Refresher on the Poisson Distribution

- X is a discrete-value random variable
- Usually a count variable
- λ is the mean and variance of X
- Write that $X \sim Poisson(\lambda)$.
- Read more here.

$$f(x) = \frac{e^{-\lambda}\lambda^{x}}{x!}$$

for $x = 0, 1, 2, ...$

Examples: (Source)

- Let equal the number of typos on a printed page.
- Let equal the number of cars passing through an intersection.
- Let equal the number of students arriving during office hours.

IPP Likelihood Derivation

IPP Likelihood

$$\log(L(\lambda)) = \sum_{i=1}^{n} \log \lambda(x_i) - \int_W \lambda(x)dx$$

- $\int_W \lambda(x)dx$ is the expected number of cases of the IPP with intensity $\lambda(x)$ in the region W

Derivation of log-likelihood (Optional)

- **Likelihood function:** probability that a particular outcome is observed given the parameter values
- Working with intensity function $\lambda(x)$
- Denote $\int_W \lambda(x)dx$ as $\lambda(W)$

What is the likelihood of each location x_i, given the n events?

$$f(x_i) = \underline{\hspace{2cm}}$$

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$$f(x_i) = \frac{}{\lambda(W)}$$

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$$f(x_i) = \frac{\lambda(x_i)}{\lambda(W)}$$

What is the joint (combined) likelihood of all of the n events? (Remember they are independent)

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$$f(x_i) = \frac{\lambda(x_i)}{\lambda(W)}$$

What is the joint (combined) likelihood of all of the n events? (Remember they are independent)

$$f(x_1, ..., x_n|n) = \prod_{i=1}^{n} \frac{\lambda(x_i)}{\lambda(W)} = \frac{1}{\lambda(W)^n} \prod_{i=1}^{n} \lambda(x_i)$$

IPP Likelihood

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  - Poisson!
- What is the mean of the Poisson distribution?
  - Not a constant λ for IPP
  - Mean $= \lambda(W)$ (Recall: $\lambda(W)$ is the "expected number of cases of the IPP with intensity $\lambda(x)$ in the region W")

IPP Likelihood

Poisson Distribution: (if $x \sim Poisson(\lambda)$)

$$f(x) = \frac{e^{-\lambda}\lambda^{x}}{x!}$$

- $n \sim Poisson(\lambda(W))$
- What is the likelihood of observing n events given rate $\lambda(W)$?

$$f(n) = \frac{e^{-\lambda(W)}\lambda(W)^{n}}{n!}$$

IPP Likelihood

- Now let's put it all together
- $L(\lambda) = f(x_1, ..., x_n|n)f(n)$

$$L(\lambda) = \left[\frac{1}{\lambda(W)^n}\prod_{i=1}^{n}\lambda(x_i)\right] \times \left[\frac{e^{-\lambda(W)}\lambda(W)^n}{n!}\right]$$

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$$\log(L(\lambda)) \propto \sum_{i=1}^{n}\log\lambda(x_i) - \lambda(W)$$