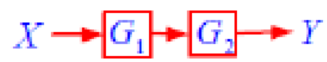
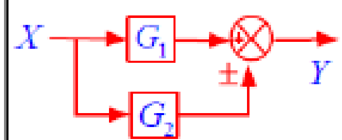
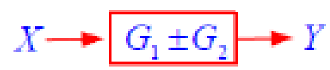
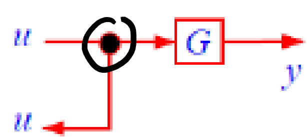
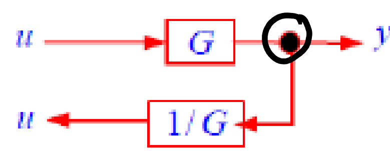
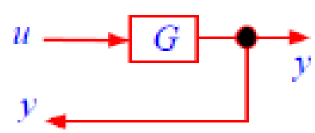
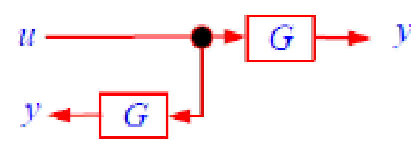
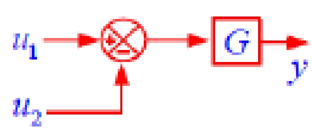
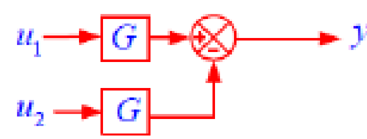
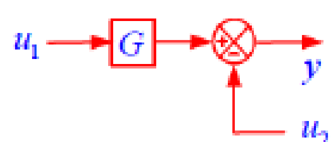
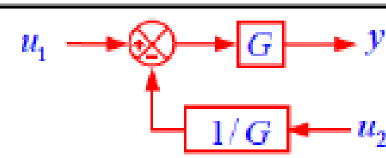
Reducing Complicated Block Diagrams:

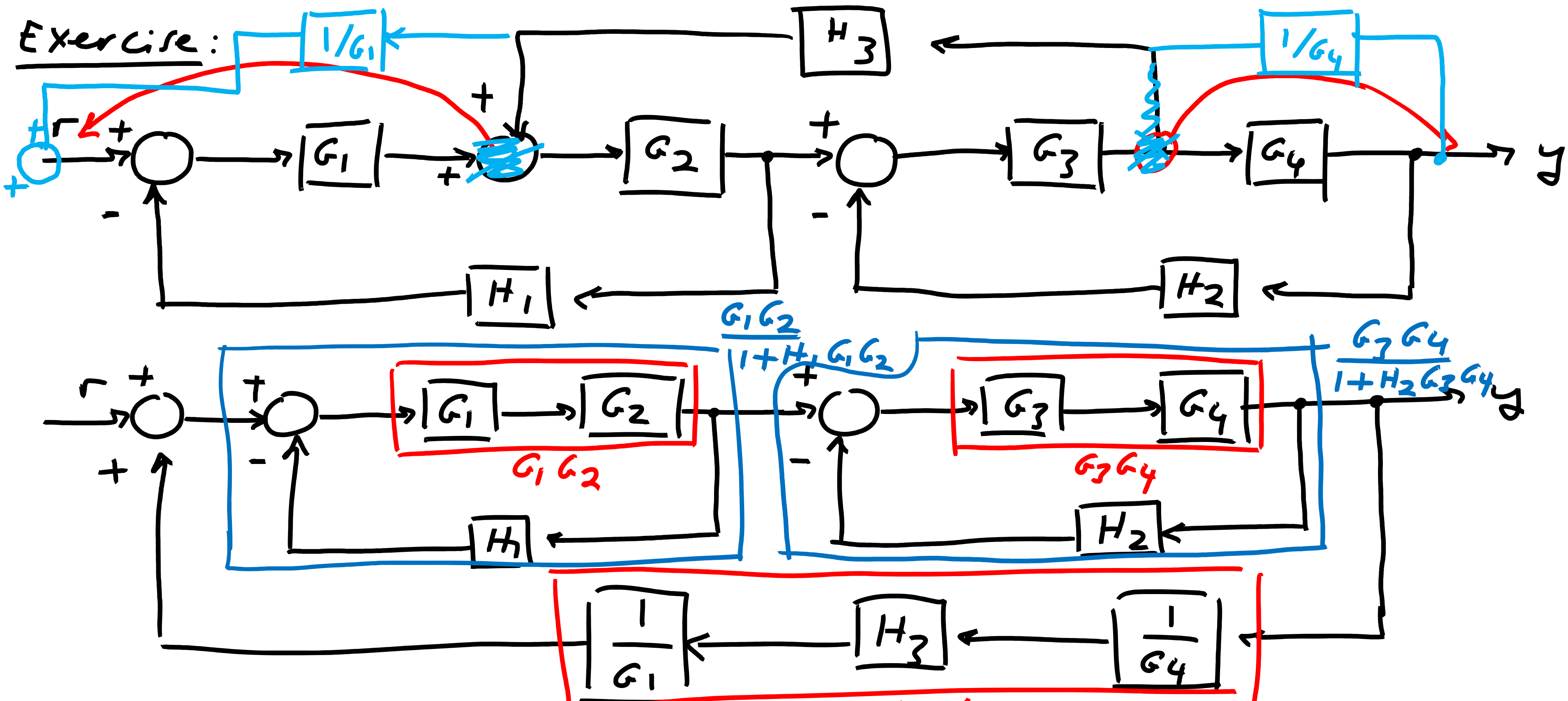
0. Move summation + take-off points to desirable positions (to avoid staggered loops)

1. Combine all cascade blocks (multiply) $(G_1 \cdot G_2)$
2. Combine all parallel blocks (add) $(G_1 + G_2)$
3. Eliminate all feedback loops $\left(\dfrac{G}{1 \mp HG}\right)$

Block Diagram Transformations

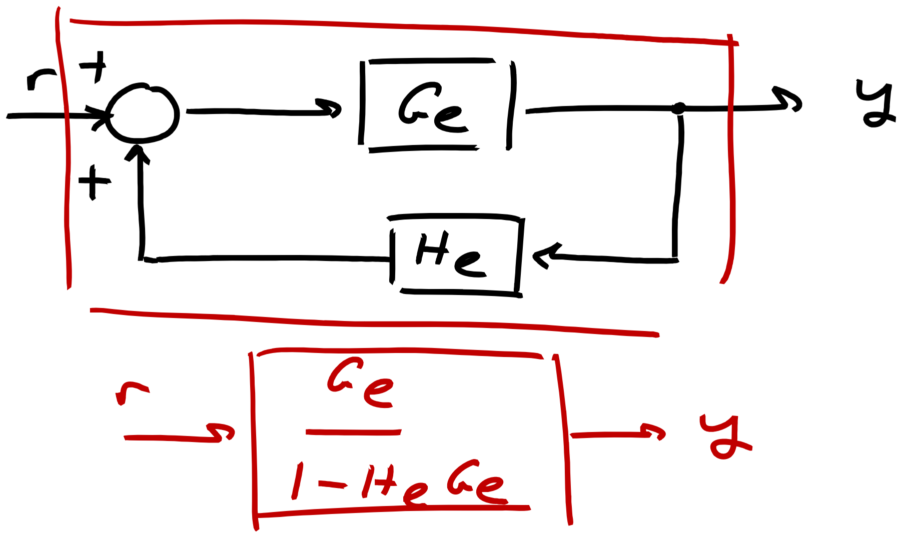
	Manipulation	Original Block Diagram	Equivalent Block Diagram	Equation
1	Combining Blocks in Cascade			$Y = (G_1 G_2) X$
2	Combining Blocks in Parallel; or Eliminating a Forward Loop			$Y = (G_1 \pm G_2) X$
3	Moving a pickoff point behind a block			$y = G u$ $u = \frac{1}{G} y$
4	Moving a pickoff point ahead of a block			$y = Gu$
5	Moving a summing point behind a block			$e_2 = G(u_1 - u_2)$
6	Moving a summing point ahead of a block			$y = Gu_1 - u_2$

Exercise:



$H_3/G_1G_4 = H_e$

$$G_e = \left(\frac{G_1 G_2}{1+H_1G_1G_2}\right)\left(\frac{G_3G_4}{1+H_2G_3G_4}\right)$$

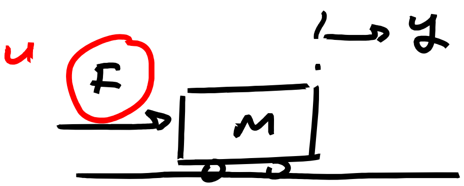

r
+
+
Ge
He
y
r
Ge / (1 − He Ge)
y

Modeling mechanical systems:

* "ALL MODELS ARE WRONG, BUT SOME ARE USEFUL"

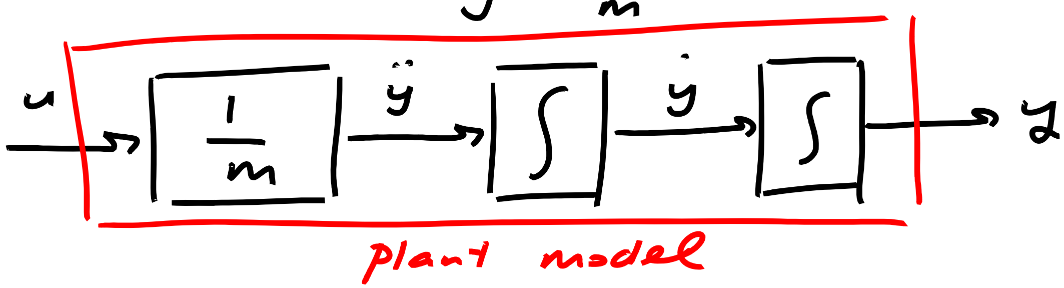
- Newton's second law. $\sum F = ma$

$$\sum M = J\ddot{\theta}$$



$$F = m\ddot{y} \quad \text{(second order system)}$$

$$\ddot{y} = \frac{1}{m}u$$



(Double integrator)

State-Space Form:

$$\dot{\vec{x}} = A\vec{x} + B\vec{u} \quad \text{(state eqn.)}$$

$$\vec{y} = C\vec{x} + D\vec{u} \quad \text{(output eqn.)}$$

$$\vec{x} = \begin{pmatrix} x_1 \\ x_2 \end{pmatrix}$$

$$\dot{\vec{x}} = \begin{pmatrix} \dot{x}_1 \\ \dot{x}_2 \end{pmatrix}$$

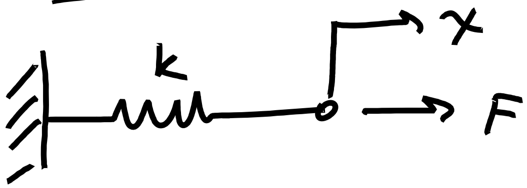
$$x_1 = y \quad \Rightarrow \quad \boxed{\dot{x}_1} = \dot{y} = \boxed{x_2}$$

$$\underline{x_2} = \dot{y} \quad \Rightarrow \quad \boxed{\dot{x}_2} = \ddot{y} = \boxed{\frac{1}{m} u}$$

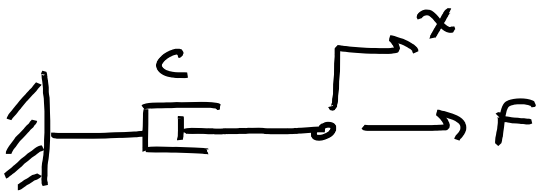
$$\underbrace{\begin{pmatrix} \dot{x}_1 \\ \dot{x}_2 \end{pmatrix}}_{\dot{\vec{x}}} = \underbrace{\begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix}}_{A} \underbrace{\begin{pmatrix} x_1 \\ x_2 \end{pmatrix}}_{\vec{x}} + \underbrace{\begin{bmatrix} 0 \\ 1/m \end{bmatrix}}_{B} \underbrace{u}_{u}$$

$$y = x_1 \quad \Rightarrow \quad y = \begin{bmatrix} 1 & 0 \end{bmatrix} \begin{pmatrix} x_1 \\ x_2 \end{pmatrix} + \begin{bmatrix} 0 \\ 0 \end{bmatrix} u = C\vec{x} + \overset{0}{D} u$$

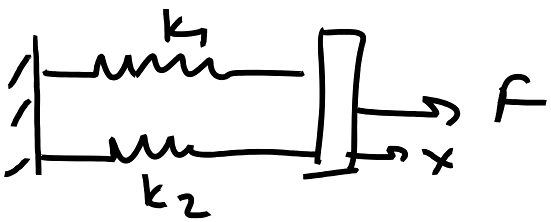
Components of a Mechanical system:



$F = kx$ (stores energy)

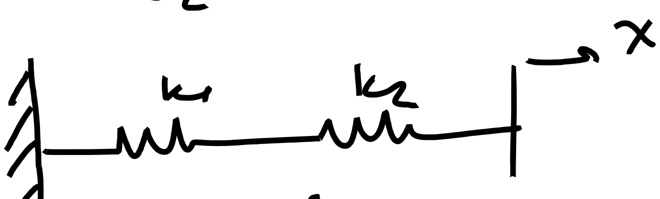


$F = c\dot{x}$ (dissipates energy)



parallel

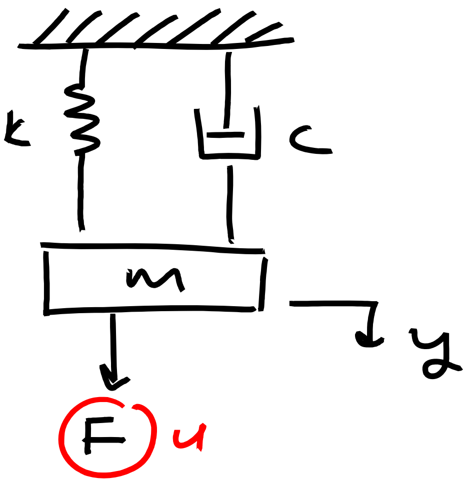
$F = (k_1 + k_2)\,x$

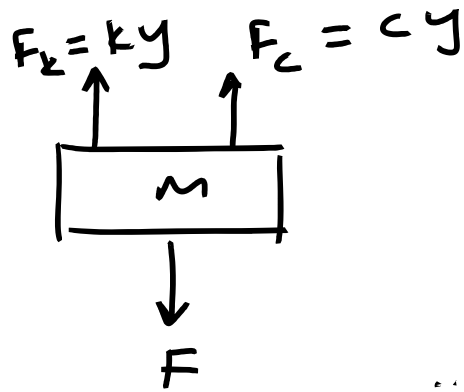


series

$$F = \frac{1}{\frac{1}{k_1} + \frac{1}{k_2}}\,x \Rightarrow \frac{1}{k_{eq}} = \frac{1}{k_1} + \frac{1}{k_2}$$

Mass-Spring-Damper





$$ma = \sum F$$

$$m\ddot{y} = F - ky - c\dot{y}$$

$$m\ddot{y} + c\dot{y} + ky = F = u$$

Rearrange:

$$\ddot{y} = \boxed{-\frac{c}{m}\dot{y} - \frac{k}{m}y + \frac{1}{m}u}$$

in state-space form:

$$x_1 = y \Rightarrow \boxed{\dot{x}_1} = \dot{y} = \boxed{x_2}$$

$$x_2 = \dot{y} \Rightarrow \boxed{\dot{x}_2} = \ddot{y} = \boxed{-\frac{c}{m}x_2 - \frac{k}{m}x_1 + \frac{1}{m}u}$$

$$\begin{pmatrix} \dot{x}_1 \\ \dot{x}_2 \end{pmatrix} = \begin{bmatrix} 0 & 1 \\ -\frac{k}{m} & -\frac{c}{m} \end{bmatrix} \begin{pmatrix} x_1 \\ x_2 \end{pmatrix} + \begin{bmatrix} 0 \\ \frac{1}{m} \end{bmatrix} u$$

$$y = \begin{bmatrix} 1 & 0 \end{bmatrix} \begin{pmatrix} x_1 \\ x_2 \end{pmatrix}$$