INVERTED PENDULUM & UPRIGHT ROBOT ENGR4220 SEMESTER PROJECT

1 Project Overview

In this semester project, you will design an inverted pendulum system. The goal is to build one of the two systems:

- The classical inverted pendulum (Figure 1): A long rod is mounted on low-friction bearings, which rest, in turn, on a movable cart. Without control action, the rod would immediately topple and drop to either side. However, the cart can be moved under the influence of a motor to provide a corrective action. A suitable sensor determines the angle of the inverted pendulum, and a controller attempts to keep it in the upright vertical position.

- The upright robot (Figure 2): A relatively lightweight two-wheeled 'cart' that is able to stand upright despite gravitational forces and other disturbances. The wheels can be coupled so that the upright robot moves in one dimension only. Most likely, the upright robot *requires* the use of digital control due to the sensors that need to be used – gyroscope ($\dot{\Theta}$) and accelerometer (Θ).

The control problem consists of reconciling disturbances on either the pendulum or the body of the robot and actuating the motor for the corrective action. Specifically, the acceleration a, which is determined by the drive of the DC motors, overcomes the disturbances acting on the pendulum/robot and keeps it upright. For this purpose of establishing feedback control, a suitable angular displacement sensor needs to be provided, and the current to the DC motors needs to be controlled in such a fashion that the angle of displacement of the pendulum or robot is near zero. The semester project is largely a design project, and the path to a viable solution is not prescribed. There are some mandatory components, however, and those are specifically highlighted. To provide an overview, the design steps comprise:

1. Design and fabrication of the *process*:

- For the pendulum, this involves the movable cart, which can be either running on free wheels or on rails of some form; a low-friction bearing for the pivot point; and a rod whose center of gravity lies above the pivot point.
- For the robot, this primarily involves the robot frame, the wheels, and the motor or motors.

2. Design of an angular displacement sensor. Note that we need both Θ and $\dot{\Theta}$; optical or magnetic sensors are possible options for the inverted pendulum. The upright robot *necessitates* [1] the use of digital accelerometer and gyroscope. If the sensor is analog, the derivative $\dot{\Theta}$ can be obtained with an analog differentiator or with a digital control system.

3. Mathematical description of the inverted pendulum process and the sensor

4. Design of a controller

5. Analysis of the closed-loop system (transient response, disturbance rejection, stability) with possible changes to the controller design to improve the quality of the control system

6. Demonstration of the working closed-loop system

The grading breakdown is described in Section 9. Please consider from the get-go that there will be bonus point awards for exceptionally good designs (see Section 9).

Grading milestone #1 (3 points): Identify your team members (4-5 members per team, points deducted for deviations from that number). Select a team leader. Make a decision which of the two models (classical pendulum or upright robot) you want to build. Prepare as the first page of your report the list of team members and the model. Turn in that page.

2 Design of the Process

The first step in this semester project is the design of the *process*, which should be functionally similar to one of the sketches shown in Figures 1 or

[1] If you find an alternative solution, you are welcome to use it

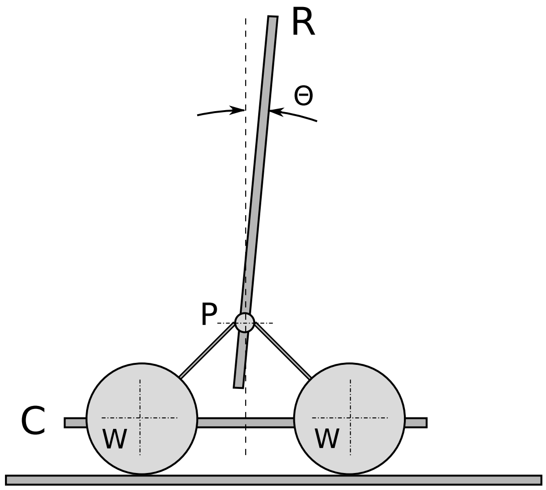

Figure 1: Sketch of the 'classical' *inverted pendulum* process: A long rod R is mounted on low-friction bearings (pivot point P), which rest, in turn, on a movable cart C. The cart is mounted on wheels W and can be moved under the influence of a motor to provide a corrective action that drives the angle Θ to zero.

2. It is suggested that you begin by selecting the physical components of device, which include the motors and motor drivers to drive the system, the cart wheels or rails, and cart body. Remember to select motors that may operate on power devices stored onboard the device, and that can react suitably fast enough to keep the pendulum upright. Also keep in mind that you can use the 3D printer to manufacture mechanical parts, the cart, or the robot itself.

Next, find the mathematical relationship between the length of the cart (pendulum), the angle Θ (that describes the angular displacement of the pendulum or robot from an upright position), gravity, disturbances to the system. Hint: The equations are nonlinear. Ideally, you should support this relationship with experimental data, because this will help your design later.

Grading milestone #2 (27 points total): A completed assembly that consists of the robot body or the movable cart with the mounted pendulum (low friction is a requirement – the pendulum needs to topple on its own), the motors, and the wheels (13 points). The motors should be strong enough that, when energized with DC current, the wheels turn with sufficient speed to move the cart (5 points). Equations that relate the net force on the cart to the angle of displacement must accompany this milestone (10 points). Turn in your amended report.

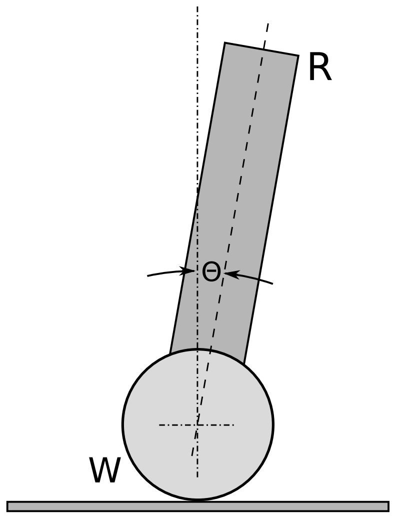

Figure 2: Sketch of the *upright robot* process: The upright robot body R is suspended vertically by two wheels W, which are in turn driven by DC motors. The DC motors are mounted on the robot body. The angle of displacement Θ of the cart must be measured by a sensor. The upright robot needs its power source on the cart (this is an advantage as the additional mass makes balancing easier)

3 Mathematical Description of the Process

Provide the Laplace-domain transfer function of the *process*. Include a time-variable Force $F(s)$ that acts on the inverted pendulum as disturbance. You will want to start with balancing the forces of the system to develop a constituent differential equations. The transfer function for the pendulum will relate the disturbance $F(s)$ to the corrective action, i.e., the cart acceleration $A(s)$. In fact, through the motor drive current, you will control the acceleration of the cart: The motor torque is proportional to the motor drive current I_M, and the acceleration A emerges.

Grading milestone #3 (10 points): Transfer function(s) of the process with the input variables $F(s)$ (disturbance) and $I_M(s)$ and with the output variable $\Theta(s)$. Turn in your amended report.

4 Design of the Sensor

In this step, you need to design and build the sensor that measures the angular displacement Θ from the vertical direction. With the cart-mounted inverted pendulum (Figure 1), a number of options exist, including magnetic or optical sensors, inductive sensors, the accelerometer-gyroscope combination, or a quadrature encoder. It is important that the sensor does not introduce friction, and that the pendulum is still toppling on its own.

With the upright robot (Figure 2), it is more difficult to access a reference surface, and a digital accelerometer-gyroscope combination would be the primary choice for consideration. Other designs, such as e.g., a "tail" that the robot drags and that touches the floor, are possible. Similar to the classical pendulum, the sensor must not introduce friction, and the robot can still topple on its own.

The list of sensor suggestions above is not exhaustive. The sensor would either provide a digital signal (this will require a digital control solution) or a voltage that is proportional to the angle Θ.

When you choose your sensor, consider several parameters, such as linear response, the dynamic range (i.e., distance between the maximum and minimum voltage output), noise and external influences (e.g., 60Hz EMI or 120Hz flicker), and the transfer function (e.g., inertia or a delay between input and output). Lastly, consider how you will obtain $\dot{\Theta}(t)$. A gyroscope provides this signal directly; a digital solution can include discrete differentiation, and an analog solution can employ an op-amp based differentiator.

You need to determine the sensor characteristic curve, that is, the quantitative relationship between $\Theta(t)$ (input) and $V_\Theta(t)$ (output) by measurement. Add the description of the sensor principle and the measured sensor characteristic to your report. Keep in mind that the senor needs to provide a signed output, where the sign depends on the direction of the angular deviation. For a digital sensor, provide a similar curve with the digital output values as a function of Θ.

To successfully complete this Milestone, your report must contain:

- The sensor transfer curve as specified above
- Evidence that the sensor's output follows the measured variable *instantly* or that the time constant can be neglected against the time constant of the inertia
- Evidence that the sensor's output data rate is order-of-magnitude higher than the sampling rate (or at least the time constant of the inertia) *or* that the sensor is time-continuous
- Evidence that the sensor can pick up angular deviations as small as 0.5 degrees.
- Optional: Evidence that you can measure the first derivative with an estimate of the signal-to-noise ratio. This element is optional, but it puts you in an advantageous situation if you complete it.

Grading milestone #4 (20 points): A working sensor for Θ and a measured sensor transfer function with the input variable Θ and with the output variable V_Θ. Turn in your amended report.

5 Design of the Motor Driver

The motor driver deserves special consideration, because here you split paths between a pure analog solution and a semi-analog solution with digital option. A transistor, configured as emitter follower, serves as voltage source. The motor has predominantly resistive behavior, and voltage and current are proportional. Since there is a voltage drop across the transistor, it may need a heatsink for cooling. Alternatively, you may opt for pulse width modulation (PWM). Many microcontrollers have on-chip PWM capability. Analog ICs exist that provide a pulse-width modulated output signal. Moreover, complete integrated PWM feedback controllers exist, such as the UC3843 or the LM494. You need to decide at this point – at least tentatively – which of the following three options you prefer:

1. Purely analog solution where two transistors serve as variable voltage source with the ability to reverse polarity and thus direcion. You can directly use the output of a controller op-amp to feed the transistors.

2. Analog solution with PWM: You can still use a purely op-amp based solution, but you use the transistor as a switch. This will require the use some form of oscillator and comparator to generate the PWM signal. It is possible to use discrete op-amps to build this unit, but integrated chips (e.g., LM494) are an interesting alternative, because you not only have the PWM generator integrated, but also a complete error amplifier around which you can build your controller. Note that you need to be able to reverse the motor direction. Use of a H-bridge (e..g, LMD18201) is recommended.

3. Digital solution: This solution uses PWM and a microcontroller. Use of a H-bridge (e..g, LMD18201) is recommended. You minimize the circuit efforts, but you need to realize the controller in software.

Irrespective of whether you use pure analog control or whether you prefer a digital option with PWM, the overall effort is not fundamentally different. In addition, you may change from PWM to purely analog (and *vice-versa*) at a later point if needed. In all cases, you need to be able to reverse the motor's direction.

In addition to the driver itself, you should attempt to drive the motors with some scaled or amplified version of your sensor output (P-control) and demonstrate that a deviation from the vertical orientation causes the motors to rotate in such a fashion that it would correct the angle mismatch. A larger angle mismatch should cause the motors to turn faster. Without evidence that you achieved this behavior, only up to 50% of this Milestone's score points can be earned.

Grading milestone #5 (20 points): Proof that you have a working motor driver with a control voltage input and a voltage-proportional motor angular velocity or torque. Proof that increased angle mismatch causes faster motor speed in the correct direction. Submit your amended report.

6 Design of the Controller – Theory

You are now ready to build the controller and close the loop. Most likely, you'll have to switch back and forth between open-loop and closed-loop configurations to test the component's performance and to optimize the controller. For this reason, the following outline is merely a suggestion. Ideally, you begin the controller design with a good description of the process, and this will lead to the next two grading milestones (theory and working closed-loop model).

Analyze the components you built up to this point (i.e., the pendulum or robot body and the sensor). Where are the poles of the open loop components? What do you observe with respect to stability? What is the frequency response? What is the dynamic impulse or step response? Most importantly, where do you want to place the closed-loop poles to get the optimum dynamic response? Where would you introduce a control voltage that controls the nominal vertical reference (setpoint)?

Based on these observations, propose a controller transfer function. How does your pole placement influence the loop gain and thus the suppression of disturbances? Sketch a circuit that realizes your transfer function (for the analog solution, see Chapter 3.6 of our book).

Note: If you use a microcontroller-based system (i.e., a time-discrete controller), your analysis will have to use the z-domain. Any purely analog solutions, and this includes integrated PWM controllers, may use Laplace-domain methods.

Provide an analysis of the behavior of the open-loop system, the controller, and the closed-loop system. Use equations, pole-zero diagrams, simulations, or any other tool you deem suitable. At this point at the latest, you need to know your process constants, such as the sensor gain in the operating point and your voltage-dependent cart acceleration. Amend your report with the proposed controller, your controller circuit diagram, and the analysis of the controller behavior. As the absolute minimum, you need to address the following points:

- Location of the open-loop poles of the process and the sensor.
- Desired location of the closed-loop poles, justified by the desired dynamic response.
- How does the proposed controller transfer function lead to the desired closed-loop pole location? What coefficients does the controller have, and how do they influence the pole location?
- Stability analysis: What range of controller coefficients leads to a stable system? What range of controller coefficients leads to an unstable system?
- Robustness: Which process constants (e.g., cart mass, motor efficiency, drive voltage) reduce relative stability or lead to an unstable system? Moreover, how does the nonlinear behavior of the process and the sensor influence the loop gain and thus the dynamic response, the absolute or relative stability?
- Steady state: Does your controller reach the equilibrium $\Theta(t \to \infty) = 0$?

Most of these questions can be answered with equations. A simulation could complement the theory and can even consider the nonlinearities, such as motor slack or stick friction. The overarching goal here is to demonstrate how the theory leads to a rational controller design.

Grading milestone #6 (20 points): Proposed controller and theoretical analysis of the expected closed-loop behavior. Acceptable closed-loop behavior of the physical system is not necessary for this grading milestone. Turn in your amended report.

7 Design of the Controller – Practice

This Milestone involves completing your system. Build the controller you proposed in the previous section. Complete your system by feeding the sensor signal into the controller and using the controller's output voltage (or PWM signal) to drive the motor(s). Perform any fine-tuning of the controller that may be necessary to obtain stable, non-oscillatory behavior.

Note: Individual help will be provided for this step. Try to make some coefficients of the controller adjustable, for example, the placement of pole(s) and zero(s), and the controller gain. Use either a sine-wave frequency sweep (Bode diagram) or a square-wave signal (step response) to verify your controller's transfer function in an open-loop configuration. Most importantly, you will need an adjustable setpoint. Due to mechanical tolerances, you cannot assume that $\Theta = 0$ in the exact vertical position. Any minor deviation will cause the cart to accelerate.

Voltmeters, a frequency generator and – most importantly – an oscilloscope are valuable tools for this step.

This step is successful if you can keep the robot or pendulum upright for a minimum of ten seconds without any additional support. The functionality of your final system can be demonstrated in the presentations. If you are applying for one of the quantitative awards, you are responsible for demonstrating the specified performance feature. If you can demonstrate limited stability as defined above, you receive the score of 40 points for this grading milestone. If the control system is not operating, a partial credit of up to 10 points (for a valiant attempt) and up to 25 points (for a system that is close to functioning) will be awarded for this section.

Grading milestone #7 (40 points): The operational feedback control system, demonstrated in video or presentation. In addition, turn in your report, now amended by any design changes you have made. The amended report must now include all drawings, circuit diagrams, code, and performance reports (or at least measurements).

8 Presentation, Demonstration, Finalized Report

Due to the large number of teams, presentations should be submitted as short (5 minute) videos, which should be approximately split into 3 minutes for the presentation of the design and realization of the controller, followed by 2 minutes of practical demonstration.

At this time, the report should be completed with any new findings, and the design and realization updated. Award-winning performance should be recorded. You may include photographs of your system if you wish. The report *must contain* all circuit diagrams with component values and, for digital systems, the full source code.

Each student will be handed out a score sheet so that the audience can award each presentation up to 20 points. A good presentation can earn points even when the control system is not functional. The audience will also record any awards (and award votes) on the score sheet.

Grading milestone #8 (20 points): Score awarded by the audience for the presentations.

Grading milestone #9 (10 points): Turn in your finalized report.

9 Grading and Award Points

A summary of the grade points (maximum achievable score is 170 points) is below:

- Grading milestone 1 (team nomination): 3 points
- Grading milestone 2 (design and assembly of the process): 27 points
- Grading milestone 3 (transfer function of the process): 10 points
- Grading milestone 4 (design of the sensor): 20 points
- Grading milestone 5 (motor driver, rudimentary P-control): 20 points
- Grading milestone 6 (theory of the controller): 20 points
- Grading milestone 7 (practical realization of the control system): 40 points
- Grading milestone 8 (presentation and demonstration): 20 points
- Grading milestone 9 (complete typewritten report): 10 points

Score points for grading milestone #8 will be awarded by the audience, i.e., the students. Points awarded are the average score from all grade sheets.

On top of the regular score points, each project *with a fully functional control system* [2] eligible for the awards listed below. Each award comes with a bonus score of 10 points or more as specified below. One team can receive multiple awards.

1. An automatic 5-point award is given to any team that uses LaTeX for their report (Milestone 9)
2. Award for the design with the largest inert mass (moment of inertial estimated from mass and length of balanced body): 10 points. Competing: All inverted pendulum projects.
3. Award for the design with the best stability, measured as the time elapsed from a stable, upright initial position to the toppling of the pendulum: 15 points. Competing: All inverted pendulum projects. For a tie, award points are shared, but all projects with $t \to \infty$ receive this award even when tied.
4. Award for the design with the best disturbance rejection, defined as the maximum angular deviation from the vertical position from which the inverted pendulum can recover: 15 points. Competing: All inverted pendulum projects.
5. Award for the design with the highest variation in mass. The pendulum must be able to accommodate added weights (bolts, iron plates or similar) and remain stable for at least 10 seconds. 10 points. Competing: All inverted pendulum projects.
6. Award for the best feature above and beyond this assignment – any demonstrated, useful and justified feature that is not part of the project assignment qualifies. Voted by the audience. Note: This award refers to the built system, not to the presentation. Each student has one vote. You cannot vote for your own team. The majority of votes determines the team that gets this award: 10 points. Competing: All projects.
7. Award for the most artistic design (voted by the audience. Each student has one vote. You cannot vote for your own team. The majority of votes determines the team that gets this award): 10 points. Competing: All projects.

[2] Stability requirement is met when the body remains upright for 10 seconds or more