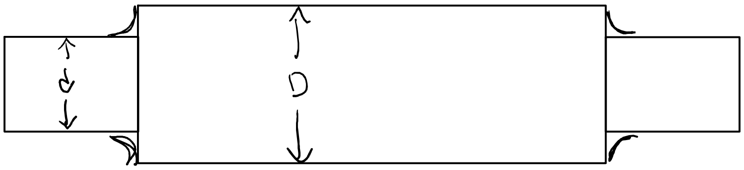
Shaft Design

Parker Knopf   A15830016

Problem Statement



FOS: 1.5   Reliability: 99.9%

$S_y = 60$ ksi   Surface: Machined

$S_{ut} = 80$ ksi

Forces:

Min/Max Bending Moment: 800 - 5000 lb·in

Min/Max Torque: 0 - 2000 lb·in

Geometry:

$D = (1.2)d$

$r = d(.1)$

Static / Dynamic Loads

$M_a = M_{max} - M_m = 5000 - 2900 = 2100$ lb·in

$M_m = \dfrac{M_{max} + M_{in}}{2} = \dfrac{5000 + 800}{2} = 2900$ lb·in

$T_a = T_{max} - T_m = 2000 - 1000 = 1000$ lb·in

$T_m = \dfrac{T_{max} + T_{min}}{2} = \dfrac{2000 - 0}{2} = 1000$ lb·in

$M_a = 2100$
$M_m = 2900$
$T_a = 1000$ lb·in
$T_m = 1000$

Initial Diameter Guess

Idea: Use static failure theory to form initial guess

$M_{max} = 5000$ lb·in $\sigma = \dfrac{Mc}{I}$ $\tau = \dfrac{Tc}{J}$ $I = \dfrac{\pi}{4}(r)^4$

$T_{max} = 2000$ lb·in

$\sigma' = \dfrac{1}{\sqrt{2}}\left[2(\sigma)^2 + 6(\tau)^2\right]^{1/2}$ $n = \dfrac{S_y}{\sigma'}$ $\Rightarrow$ $\underbrace{\dfrac{S_y}{n} = \dfrac{1}{\sqrt{2}}\left[2(\sigma)^2 + 6(\tau)^2\right]^{1/2}}_{\text{solve for } r}$

$\boxed{d_0 = 1.104 \text{ in} \quad \text{Initial Guess}}$

$$r = \left[\frac{}{\pi^2\left(\frac{S_y\sqrt{2}}{n}\right)^2}\left(32M^2 + 24T^2\right)\right]^{1/6}$$

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S_e

$S_e' = S_{ut}/2 = (80\text{ ksi})/2 = 40\text{ ksi}$ $S_e = k_a k_b k_c k_d k_e S_e'$

$k_a = a(S_{ut})^b$ $a = 2.00$ $b = -0.217$

$k_b = 0.91\, d^{(-0.157)}$ $k_c = 1$

$k_d = 1$ $k_e = 0.753$

$S_e = (2)(80)^{-0.217}(0.91)d^{-0.157}(0.753)$

$$\boxed{S_e = (0.5295)d^{-0.157}S_{ut}/2 = 20.854\text{ ksi}}$$

Stress Concentrations

Estimates	Bending	Torsional	Axial
Shoulder fillet—sharp (r/d = 0.02)	2.7	2.2	3.0
Shoulder fillet—well rounded (r/d = 0.1)	1.7	1.5	1.9

$\boxed{k_t = 1.7 \quad k_{ts} = 1.5}$ => Initial Guesses

Bending:

Table A-15-9

$$K_t = C_1 + C_2\left(\frac{2t}{D}\right) + C_3\left(\frac{2t}{D}\right)^2 + C_4\left(\frac{2t}{D}\right)^3$$

	$0.1 \le t/r \le 2.0$	$2.0 \le t/r \le 20.0$
C_1	$0.947 + 1.206\sqrt{t/r} - 0.131t/r$	$1.232 + 0.832\sqrt{t/r} - 0.008t/r$
C_2	$0.022 - 3.405\sqrt{t/r} + 0.915t/r$	$-3.813 + 0.968\sqrt{t/r} - 0.260t/r$
C_3	$0.869 + 1.777\sqrt{t/r} - 0.555t/r$	$7.423 - 4.868\sqrt{t/r} + 0.869t/r$
C_4	$-0.810 + 0.422\sqrt{t/r} - 0.260t/r$	$-3.839 + 3.070\sqrt{t/r} - 0.600t/r$

Torsional:

Table A-15-8

$$K_{tn} = C_1 + C_2\left(\frac{2t}{D}\right) + C_3\left(\frac{2t}{D}\right)^2 + C_4\left(\frac{2t}{D}\right)^3$$

	$0.25 \le t/r \le 4.0$
C_1	$0.905 + 0.783\sqrt{t/r} - 0.075t/r$
C_2	$-0.437 - 1.969\sqrt{t/r} + 0.553t/r$
C_3	$1.557 + 1.073\sqrt{t/r} - 0.578t/r$
C_4	$-1.061 + 0.171\sqrt{t/r} + 0.086t/r$

$$k_f = 1 + \frac{K_t - 1}{1 + \sqrt{a}/\sqrt{r}}$$

Bending or axial:

$$\sqrt{a} = 0.246 - 3.08(10^{-3})S_{ut} + 1.51(10^{-5})S_{ut}^2 - 2.67(10^{-8})S_{ut}^3 \qquad 50 \le S_{ut} \le 250\text{ kpsi}$$

$$\sqrt{a} = 1.24 - 2.25(10^{-3})S_{ut} + 1.60(10^{-6})S_{ut}^2 - 4.11(10^{-10})S_{ut}^3 \qquad 340 \le S_{ut} \le 1700\text{ MPa} \tag{6–35}$$

Torsion:

$$\sqrt{a} = 0.190 - 2.51(10^{-3})S_{ut} + 1.35(10^{-5})S_{ut}^2 - 2.67(10^{-8})S_{ut}^3 \qquad 50 \le S_{ut} \le 220\text{ kpsi}$$

$$\sqrt{a} = 0.958 - 1.83(10^{-3})S_{ut} + 1.43(10^{-6})S_{ut}^2 - 4.11(10^{-10})S_{ut}^3 \qquad 340 \le S_{ut} \le 1500\text{ MPa} \tag{6–36}$$

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K_f:

$\sqrt{a} = 0.0826$

$k_f = 1.63$

K_{fs}:

$\sqrt{a} = -0.0234$

$k_{fs} = 1.5163$

Solve

$$d = \left\{ \frac{16n}{\pi} \left[4\left(\frac{K_f M_a}{S_e}\right)^2 + 3\left(\frac{K_{fs} T_a}{S_e}\right)^2 + 4\left(\frac{K_f M_m}{S_y}\right)^2 + 3\left(\frac{K_{fs} T_m}{S_y}\right)^2 \right]^{1/2} \right\}^{1/3}$$

$\boxed{d_1 = 1.732 \text{ in}}$