Tutorial: Statistical Process Control

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Introduction

Introduction

- Statistical Process Control enables the user to identify variation within their process.
- Understanding this variation is the first step towards quality improvement.
- The simplest SPC techniques to implement are the run and control charts.
- The purpose of these techniques is to identify when the process is displaying unusual behaviour.

About SPC

Understanding Variation: Types of variation

- There are two types of variation
  - **Common cause variation**
    All processes have inherent variation. A process is said to be 'in control' if it exhibits only common cause variation.
  - **Special cause variation**
    caused by unexpected events/unplanned situations. A process is said to be 'out of control' if it exhibits special cause variation.
- SPC charts are a good way to identify between these types of variation.
- There are two types of processes
  - **Dynamic processes** : A process that is observed across time.
  - **Static processes** : A process that is observed at a particular point in time.

Understanding Variation: Example

Identifying Variation: SPC charts

- Two of the most popular SPC charts are the run chart and the control chart.
  - Easy to construct and interpret.
  - To identify the variation type without the need to worry too much about the underlying statistical theory.
- Useful Observations: The number of useful observations in a sample is equal to the total number of observations minus the number of observations falling on the centreline.

SPC Charts: Run chart

- Steps to create a Run chart
  1. Ideally, there should be minimum of 15 data points.
  2. Plot the data on the graph in time order and join adjacent points with solid line.
  3. Calculate the mean or median of the data and draw this on the graph.
- Run: A sequence of one or more consecutive useful observations on the same side of the centreline.
- Trend: A sequence of successive increase or decrease. An observation that falls directly on the centreline, or is the same as the preceding value is not counted.

SPC Charts: Interpret a Run chart

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SPC Charts: Control chart

- Control charts bring the addition of control limits (and warning limits-optional), with a centreline calculated by the mean.
- Steps to create a Control chart
    1. Select the most appropriate control chart for our data, which is dependent on the properties of data. See flow chart.
    2. Proceed as for the run chart, using the mean as the centreline.
    3. Calculate the standard deviation of the sample using the formula listed in the appendix.
    4. Calculate the control limit (optionally, the warning limit):

$$\bar{x} \pm 3 \cdot sd (\text{or } 2 \cdot sd).$$

SPC Charts: Types of control chart

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SPC Charts: Types of control chart

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SPC Charts: Example

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SPC Charts: Interpret a control chart

- The same rules for identifying special cause variation in run charts also apply to control charts, with the addition of two extra rules.

SPC Charts: Interpret a control chart

- The setting of control limits and warning limits are an attempt to balance the risk of committing two possible types of error:
  - Type 1 (False positives): Identifying special cause variation when there is none.
  - Type 2 (False negative): Not identifying special cause variation when there is.
- The combined risk of committing Type 1 and Type 2 errors is minimized when the control limits are set at $3 \cdot sd$ (Carey and Lloyd, 1995).
- However in some cases this may be deemed as too conservative. For example, if poor surgical performance is the process that is being investigated, in order to increase the chances of identifying possible aberrant practice, it may be beneficial to choose tighter limits.

Alternatives

Alternative SPC Charts

- Cumulative Summation (CUSUM) charts are more sensitive to small shifts than the types of SPC charts that have been discussed.
- Exponentially Weighted Moving Average (EWMA) charts, as well as being more sensitive to smaller shifts, also have the advantage of taking into account past data, which avoids biasing the process variation to the current time period.

Appendix

Appendix

- X-chart
  - Assume you have m observations, $X_1, ..., X_m$.
  - Calculate the average

$$\bar{X} = \frac{1}{m}\sum_{i=1}^{m} X_i.$$

  - Calculate the absolute moving ranges(MRs) between adjacent observations, where

$$MR\{i, i+1\} = |X_i - X_{i+1}|, \; i = 1, 2, ..., m-1.$$

  - Calculate the mean range, $\bar{R}$, as

$$\bar{R} = \frac{1}{m-1}\sum_{i=1}^{m-1} MR\{i, i+1\}.$$

  - Set the control limits at

$$\bar{X} \pm (2.66 \times \bar{R}).$$

Appendix

- c-chart
  - Assume you have m observations, $X_i \sim Poisson(\mu)$, where X_i is the number of occurrences for observation i, and μ is the process average.
  - Replace μ by the observed average, which is given by

$$\bar{X} = \frac{1}{m} \sum_{i=1}^{m} X_i.$$

  - The estimated standard deviation is easily given by

$$s = \sqrt{\bar{X}}.$$

  - Set the control limits and warning limits.

Appendix

- u-chart
  - Assume you have m observations, $X_i \sim Poisson(\mu)$, where X_i is the number of occurrences for observation i, and μ is the process average.
  - Since it is the proportion of occurrences, let $Y_i = X_i/n_i \sim Poisson(\mu/n_i)$, where n_i is simply a scaling constant that allows for the heterogeneity of the area of opportunity.
  - Replace μ by the observed average, which is given by

$$\bar{X} = \frac{1}{m}\sum_{i=1}^{m} X_i.$$

  - The estimated standard deviation of Y_i is given by

$$s(Y_i) = \sqrt{\bar{X}/n_i}.$$

Appendix

- np-chart
  - Assume you have m observations, $X_i \sim Bin(n_i, p)$.
  - In addition, let

$$N = \sum_{i=1}^{m} n_i, \qquad X = \sum_{i=1}^{m} X_i$$
$$\bar{X} = X/M, \qquad \hat{p} = X/N.$$

  - The estimated standard deviation of X_i is given by

$$s = \sqrt{n_i \hat{p}(1 - \hat{p})}.$$

  - This s is used for the calculation of the control limits.

Appendix

- p-chart
  - Assume you have m observations, $X_i \sim Bin(n_i, p)$.
  - Since it is the proportion occurrences, let $Y_i = X_i/n_i$.
  - The estimated standard deviation is given by

$$s(Y_i) = \sqrt{\hat{p}(1-\hat{p})/n_i}.$$

  - This s is used for the calculation of the control limits.