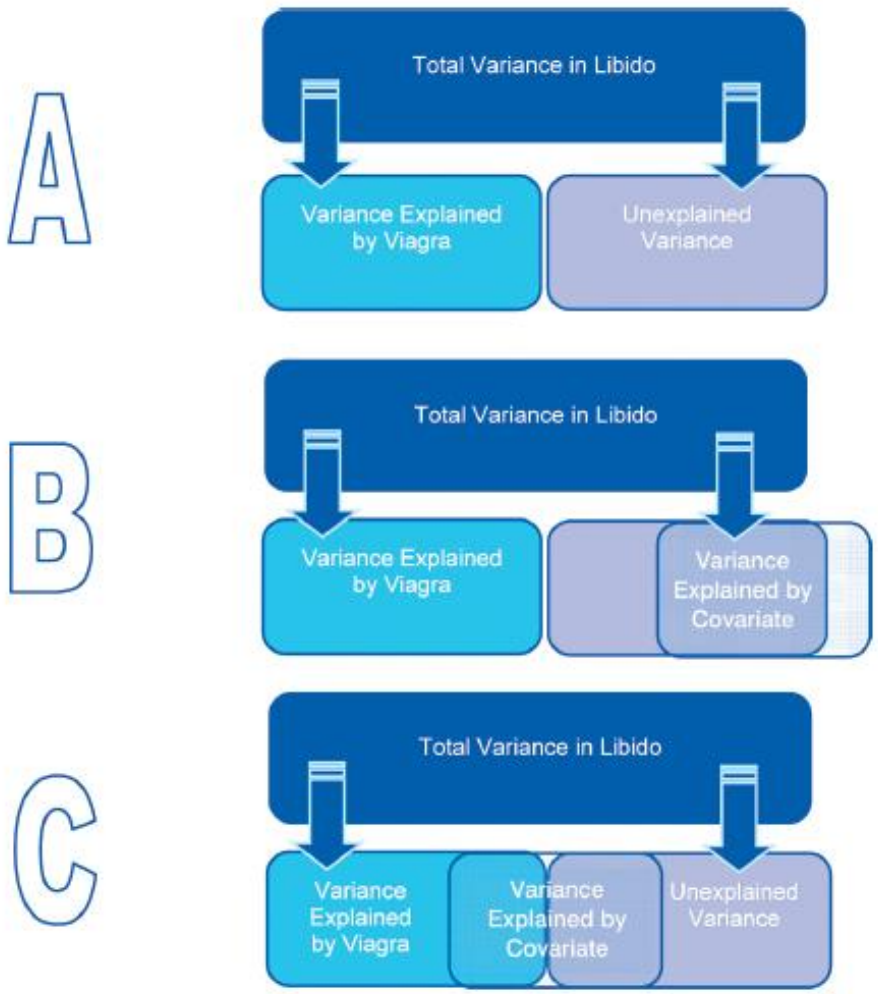

SS_T
Total Variance In The Data
SS_M
Improvement Due to the Model
SS_R
Error in Model
Covariate
SS_R

An Example

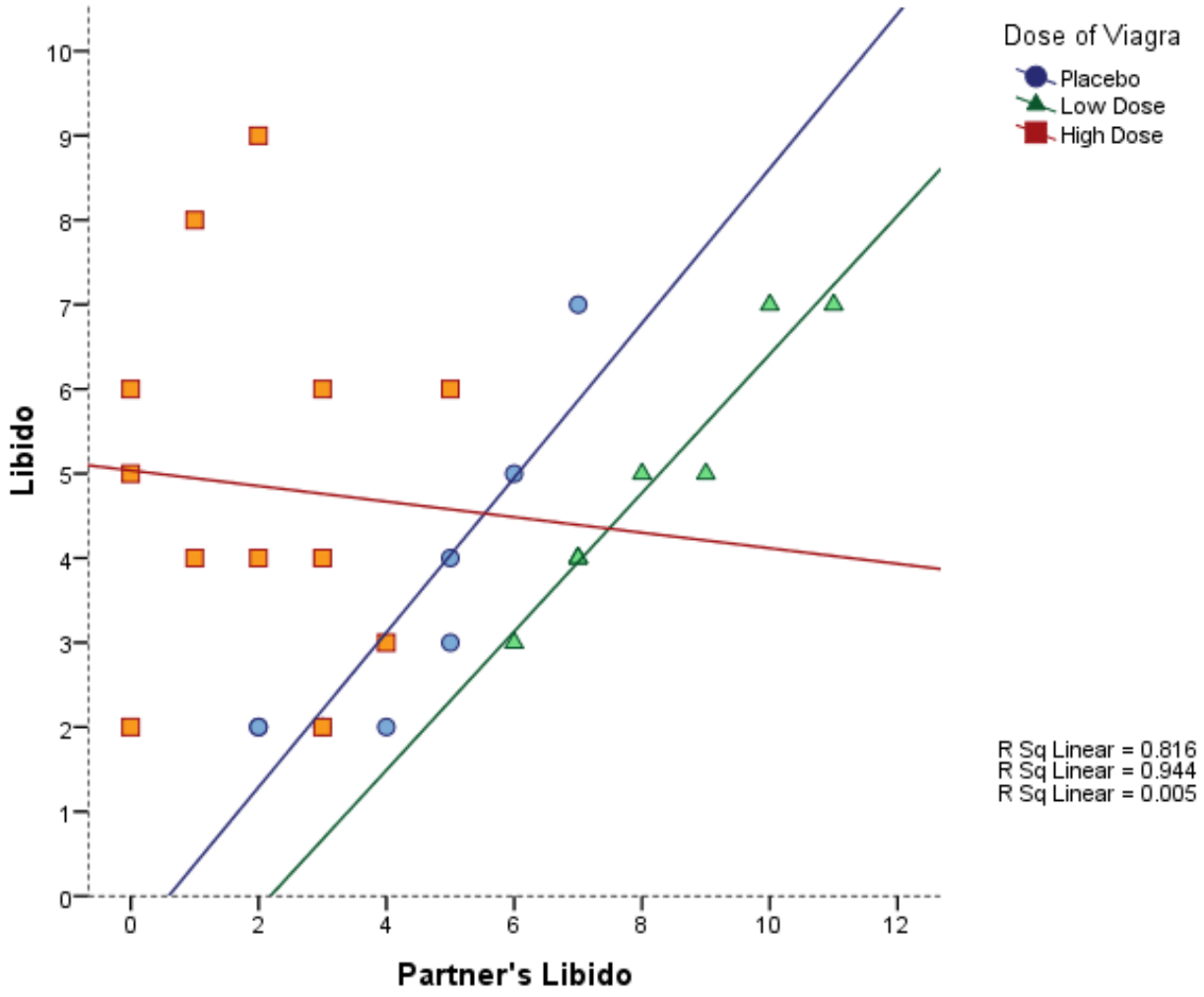
- We will use Field's (2009) Viagra example (from the ANOVA lecture).
  - There are several possible confounding variables – e.g. Partner's libido, medication.
- We can conduct the same study but measure partner's libido over the same time period following the dose of Viagra.
  - Outcome (or DV) = Participant's libido
  - Predictor (or IV) = Dose of Viagra (Placebo, Low & High)
  - Covariate = Partner's libido

Relationships between the IV and Covariate

FIGURE 11.2
The role of the covariate in ANCOVA (see text for details)



Homogeneity of Regression Slopes



Dose	Participant's Libido	Partner's Libido
Placebo	3.22 (1.79)	3.44 (2.07)
Low Dose	4.88 (1.46)	3.12 (1.73)
High Dose	4.85 (2.12)	2.00 (1.63)

How Does ANCOVA Work?

- Imagine we had just two groups:
  - Placebo
  - Low Dose
- This paradigm can be expressed as a regression equation using a dummy coding variable:

$$Y_i = b_0 + b_1 X_i$$

$$Libido_i = b_0 + b_1 Dose_i$$

Dummy Coding

- Dummy Coding
  - Placebo = 0, Low Dose = 1
  - When Dose = Placebo, Predicted Libido = mean of placebo group:

$$\bar{X}_{Placebo} = (b_1 \times 0) + b_0$$
$$\bar{X}_{Placebo} = b_0$$

  - When Dose = Low Dose, Predicted Libido = mean of Low Dose group:

$$\bar{X}_{LowDose} = (b_1 \times 1) + b_0$$
$$\bar{X}_{LowDose} = b_1 + \bar{X}_{Placebo}$$
$$\bar{X}_{LowDose} - \bar{X}_{Placebo} = b_1$$

ANOVA As Regression

- We can run a regression with Libido as the outcome and the Dose (Placebo or Low) as the predictor, Note:
  - Intercept is the mean of Placebo group
  - *b* for the Dummy Variable is the difference between the means of the placebo and low dose group (4.88-3.22 = 1.66)

Coefficients[a]

Model		Unstandardized Coefficients		Standardized Coefficients		
		B	Std. Error	Beta	t	Sig.
1	(Constant)	3.222	.547		5.888	.000
	Dummy Variable 1 (Placebo vs. Low)	1.653	.798	.472	2.072	.056

a. Dependent Variable: Libido

ANCOVA

- ANCOVA extends this basic idea.
- The covariate can be added to the regression model of the ANOVA.
- To evaluate the effect of the experimental manipulation controlling for the covariate we enter the covariate into the model first (think back to hierarchical regression).

$$Y_i = b_0 + b_1 X_i + b_2 \text{Covariate}$$

$$Libido_i = b_0 + b_1 Dose_i + b_2 \text{Partner's Libido}_i$$

To Recap

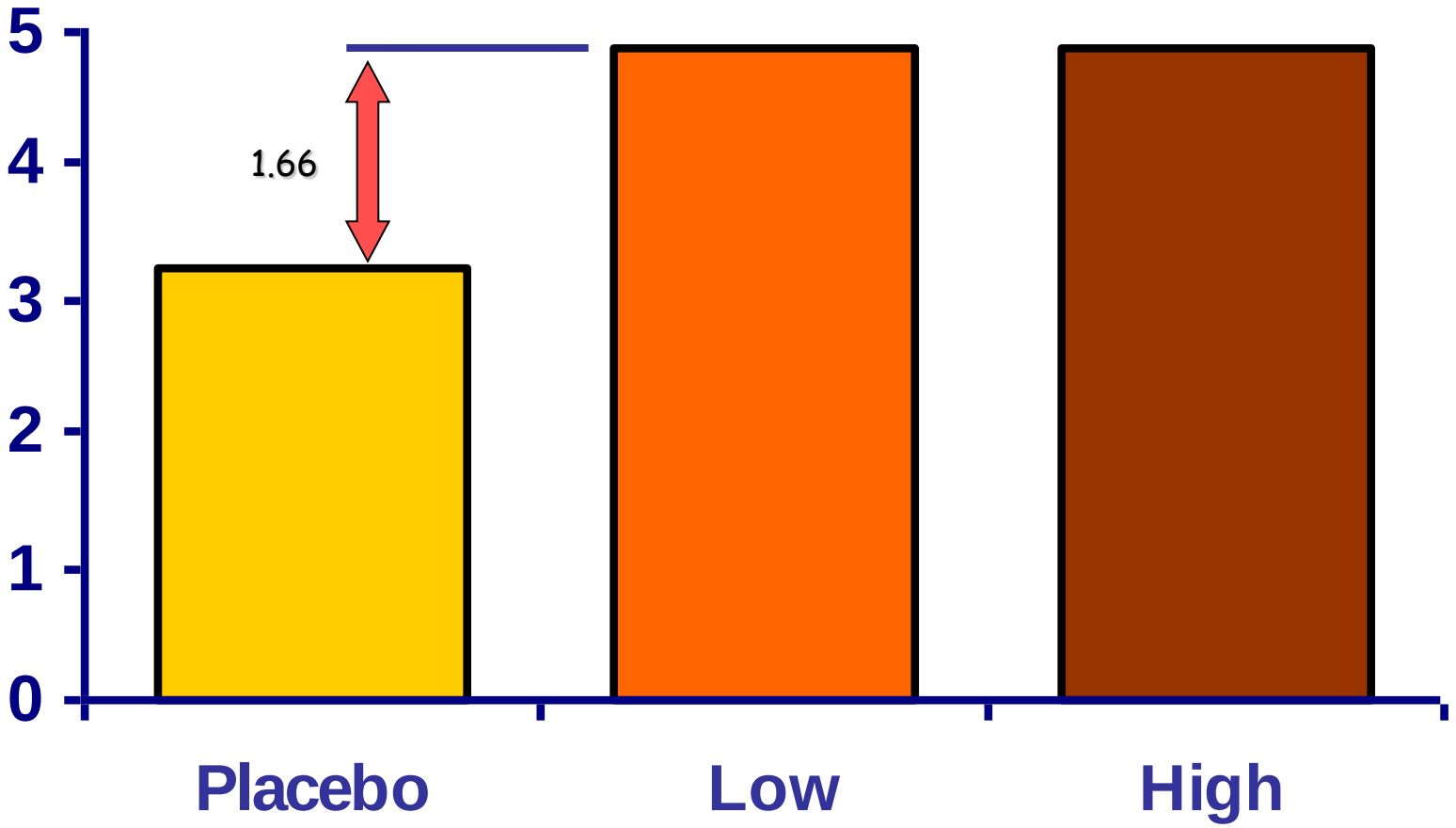
- To control for the effect of a covariate all we do is do a multiple regression in which we enter the covariate in the first step.
- We enter Dose in a second step
- The result is that we see the effect of dose above and beyond the effect of the covariate.

Coefficients[a]

Model		Unstandardized Coefficients		Standardized Coefficients		
		B	Std. Error	Beta	t	Sig.
1	(Constant)	.421	.534		.788	.443
	Partner's Libido	.596	.082	.883	7.293	.000
2	(Constant)	-.362	.440		-.824	.424
	Partner's Libido	.872	.094	1.291	9.295	.000
	Dummy Variable 1 (Placebo vs. Low)	-1.847	.487	-.527	-3.795	.002

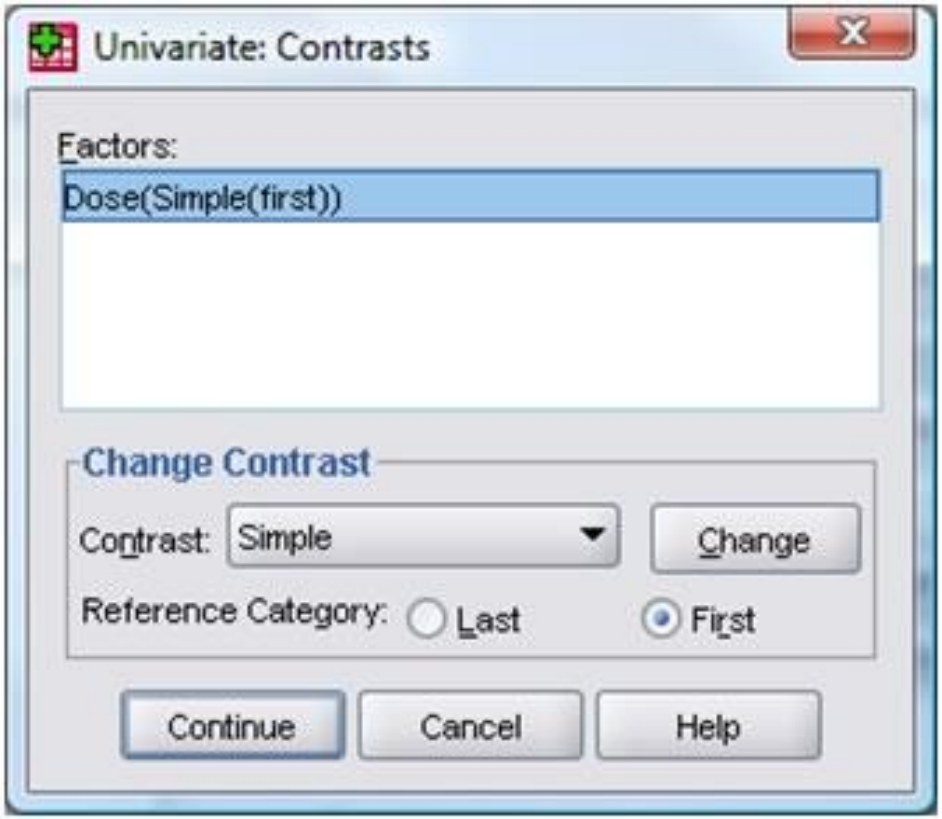
a. Dependent Variable: Libido

Dose	Participant's Libido	Partner's Libido
Placebo	3	4
	2	1
	5	5
	2	1
	2	2
	2	2
	7	7
	2	4
	4	5
Low Dose	7	5
	5	3
	3	1
	4	2
	4	2
	7	6
	5	4
	4	2
High Dose	9	1
	2	3
	6	5
	3	4
	4	3
	4	3
	4	2
	6	0
	4	1
	6	3
	2	0
	8	1
	5	0

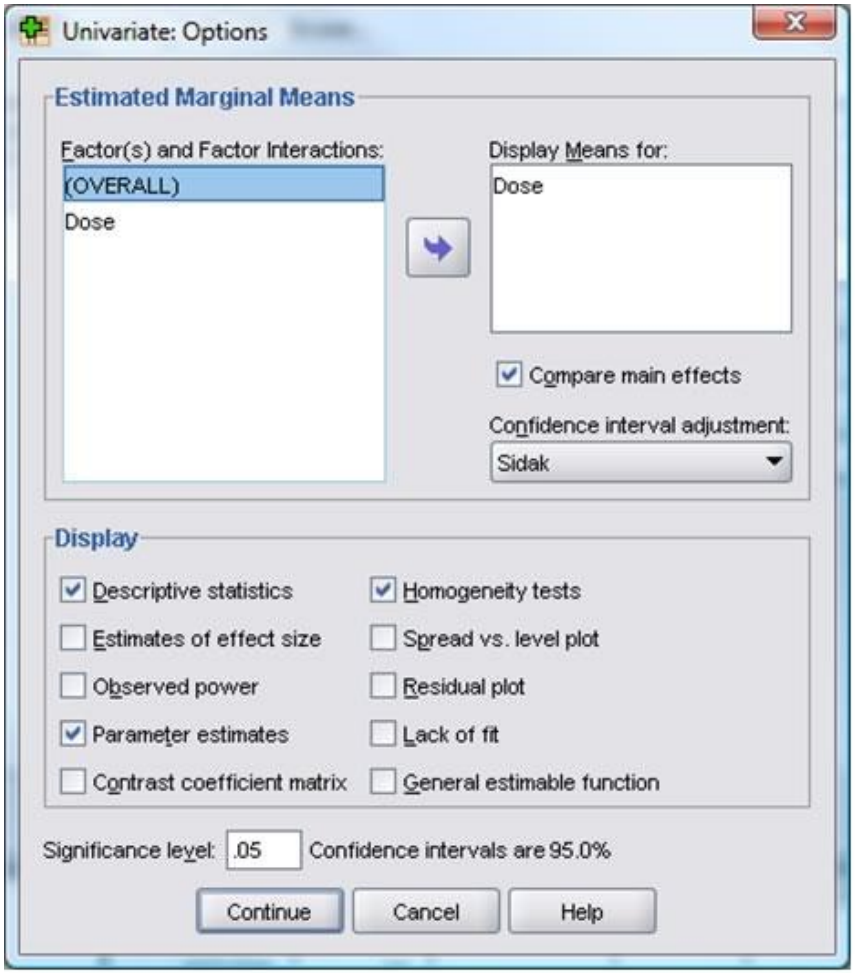

5
4
3
2
1
0
1.66
Placebo
Low
High

ANCOVA on SPSS

Contrasts


Univariate: Contrasts
Factors:
Dose(Simple(first))
Change Contrast
Contrast:
Simple
Change
Reference Category:
Last
First
Continue
Cancel
Help

Options


Univariate: Options
Estimated Marginal Means
Factor(s) and Factor Interactions:
(OVERALL)
Dose
Display Means for:
Dose
Compare main effects
Confidence interval adjustment:
Sidak
Display
Descriptive statistics
Homogeneity tests
Estimates of effect size
Spread vs. level plot
Observed power
Residual plot
Parameter estimates
Lack of fit
Contrast coefficient matrix
General estimable function
Significance level: .05
Confidence intervals are 95.0%
Continue
Cancel
Help

Without the Covariate

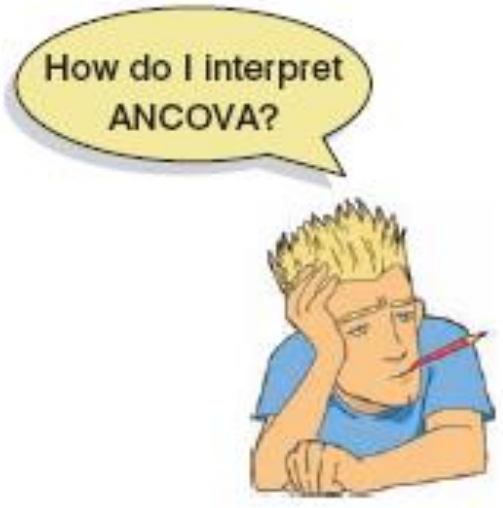
Tests of Between-Subjects Effects

Dependent Variable: Libido

Source	Type III Sum of Squares	df	Mean Square	F	Sig.
Corrected Model	16.844[a]	2	8.422	2.416	.108
Intercept	535.184	1	535.184	153.522	.000
DOSE	16.844	2	8.422	2.416	.108
Error	94.123	27	3.486		
Total	683.000	30			
Corrected Total	110.967	29			

a. R Squared = .152 (Adjusted R Squared = .089)

Output



Levene's Test of Equality of Error Variances[a]

Dependent Variable: Libido

F	df1	df2	Sig.
5.525	2	27	.010

Tests the null hypothesis that the error variance of the dependent variable is equal across groups.

a. Design: Intercept+PARTNER+DOSE

Tests of Between-Subjects Effects

Dependent Variable: Libido

Source	Type III Sum of Squares	df	Mean Square	F	Sig.
Corrected Model	34.750[a]	3	11.583	3.952	.019
Intercept	12.171	1	12.171	4.152	.052
PARTNER	17.906	1	17.906	6.109	.020
DOSE	28.337	2	14.169	4.833	.016
Error	76.216	26	2.931		
Total	683.000	30			
Corrected Total	110.967	29			

a. R Squared = .313 (Adjusted R Squared = .234)

SPSS Output: Contrasts

Contrast Results (K Matrix)

Dose of Viagra Simple Contrast[a]			Depende... Libido
Level 2 vs. Level 1	Contrast Estimate		1.786
	Hypothesized Value		0
	Difference (Estimate - Hypothesized)		1.786
	Std. Error		.849
	Sig.		.045
	95% Confidence Interval for Difference	Lower Bound	.040
		Upper Bound	3.532
Level 3 vs. Level 1	Contrast Estimate		2.225
	Hypothesized Value		0
	Difference (Estimate - Hypothesized)		2.225
	Std. Error		.803
	Sig.		.010
	95% Confidence Interval for Difference	Lower Bound	.575
		Upper Bound	3.875

a. Reference category = 1

Output

Estimates

Dependent Variable:Libido

Dose of Viagra	Mean	Std. Error	95% Confidence Interval	
			Lower Bound	Upper Bound
Placebo	2.926[a]	.596	1.701	4.152
Low Dose	4.712[a]	.621	3.436	5.988
High Dose	5.151[a]	.503	4.118	6.184

Pairwise Comparisons

Dependent Variable:Libido

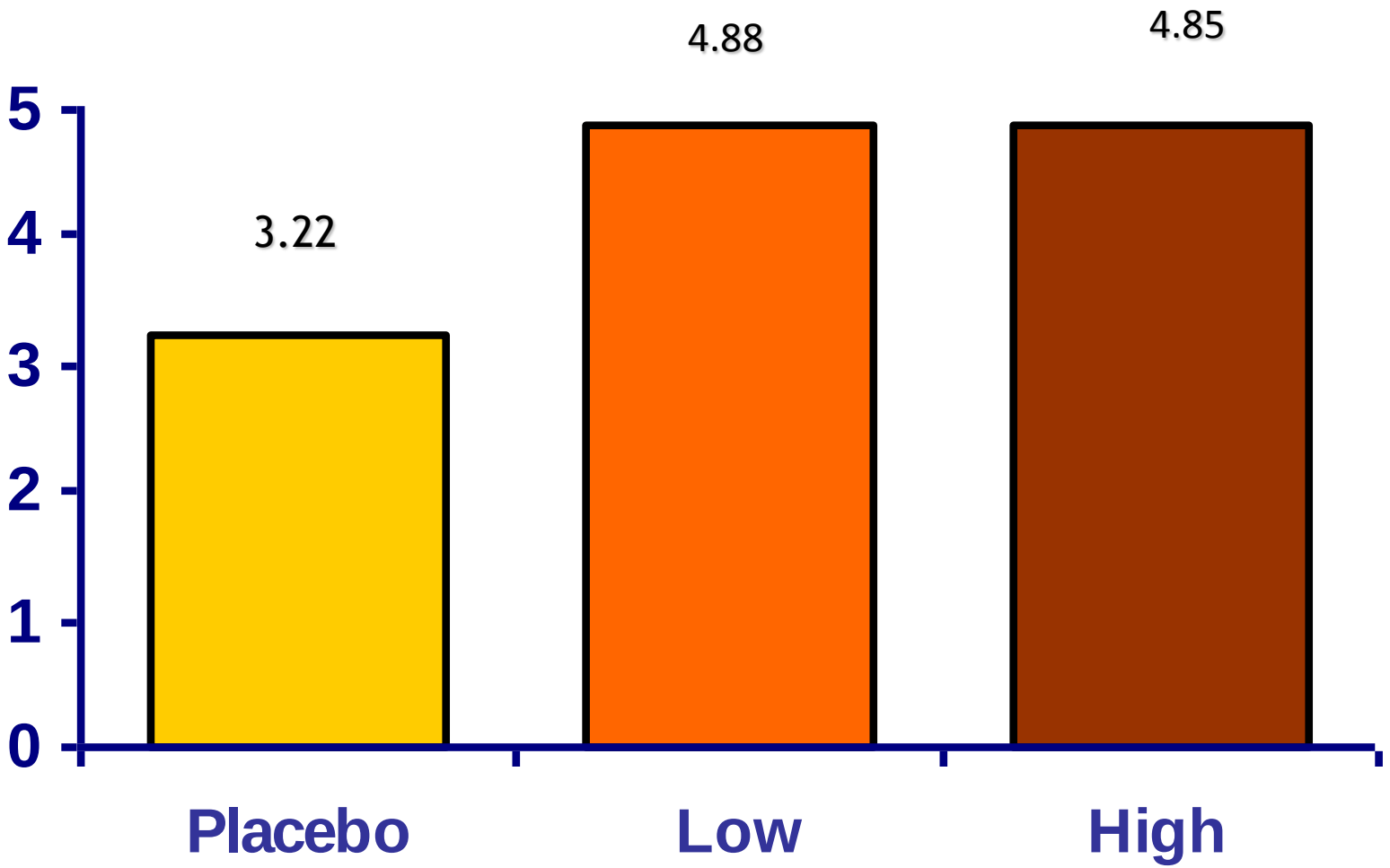
(I) Dose of Viagra	(J) Dose of Viagra	Mean Difference (I-J)	Std. Error	Sig.[a]	95% Confidence Interval for Difference[a]	
					Lower Bound	Upper Bound
Placebo	Low Dose	-1.786	.849	.130	-3.953	.381
	High Dose	-2.225*	.803	.030	-4.273	-.177
Low Dose	Placebo	1.786	.849	.130	-.381	3.953
	High Dose	-.439	.811	.932	-2.509	1.631
High Dose	Placebo	2.225*	.803	.030	.177	4.273
	Low Dose	.439	.811	.932	-1.631	2.509

Based on estimated marginal means

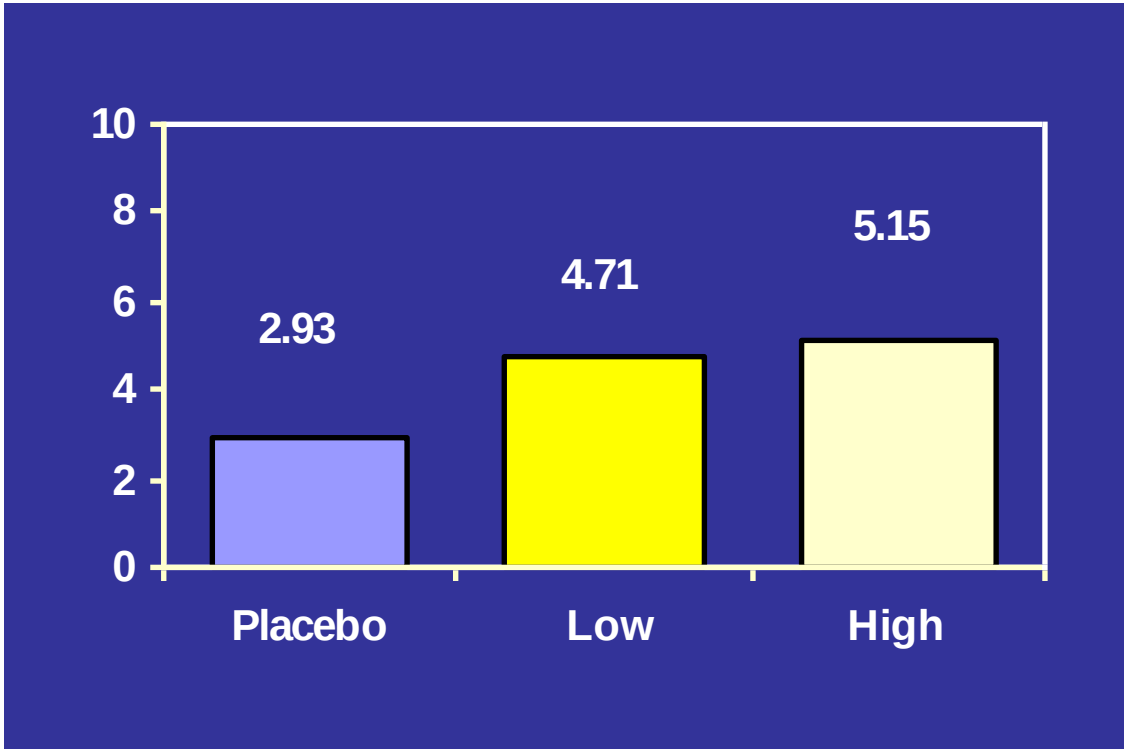
a. Adjustment for multiple comparisons: Sidak.

*. The mean difference is significant at the .05 level.

Unadjusted Means

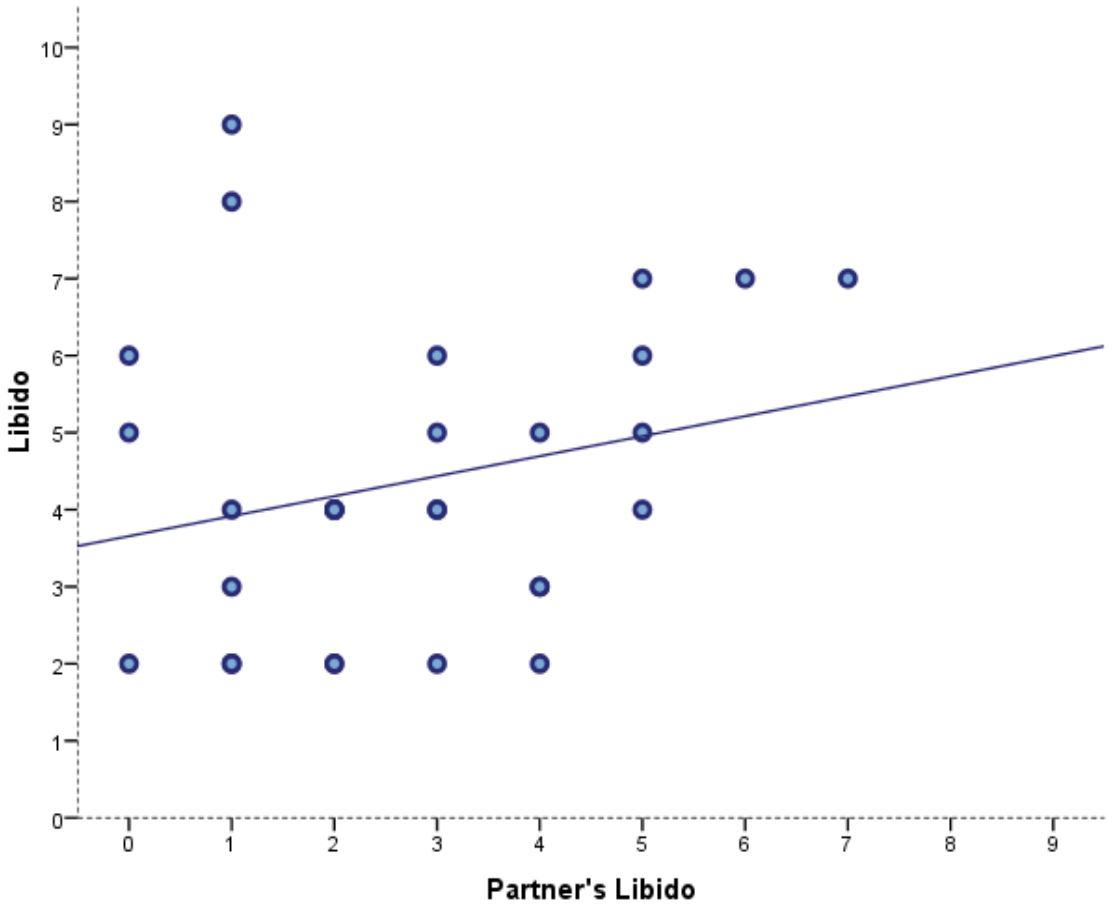


The Main Effect



$F(2, 26) = 4.14, p < .05$

The Covariate



$F(1, 26) = 4.96, p < .05$