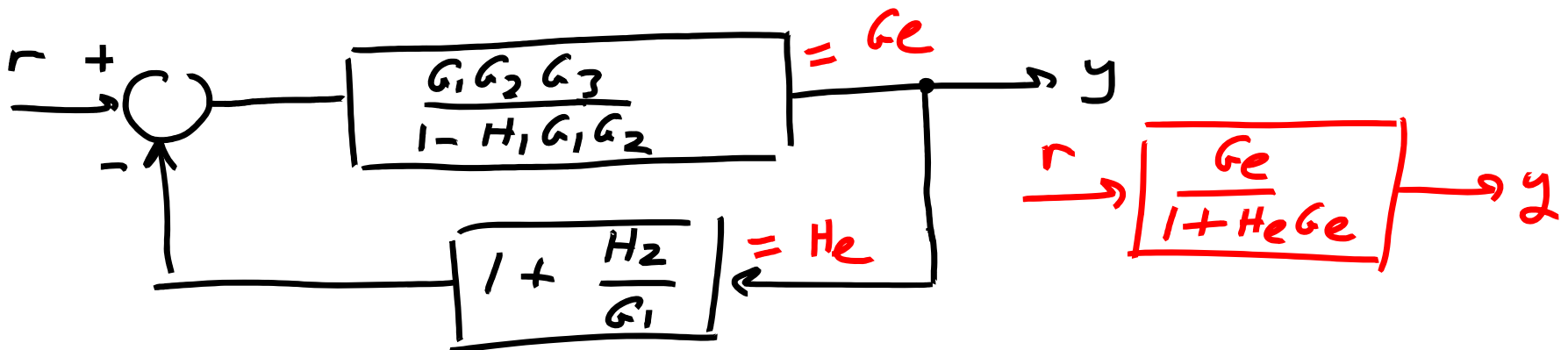
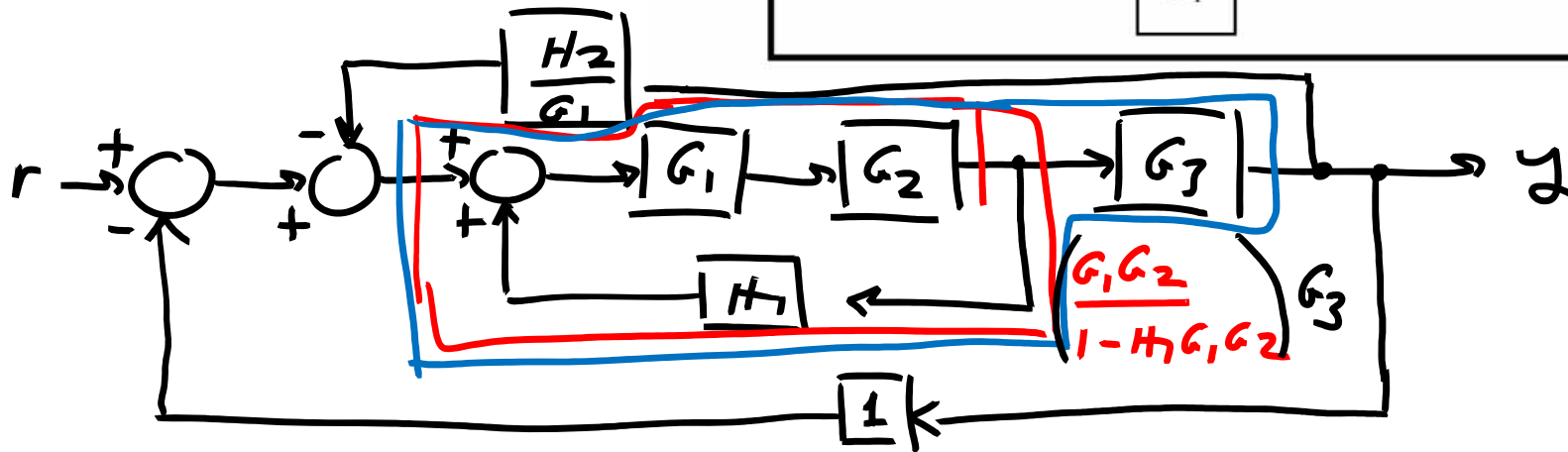
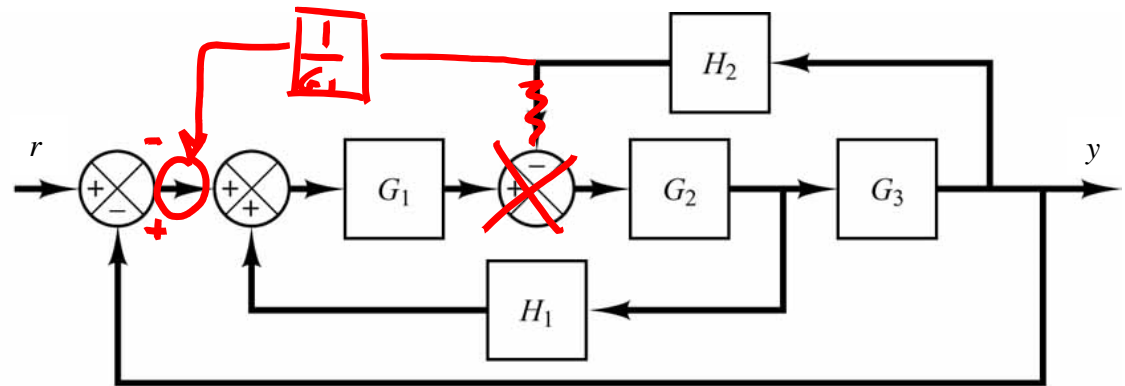
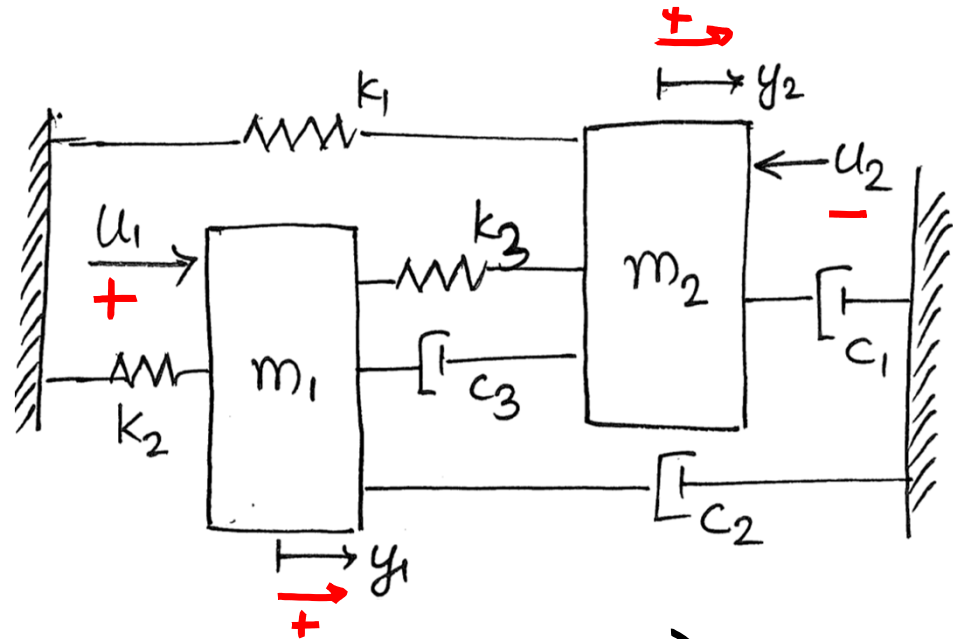


Please reduce the block diagram to find the relationship between  $r$  and  $y$ .



a) Find the differential equation(s) that model this system.

b) Identify state variables, and represent your model in the state-space form.



A)

Mass 1:

$$m_1 \ddot{y}_1 = u_1 - k_2 (y_1 - 0) - k_3 (y_1 - y_2) - c_3 (\dot{y}_1 - \dot{y}_2) - c_2 (\dot{y}_1 - 0)$$

Mass 2:

$$m_2 \ddot{y}_2 = -u_2 - k_1 (y_2 - 0) - c_1 (\dot{y}_2 - 0) - k_3 (y_2 - y_1) - c_3 (\dot{y}_2 - \dot{y}_1)$$

B) Mass 1:

$$m_1 \ddot{y}_1 = u_1 - k_2 \underline{y_1} - k_3 \underline{y_1} + k_3 y_2 - c_3 \underline{\dot{y}_1} + c_3 \dot{y}_2 - c_2 \underline{\dot{y}_1}$$

$$\underline{m_1 \ddot{y}_1 = u_1 - (k_2 + k_3) y_1 + k_3 y_2 - (c_2 + c_3) \dot{y}_1 + c_3 \dot{y}_2}$$

Mass 2:

$$m_2 \ddot{y}_2 = -u_2 - (k_1 + k_3) y_2 + k_3 y_1 - (c_1 + c_3) \dot{y}_2 + c_3 \dot{y}_1$$

Output variables:  $y_1, y_2$

Control inputs:  $u_1, u_2$

State variables:

$$\begin{aligned} x_1 &= y_1 \\ x_2 &= \dot{y}_1 \\ x_3 &= y_2 \\ x_4 &= \dot{y}_2 \end{aligned} \quad \begin{pmatrix} \dot{x}_1 \\ \dot{x}_2 \\ \dot{x}_3 \\ \dot{x}_4 \end{pmatrix} = \begin{bmatrix} 0 & 1 & 0 & 0 \\ \frac{-(k_2+k_3)}{m_1} & \frac{-(c_2+c_3)}{m_1} & \frac{k_3}{m_1} & \frac{c_3}{m_1} \\ 0 & 0 & 0 & 1 \\ \frac{k_3}{m_2} & \frac{c_3}{m_2} & \frac{-(k_1+k_3)}{m_2} & \frac{-(c_1+c_3)}{m_2} \end{bmatrix} \begin{pmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{pmatrix} + \begin{bmatrix} 0 & 0 \\ \frac{1}{m_1} & 0 \\ 0 & 0 \\ 0 & -\frac{1}{m_2} \end{bmatrix} \begin{pmatrix} u_1 \\ u_2 \end{pmatrix}$$

Derivatives of state var:

$$\begin{aligned} \dot{x}_1 &= \dot{y}_1 = 1 x_2 \\ \dot{x}_2 &= \dot{y}_1 = \frac{1}{m_1} \left( 1 \cdot u_1 - \underbrace{(k_2+k_3)}_{x_1} y_1 + \underbrace{k_3}_{x_3} y_2 - \underbrace{(c_2+c_3)}_{x_2} \dot{y}_1 + \underbrace{c_3}_{x_4} \dot{y}_2 \right) \\ \dot{x}_3 &= \dot{y}_2 = 1 x_4 \\ \dot{x}_4 &= \dot{y}_2 = \frac{1}{m_2} \left( -u_2 - \underbrace{(k_1+k_3)}_{x_3} x_3 + \underbrace{k_3}_{x_1} x_1 - \underbrace{(c_1+c_3)}_{x_4} x_4 + \underbrace{c_3}_{x_2} x_2 \right) \end{aligned}$$

output eqn

$$\begin{pmatrix} y_1 \\ y_2 \end{pmatrix} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix} \begin{pmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{pmatrix}$$

Please find the differential equation(s) that model the electrical system shown.

$$i_- = i_+ = 0$$

KCL @ A:

$$i_{R1} = i_{R2} + i_C$$

$$\frac{V_{in} - 0}{R_1} = \frac{0 - V_{out}}{R_2} + C \frac{d(0 - V_{out})}{dt}$$

$$\boxed{\frac{V_{in}}{R_1} = -\frac{V_{out}}{R_2} - C \frac{dV_{out}}{dt}}$$

