Two-Way Independent ANOVA (GLM 3)

Lecture 11



Aims

- Rationale of Factorial ANOVA
- Partitioning Variance
- Interaction Effects
 - Interaction Graphs
 - —Interpretation





What is Two-Way Independent ANOVA?

- Two Independent Variables
 - Two-way = 2 Independent variables
 - Three-way = 3 Independent variables
- Different participants in all conditions.
 - Independent = 'different participants'
- Several Independent Variables is known as a factorial design.



Benefit of Factorial Designs

- We can look at how variables Interact.
- **Interactions**
 - Show how the effects of one IV might depend on the effects of another
 - Are often more interesting than main effects.
- Examples
 - Interaction between hangover and lecture topic on sleeping during lectures.
 - A hangover might have more effect on sleepiness during a stats lecture than during a clinical one.



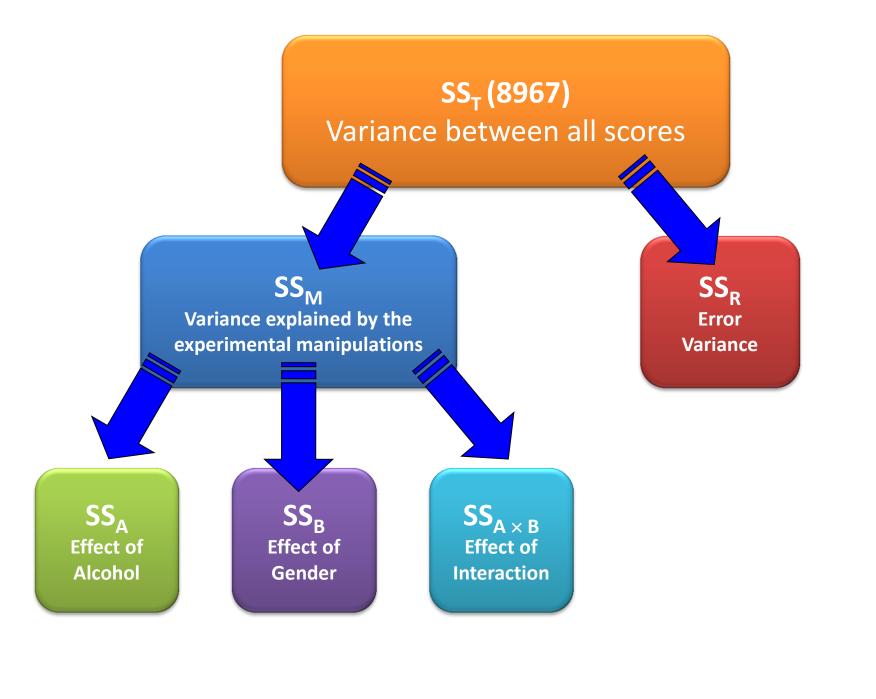
An Example

- Field (2009): Testing the effects of Alcohol and Gender on 'the beer-goggles effect':
 - IV 1 (Alcohol): None, 2 pints, 4 pints
 - IV 2 (Gender): Male, Female
- Dependent Variable (DV) was an objective measure of the attractiveness of the partner selected at the end of the evening.



Alcohol	None		2 Pints		4 Pints	
Gender	Female	Male	Female	Male	Female	Male
	65	50	70	45	55	30
	50	55	65	60	65	30
	70	80	60	85	70	30
	45	65	70	65	55	55
	55	70	65	70	55	35
	30	75	60	70	60	20
	70	75	60	80	50	45
	55	65	50	60	50	40
Total	485	535	500	535	460	285
Mean	60.625	66.875	62.50	66.875	57.50	35.625
Variance	24.55	106.70	42.86	156.70	50.00	117.41





Step 1: Calculate SS_T

					_
65	50	70	45	55	30
50	55	65	60	65	30
70	80	60	85	70	30
45	65	70	65	55	55
55	70	65	70	55	35
30	7 5	60	70	60	20
70	7 5	60	80	50	45
55	65	50	60	50	40

Grand Mean = 58.33

$$SS_T = s_{grand}^2(N-1)$$

= 190.78 (48 - 1)
= 8966.66

Step 2: Calculate SS_M

$$SS_{M} = \sum n_{i} (\overline{x}_{i} - \overline{x}_{grand})^{2}$$

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\begin{split} SS_{M} &= 8(60.625 - 58.33)^{2} + 8(66.875 - 58.33)^{2} + 8(62.5 - 58.33)^{2} \\ &\quad + 8(66.875 - 58.33)^{2} + 8(57.5 - 58.33)^{2} + 8(35.625 - 58.33)^{2} \\ &= 8(2.295)^{2} + 8(8.545)^{2} + 8(4.17)^{2} + 8(8.545)^{2} + 8(-0.83)^{2} + 8(-22.705)^{2} \\ &= 42.1362 + 584.1362 + 139.1112 + 584.1362 + 5.5112 + 4124.1362 \\ &= 5479.167 \end{split}
```



A ₁ : Female					
65	70	55			
70	65	65			
60	60	70			
60	70	55			
60	65	55			
55	60	60			
60	60	50			
55	50	50			

Mean Fema	e = 60.21
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	A ₂ : Male	
50	45	30
55	60	30
80	85	30
65	65	55
70	70	35
75	70	20
75	80	45
65	60	40

Mean Male = 56.46

$$SS_{Gender} = 24(60.21 - 58.33)^2 + 24(56.46 - 58.33)^2$$

= $24(1.88)^2 + 24(-1.87)^2$
= $84.8256 + 83.9256$
= 168.75

Step 2b: Calculate SS_B

B ₁ : None				
65	50			
70	55			
60	80			
60	65			
60	70			
55	7 5			
60	7 5			
55	65			

Mean	None =	63.75
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B ₂ : 2 Pints				
70	45			
65	60			
60	85			
70	65			
65	70			
60	70			
60	80			
50	60			

Mean 2 Pints = 64.6875

B ₃ : 4 Pints				
55	30			
65	30			
70	30			
55	55			
55	35			
60	20			
50	45			
50	40			

Mean 4 Pints = 46.5625

```
SS_{alcohol} = 16(63.75 - 58.33)^2 + 16(64.6875 - 58.33)^2 + 16(46.5625 - 58.33)^2
= 16(5.42)^2 + 16(6.3575)^2 + 16(-11.7675)^2
= 470.0224 + 646.6849 + 2215.5849
= 3332.292
```

Step 2c: Calculate SS_{A × B}

$$SS_{A\times B} = SS_M - SS_A - SS_B$$

$$SS_{A\times B} = SS_M - SS_A - SS_B$$

= 5479.167 - 168.75 - 3332.292
= 1978.125

Step 3: Calculate SS_R

$$SS_R = s_{group1}^2(n_1 - 1) + s_{group2}^2(n_2 - 1) + s_{group3}^2(n_3 - 1) ... s_{groupn}^2(n_n - 1)$$

$$SS_{R} = s_{group1}^{2}(n_{1} - 1) + s_{group2}^{2}(n_{2} - 1) + s_{group3}^{2}(n_{3} - 1)$$

$$+ s_{group4}^{2}(n_{4} - 1) + s_{group5}^{2}(n_{5} - 1) + s_{group6}^{2}(n_{6} - 1)$$

$$= (24.55 \times 7) + (106.7 \times 7) + (42.86 \times 7)$$

$$+ (156.7 \times 7) + (50 \times 7) + (117.41 \times 7)$$

$$= 171.85 + 746.9 + 300 + 1096.9 + 350 + 821.87$$

$$= 3487.52$$

Summary Table

Tests of Between-Subjects Effects

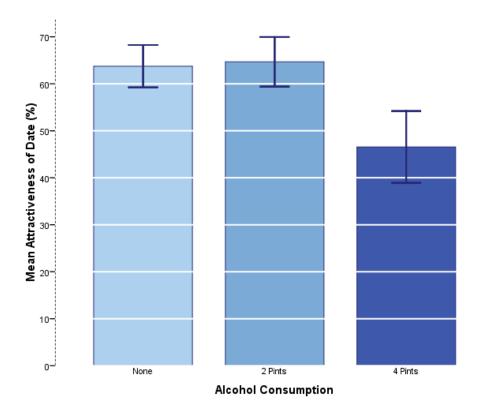
Dependent Variable: Attractiveness of Date

Dependent variable.Attractiveness of Date						
Source	Type III Sum of Squares	df	Mean Square	F	Siq.	
Gender	168.750	1	168.750	2.032	.161	
Alcohol	3332.292	2	1666.146	20.065	.000	
Gender * Alcohol	1978.125	2	989.062	11.911	.000	
Error	3487.500	42	83.036			
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a. R Squared = .611 (Adjusted R Squared = .565)

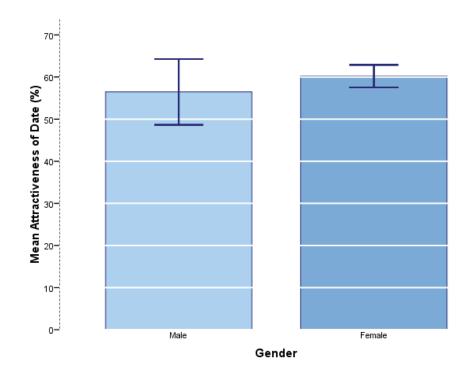


Interpretation: Main Effect Alcohol

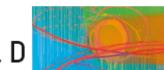


There was a significant main effect of the amount of alcohol consumed at the night-club, on the attractiveness of the mate that was selected, F(2, 42) = 20.07, p < .001.

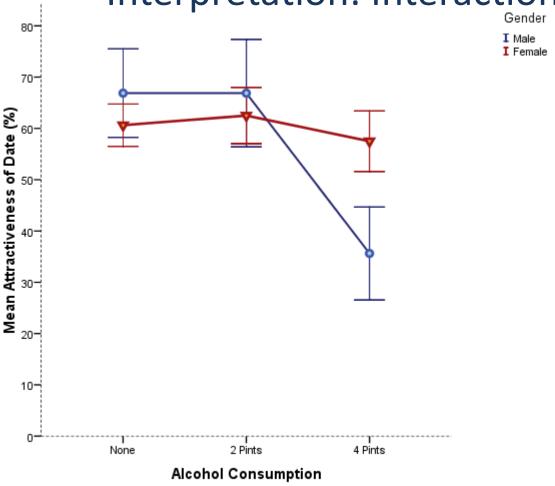
Interpretation: Main Effect Gender



There was a nonsignificant main effect of gender on the attractiveness of selected mates, F(1, 42) = 2.03, p = .161.

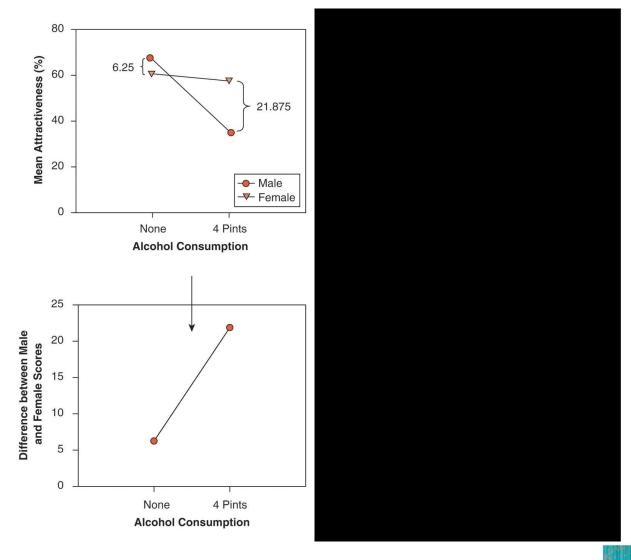


Interpretation: Interaction

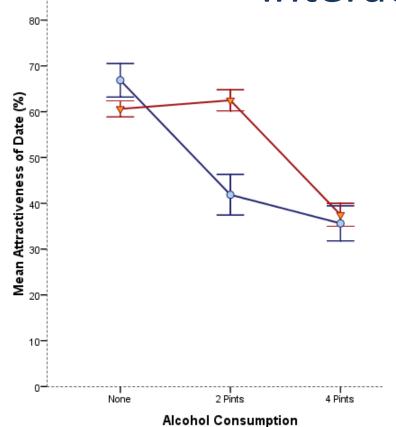


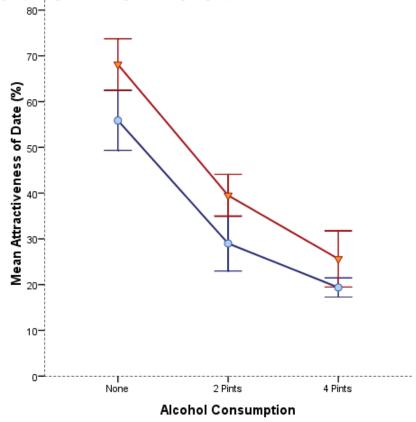
There was a significant interaction between the amount of alcohol consumed and the gender of the person selecting a mate, on the attractiveness of the partner selected, F(2, 42) = 11.91, p < .001.

What is an Interaction?



Is there likely to be a significant interaction effect?









Gender

I Male I Female

Is there likely to be a significant interaction effect?

