Analysis on Stress Loss in Prestressed Concrete Structure

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Abstract—**This paper aims at the problem of prestress loss caused by friction between prestressed tendon and curved duct for the design of prestressed concrete structure. Formulas of normal pressure between prestressed tendon and curved duct and tension of prestressed tendon are derived based on the Hertz contact theory. And also the results come from experiments, derived formulas and current norms are compared in three cases.**

Keywords- Prestress loss; prestressed concrete structure; prestressed tendon

I. INTRODUCTION

A lot of test results show that the actual prestress loss is larger than that given by the current bridge design codes for long span prestressed concrete structure. Some uncertain factors, such as the deviation of the duct wall, affect the stress distribution of concrete structure. Bridge Engineers have done a great deal of experimental research on prestress loss caused by the deviation of concrete duct wall, and it is usually considered that the deviation coefficient of duct wall given by current design codes is slightly smaller than the measured value.

This paper discusses the problem of prestress loss caused by the friction between prestressed tendon and curved duct wall for the prestressed concrete structure design. Formulas of normal pressure between prestressed tendon and curved duct and tension of prestressed tendon are derived based on the Hertz contact theory. Physical experiments have been carried out to explore the relationship between friction moment and tension of prestressed tendon for different angles of contact surfaces. The results have been compared with that of the current design theory of prestressed concrete structure.

II. THEORETICAL ANALYSIS OF PRESTRESS LOSS CAUSED BY FRICTION IN CURVED DUCT

A. Current method of prestress loss caused by friction in curved duct

In current theories or norms of prestressed structure design, prestress loss caused by the friction between prestressed tendon and curved wall is

$$\sigma_s = \sigma_k(1 - e^{-\mu\theta + kx}) \tag{1}$$

Where

σ_s is the friction loss caused by deviation of the duct wall (MPa),

σ_k is the control stress under anchor (MPa),

μ is the friction coefficient between prestressed tendon and duct wall,

θ is the tangent angle of curved duct from tension end to the calculated cross-section(rad),

k is the influence coefficient of the frictional resistance caused by local deviation in unit length of the duct,

x is the duct length from tension end to calculated cross-section(m).

B. Fomulas derivation of prestress loss caused by friction in curved duct based on Hertz theory

The static equilibrium condition is used in the course of deriving formula (1). The Contact stress depends primarily on tension T, which follows the function of exponential distribution with θ. In case that there is no relative slip between prestressed tendon and curved duct, normal pressure p in unit length of duct wall is uniformly distributed on the contact surface. According to Hertz contact theory, when two elastic bodies contact and extrude with each other, elastic deformation will happen. The contact stress on contact surface is in ellipsoidal distribution, and it is a function of the radius of curvature and the elastic modulus of contact bodies. Figure 1 is the sketch of distribution of contact normal stress of the above theories in curved duct.

In fact, in the process of tension, strands will produce large normal pressure on the curved part of the concrete, making concrete structure deform locally, thus change the size of radius of curvature R of the contact surface, and then impact on the distribution of the normal pressure distribution p. Taking elastic deformation of the curved duct wall into account, the distribution of the normal contact stress between curved duct and prestressed strands should be between uniform and ellipsoidal. According to Coulomb friction law, the relative sliding friction force between curved duct and prestressed strands is proportional to contact pressure; the non-linear distribution of p will make frictional reststance change non-uniformly in curved duct either.

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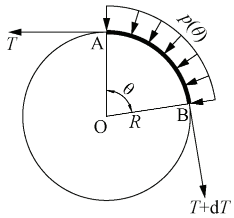
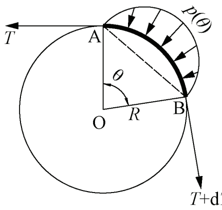


(a) contact stress in current design theory



(b) contact stress in elastic theory

Figure 1. Sketch of the distribution of contact normal stress in curved duct

In follewing research, the contact problem between prestressed strands and curved duct wall is reduced to the special case that two curved-surface bodies contacted in a small area of surface, that is the case of two spheres contact with each other.

Assuming that the radius of the two spheres are R1 and R2 respectively (shown in Figure 2), two spheres touch only at the point of O if there is no force acting on. When two spheres touch each other with a force P, local deformation will happen, which causes a circular contact surface with radius of a (Figure 3) formed. If a half sphere is drawn at the boundary of the contact surface, the height of one point on the spherical surface just represents the value of stress q of the point on the contact surface correspondingly. It is easy to know that the maximum pressure q_0 happens at the center of contact surface. The expression is

$$q_0 = \frac{3P}{2\pi a^2} \tag{2}$$

$$a = \left[\frac{3\pi P(k_1 + k_2) R_1 R_2}{4(R_1 + R_2)} \right]^{\frac{1}{2}} \tag{3}$$

Where

$$k_1 = \frac{1-\mu_1^2}{\pi E_1}, \; k_2 = \frac{1-\mu_2^2}{\pi E_2} \tag{4}$$

μ_1 , E_1 , μ_2 and E_2 are the elastic constants of of the two spheres respectively.

Since the radius a is much smaller than the radius R_1 and R_2, it is assumed that pressure distribution in the local area of the contact point is average for simplify.

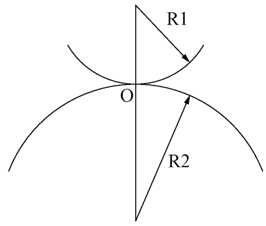


Figure 2. Two spheres contact

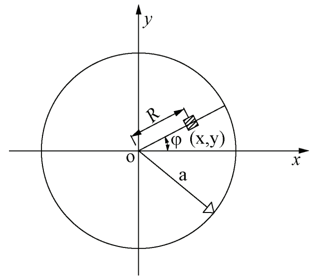


Figure 3. Contact surface

The average pressure $\bar{q}$ can be used as the pressure of every point in the contact surface.

$$q = \bar{q} = \frac{P}{\pi a^2} \tag{5}$$

Assuming prestressed tendon as the upper sphere, and curved duct as the lower sphere, and the initial tension as T(shown in Figure 4), then the formula of force *P* in the contact surface between curved duct and prestressed strands can be derived.

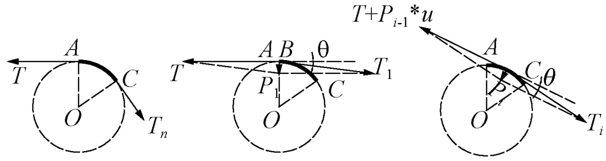


Figure 4. Sketch of derivation of force P

The initial tension T acts at point A. There is a point B in the arc AC. Draw line OA and line OB, the angle between line OA and line OB is θ . Assuming angle θ is small, the force P_1 is

$$P_1 = T \times \tan\theta \tag{6}$$

There is $\theta = \tan\theta$, as angle θ is small.

So

$$P_1 \approx T \times \theta \tag{6'}$$

Assuming the friction coefficient of all the points of contact surface is μ

$$P_2 = (\mu \times P_1 + T) \times \theta \tag{7}$$

The force of any point can be written as

$$P_i = (\mu \times P_{i-1} + T) \times \theta \quad i=2,\ldots,n \quad n = \theta_0 / (2\theta) \tag{7'}$$

Where θ_0 is the central angle on the contact surface between curved duct and prestressed strands.

$$P_i = \begin{cases} P_{2n+1-i} & n \text{ is even} \\ P_{2n-i} & n \text{ is odd} \end{cases} \tag{8}$$

The force P_n of point C is

$$P_n = (\lambda \times P_{n-1} + T) \times \theta \tag{9}$$

Thinking the friction loss in curved duct is caused by rigid bodies and elastic bodies, then the prestress loss σ_w caused by friction is

$$\sigma_w = \sigma_s + \lambda q_n \theta_0 = \sigma_k [1 - e^{-(\mu\theta + kx)}] + \frac{\mu P_n \theta_0}{\pi a^2} \tag{10}$$

III. FRICTION EXPERIMENT IN CURVED DUCT

A. Experimental device

Figure 5 shows the experimental device. The radius of the friction wheel is 173mm, and the angle between steel rope and the contact surface of wheel is θ, a thin layer of rubber is pasted around the edge of wheel to increase the friction. When rotating the wheel clockwise, a relative slide between steel rope and the wheel will happen. Given a tension T, the other tension T' can be got by force sensor accordingly, there is T＞T'. Thus the friction torque caused by the friction force between the steel wire rope and the wheel could be calculated. Formula (2) shows the friction torque

$$M_f = TR - T'R \tag{11}$$

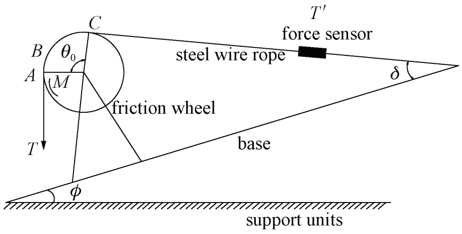


Figure 5. Diagram of experiment device

B. Experiment cases and data processing

Figures 6 and Figures 7 shows two kinds of curved duct respectively: continuous curved duct and discontinue curved duct, where $\theta = \theta_1 + \theta_2$. The initial tension *T,* corresponding to angle θ shown in Figure 6, is equal to T_1. And the initial tension T_2, corresponding to angle θ_2, is equal to $T_1^{'}$, corresponding to the terminal end of angle θ_1 shown in Figure 7, which means $T = T_1^{'}$ and $T_1^{'} = T_2$. For multiple discontinue curved ducts, the initial tensions corresponding to each angles could be got by analogy.

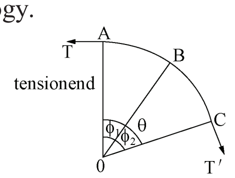


Figure 6. Tension of the continuous curved duct

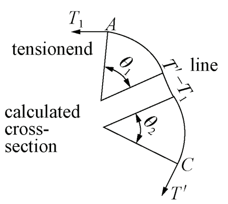


Figure 7. Tension of the discontinue curved duct

There are three cases in different tensions and different angles with its sub-angles. Table 1 shows the results of friction torque in the continuous and discontinuous curved duct.

Table 1. Friction torque of different cases (N/N·m)

case	θ	T_1=T	T_1=T_2	T_2=T_3	T_3/T	M_f	ΔM_f
1	3×20°	588	490.4	414.8	355.1	40.29	6.14
	60°	588			319.6	46.43	
	3×20°	784	653.6	550.8	474.5	53.54	6.28
	60°	784			438.2	59.82	
	3×20°	882	736.2	612.8	512.3	63.97	7.18
	60°	882			470.7	71.15	
2	2×30°	588	430.7		321.9	46.05	0.38
	60°	588			319.6	46.43	
	2×30°	784	596.2		451.1	57.59	2.23
	60°	784			438.2	59.82	
	2×30°	882	657.4		489.6	67.88	3.27
	60°	882			470.7	71.15	
3	2×40°	588	412.6		280.3	53.23	0.26
	80°	588			278.8	53.49	
	2×40°	784	547.8		365.7	72.37	2.35
	80°	784			352.1	74.72	
	2×40°	882	590.1		394.4	84.36	3.78
	80°	882			372.5	88.14	

C. Analysis of experimental results

Several conclusions can be got from Table 1 as follows:

(1) If the normal contact pressure between curved duct wall and strands distributes follows the current design theory, there is $M_f^{\theta_1} + M_f^{\theta_2} = M_f^{\theta}$, where M_f^{θ} is defined as the friction torque corresponding to angle θ. Then when the sum of curved angles of discontinue ducts is equal to that of continuous ducts, there is $\sum M_f^{\theta_i} = M_f^{\theta}$. In fact, friction torque of the same curved angle of the continuous curved duct is not equal to that of discontinue ducts.

(2) Friction torque of the continuous curved duct is larger than that of discontinue ducts in the same curved angle. Thus the assumption that the contact stress between curved duct and strands is elliptically distributed is reasonable.

IV. ANALYSIS AND COMPARISON OF PRESTRESS LOSS CALCULATION

The experimental results in Section 3 are used to verify the derived formula in Section 2. The results by three methods are compared in three cases. By normalizing, the maps of three results of prestress loss are shown in Figure 8.

As shown in Figure 8 to Figure 10, the prestress loss increases fast while the tension increasing, and the growth rate follows the quadratic curve. It shows that three results of prestress loss got from three methods are not the same while the curved angle increasing. Experimental results and derived formula results are significantly larger than the results from current theories. And derived formula results are closer to the experimental results.

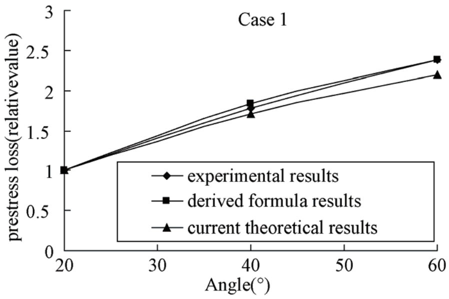


Figure 8. Comparison of theoretical, derived formula and experimental results in Case 1

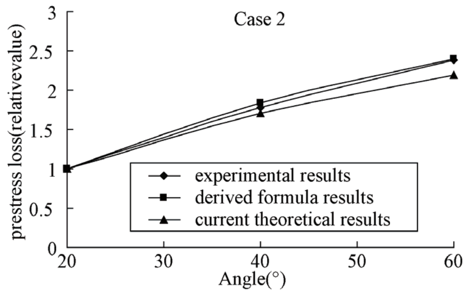


Figure 9. Comparison of theoretical, derived formula and experimental results in Case 2

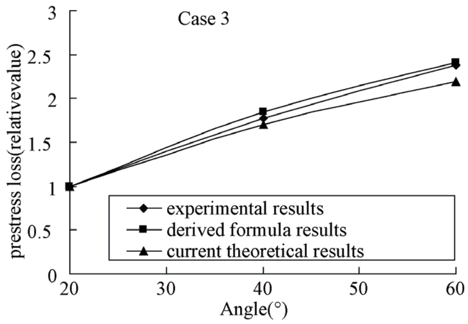


Figure 10. Comparison of theoretical, derived formula and experimental results in Case 3

V. CONCLUSIONS

This article explored the prestress loss caused by the prestressed strands and curved duct wall, and compared the experimental results with the results from the current theories of prestressed concrete structural design, and those of derived formula based on Hertz theory. The conclusions as follow:

(1) According to elastic contact theory and experimental results, distribution of normal stress between strands and curved duct wall concerns with not only tension but also elastic modulus and radius of curvature of touch bodies.

(2) Friction torque of discontinue curved duct is less than continuous curved ducts with the same angle, which is consistent with distribution of contact stress in elastic contact theory.

(3) With the tension increasing in curved duct, the normal stress and the prestress loss increases fastly.

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