

Study on creep stress field of concrete

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Abstract—The exact prediction of creep is of great significance to the simulation of the stress field of concrete creep. As the limitation of the current prediction model of creep, by considering the characteristic of the concrete material, the available test data and the pertinent models abroad, the creep coefficient formulae is obtained. In the creep prediction of the high performance concrete, all the influencing factors are quantified, while the calculation is easy and convenient with good fitting accuracy. According to the fit of the creep degree, the implicit method is usually used in the calculation of creep stress field, and based on which, the creep coefficient formulae are established in this paper and used in the strain increment calculation, at the same time, the finite element expression to calculate the creep stress field with creep coefficient is deduced.

Keywords- creep coefficient; prediction model; standard state; nonstandard state ; creep stress field;

I. INTRODUCTION

In this paper, different influencing factors caused by different materials to the performance of concrete are considered, and the influencing factors which are not referred to in ordinary concrete are taken into consideration. By the HPC creep coefficient formula under the NSC standard state, using the spreading models of CEN Eurocode 2(1991) and CEB Model 90(1993), based on the calculation result of the creep coefficient by the evolution type of CEB model(1993), the data stylebook of creep coefficient under standard condition is established, and by which the creep coefficient formula under the standard state is set up, too. Compared with the creep coefficient model of ordinary concrete, all the influence factors are numerical represented in this model, with no need of lots of test data. The model established in the paper enriches the creep forecast model of the normal concrete and can precisely forecast the creep of the high-performance concrete.

This paper can be divided into two parts, with the first part to establish the creep coefficient prediction model for the present concrete materials and check the model, and the second part to use the creep coefficient formula in the incremental strain calculation, and derive the FEM expression of creep stress field by using the creep coefficient.

II. BASIC EQUATIONS OF CREEP COEFFICIENT UNDER STANDARD STATE

The "standard state" in this paper is different from that recommended by the United States Concrete Association (ACI) 209 Committee. In China's power industry standard

DL / T 5150-2001 "hydraulic concrete test order", there isn't involved the environmental humidity in the orders of compression and tension for concrete tests, as a result, by China's normative GBJ 82-85 "long-term performance of ordinary concrete and the durability test method", the standard state is set as: the concrete strength grade is C30, with no fly ash and admixtures, the cross-section size of cube specimens of 150mm × 150mm, and after 3 day's standard curing, creep test is carried out in the constant temperature and humidity room (temperature of 20±2 °C , relative humidity of 60±5%), with the loading creep stress 30% of the damage load, which means that the stress level is 0.3.

The holding time is the major variable of the creep coefficient, and the basic equation of creep coefficient under the standard state is^[1]:

$$\varphi(t, \tau)_{\text{basic}} = \varphi(\infty, 3)k_{\tau} \quad (1)$$

Here, $\varphi(\infty, 3)$ is the creep coefficient under the standard state, when the loading age $t_0=3d$ and the holding time is ∞ ; k_{τ} is the influence function of the holding time; $\tau = t-t_0$ is the holding time.

III. THE DESIGN CREEP COEFFICIENT UNDER NON-STANDARD STATE

There are many influencing factors of the concrete creep^[2-7], and these factors contact with and restrict each other, so its regular should be studied on the basis of large amount of data. The most important factors are load duration, loading age, the characteristic of basic material composing concrete (including the cement variety, aggregate variety and water-cement ratio, etc.), admixture, fly ash content, volume-surface ratio of component, curing condition. Among these factors, the effect of the load duration has been considered under the standard state. The characteristic of the basic material component of concrete is represented by the concrete strength. The above factors are used to determine the influence function, respectively. It is very complex to do nonlinear influence analysis under multivariate and multi-level condition. Therefore, when determining the effect of a factor to the concrete creep function, this factor is changed while the other factors are controlled in the standard state, with all the factors independent of each other.

To sum up, the creep coefficient of the common concrete in the non-standard state can be calculated by the following Eq.(2)^[1]:

$$\varphi(t, \tau) = \varphi(t, 3)_{\text{basic}} \cdot \beta(\tau) \beta(h_0) \beta(f_{ck}) \beta_f \quad (2)$$

Here, $\beta(\tau)$ is the influence function of loading age, $\beta(h_0)$ is the influence coefficient of component theoretical thickness; $\beta(f_{ck})$ is the strength-influence function; β_f is the influence coefficient of fly ash;

IV. THE CALCULATION DERIVATION OF CREEP INCREMENTS

The creep of concrete is related to both the current stress and the historical stress, and the stress history has to be considered in the calculation, so how to compact the memory capacity of the computer is the key point while the finite element numerical method^[9-11], etc are adopted to analyze the concrete structure. For the creep degree which is expressed by the exponential function, Zienkiewicz et has ever provided a explicit method of equal step length and ZHU BF Academician gave the thought of implicit method, on the basis of which, the formula of creep coefficient developed in the paper is used in the calculation of strain increment, and the implicit method of creep equation expressed by creep coefficient is deduced.

Loading from t_0 to t , the strain of concrete creep is:

$$\begin{aligned}\varepsilon^c(t) &= \Delta\sigma_0 C(t, \tau_0) + \int_{\tau_0}^t C(t, \tau) d\sigma = \Delta\sigma_0 C(t, \tau_0) + \sum_i \int_{\tau_{i-1}}^{\tau_i} C(t, \tau) \frac{d\sigma}{d\tau} d\tau \\ &= \Delta\sigma_0 C(t, \tau_0) + \sum_i \left(\frac{d\sigma}{d\tau} \right)_i \int_{\tau_{i-1}}^{\tau_i} C(t, \tau) d\tau\end{aligned}\quad (3)$$

When the time step length is not very large, according to the integral mean value theorem, the creep degree $C(t, \bar{\tau}_i)$ at the mid-point age is used to replace $C(t, \tau)$, then the following formula is obtained:

$$\varepsilon^c(t) = \Delta\sigma_0 C(t, \tau_0) + \sum_i \left(\frac{d\sigma}{d\tau} \right)_i C(t, \bar{\tau}_i) \Delta\tau_i = \Delta\sigma_0 C(t, \tau_0) + \sum_i C(t, \bar{\tau}_i) \Delta\sigma_i \quad (4)$$

According to the relational expression between creep coefficient and creep degree:

$$\varphi(t, \tau) = C(t, \tau) E(\tau) \quad (5)$$

There is:

$$C(t, \tau) = \frac{\varphi(t, \tau)}{E(\tau)} \quad (6)$$

According to Eq.(2), for the particular concrete and curing condition, $\beta(h_0)$, $\beta(f_{ck})$, β_a , β_f , are constant values. There is:

$$\beta = \beta(h_0) \beta(f_{ck}) \beta_a \beta_f \quad (7)$$

Put Eq.(7) into Eq.(6), then there is:

$$C(t, \tau) = \frac{\beta(\tau) \beta}{E(\tau)} \varphi(t - \tau, 3)_{\text{basic}} \quad (8)$$

Put Eq.(6) into Eq.(4), then there is:

$$\begin{aligned}\varepsilon^c(t) &= \Delta\sigma_0 \frac{\beta(\tau_0) \beta}{E(\tau_0)} \varphi(t - \tau_0, 3)_{\text{basic}} + \sum_i \frac{\beta(\bar{\tau}_i) \beta}{E(\bar{\tau}_i)} \varphi(t - \bar{\tau}_i, 3)_{\text{basic}} \Delta\sigma_i \\ &= \Delta\sigma_0 \frac{\beta(\tau_0) \beta}{E(\tau_0)} 3.7419[1 - 0.72e^{-0.006(t-\tau_0)}] + \sum_i \frac{\beta(\bar{\tau}_i) \beta}{E(\bar{\tau}_i)} 3.7419[1 - 0.72e^{-0.006(t-\bar{\tau}_i)}] \Delta\sigma_i\end{aligned}\quad (9)$$

Choose three adjacent time t_{n+1} , t_n , and t_{n-1} , with time step according to $\Delta\tau_n = t_n - t_{n-1}$, $\Delta\tau_{n+1} = t_{n+1} - t_n$, then there is:

$$\varepsilon_{n+1}^c = \Delta\sigma_0 \frac{\beta(\tau_0) \beta}{E(\tau_0)} 3.7419[1 - 0.72e^{-0.006(t_{n+1}-\tau_0)}]$$

$$+ \sum_i \frac{\beta(\bar{\tau}_i) \beta}{E(\bar{\tau}_i)} 3.7419[1 - 0.72e^{-0.006(t_{n+1}-\bar{\tau}_i)}] \Delta\sigma_i \quad (10)$$

$$\begin{aligned}\varepsilon_n^c &= \Delta\sigma_0 \frac{\beta(\tau_0) \beta}{E(\tau_0)} 3.7419[1 - 0.72e^{-0.006(t_n-\tau_0)}] \\ &+ \sum_i \frac{\beta(\bar{\tau}_i) \beta}{E(\bar{\tau}_i)} 3.7419[1 - 0.72e^{-0.006(t_n-\bar{\tau}_i)}] \Delta\sigma_i\end{aligned}\quad (11)$$

$$\begin{aligned}\varepsilon_{n-1}^c &= \Delta\sigma_0 \frac{\beta(\tau_0) \beta}{E(\tau_0)} 3.7419[1 - 0.72e^{-0.006(t_{n-1}-\tau_0)}] \\ &+ \sum_i \frac{\beta(\bar{\tau}_i) \beta}{E(\bar{\tau}_i)} 3.7419[1 - 0.72e^{-0.006(t_{n-1}-\bar{\tau}_i)}] \Delta\sigma_i\end{aligned}\quad (12)$$

Subtract Eq. (10) with (11), then there is:

$$\begin{aligned}\Delta\varepsilon_{n+1}^c &= \Delta\sigma_0 \frac{\beta(\tau_0) \beta}{E(\tau_0)} 2.694[e^{-0.006(t_n-\tau_0)} - e^{-0.006(t_{n+1}-\tau_0)}] \\ &+ \sum_i \frac{\beta(\bar{\tau}_i) \beta}{E(\bar{\tau}_i)} 2.694[e^{-0.006(t_n-\bar{\tau}_i)} - e^{-0.006(t_{n+1}-\bar{\tau}_i)}] \Delta\sigma_i \\ &+ \frac{\beta(\bar{\tau}_{n+1}) \beta}{E(\bar{\tau}_{n+1})} 3.7419[1 - 0.72e^{-0.006(t_n+\Delta\tau_{n+1}-\bar{\tau}_{n+1})}] \Delta\sigma_{n+1}\end{aligned}\quad (13)$$

$$\psi(\tau) = \frac{\beta(\tau) \beta}{E(\tau)} 2.694 \quad (14)$$

Put Eq.(14) into (13), then (14) can be simplified as:

$$\begin{aligned}\Delta\varepsilon_{n+1}^c &= (1 - e^{-0.006\Delta\tau_{n+1}}) \{ \Delta\sigma_0 \psi(\tau_0) e^{-0.006(t_n-\tau_0)} + \sum_i \Delta\sigma_i \psi(\bar{\tau}_i) e^{-0.006(t_n-\bar{\tau}_i)} \} \\ &+ \frac{\beta(\bar{\tau}_{n+1}) \beta}{E(\bar{\tau}_{n+1})} 3.7419[1 - 0.72e^{-0.006(t_n+\Delta\tau_{n+1}-\bar{\tau}_{n+1})}] \Delta\sigma_{n+1}\end{aligned}\quad (15)$$

Subtract formula(11) with(12), then there is:

$$\begin{aligned}\Delta\varepsilon_n^c &= (1 - e^{-0.006\Delta\tau_n}) \{ \Delta\sigma_0 \psi(\tau_0) e^{-0.006(t_n-\tau_0)} + \sum_i \Delta\sigma_i \psi(\bar{\tau}_i) e^{-0.006(t_n-\bar{\tau}_i)} \} \\ &+ \frac{\beta(\bar{\tau}_n) \beta}{E(\bar{\tau}_n)} 3.7419[1 - 0.72e^{-0.006(t_n-\bar{\tau}_n)}] \Delta\sigma_n\end{aligned}\quad (16)$$

From the Eq.(15) and (16), there is the recursion formula:

$$\Delta\varepsilon_n^c = (1 - e^{-0.006\Delta\tau_n}) \vartheta_n + \Delta\sigma_n C(t_n, \bar{\tau}_n) \quad (17)$$

Here,

$$\vartheta_n = \vartheta_{n-1} e^{-0.006\Delta\tau_{n-1}} + \Delta\sigma_{n-1} \psi(\bar{\tau}_{n-1}) e^{-0.003\Delta\tau_{n-1}}, \vartheta_1 = \Delta\sigma_0 \psi(\tau_0).$$

When the concrete structure is analyzed with numerical method, these recursion Equation don't need to record the stress history, but only need to store ϑ_n .

V. CONCLUSION

Correctly predicting the creep model of high performance concrete is very important for the engineering design and construction. For this reason, some exploration about how to develop a set of Equation of creep prediction model that are convenient, practical and precise are done, and some conclusions are obtained as follows:

(1) Under the circumstances of lacking of tests, not only the creep of common concrete can be predicted, but also the creep of high performance concrete can be simply estimated with certain accuracy according to the material and the circumstance by the Equation under non-standard state.

(2) In the calculation of creep stress field, the implicit method is usually adopted to calculate the creep by fitting creep degree, and on its basis, the creep coefficient Equation

are used in the calculation of strain increment, while the finite element expression to calculate the creep stress field with creep coefficient is deduced, with the advantage of no need to store stress period.

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