Electromagnetic Theory

EEPC-203 (3-1-0)

Electrostatics - Magnetostatics -Time varying fields

Subject: Electromagnetic field theory

Subject Code: EEPC203

Semester: III

Lecture No: 02

Topic: Coulomb’s Law

L-02

Maxwell's Equation

$$\oint_s \vec{E}.d\vec{S} = \frac{Q}{\epsilon_0} \tag{1}$$

$$\oint_c \vec{E}.d\vec{l} = -\frac{d}{dt}\int_s \vec{B}.d\vec{S} \tag{2}$$

$$\oint_s \vec{B}.d\vec{S} = 0 \tag{3}$$

$$\oint_c \vec{B}.d\vec{l} = \mu_0 I_{encl} + \mu_0\epsilon_0 \frac{d}{dt}\int_s \vec{E}.d\vec{S} \tag{4}$$

L-02

What is this course??

- Fundamental understanding of how electricity and magnetism works
- Understanding of these equations one by one, piece by piece
- How these equations describe electric and magnetic field? where those fields come from? how they are made!
- What they do? how they affect charges and currents.

L-02

Why this course??

These equations were written down by this gentleman in 1864

L-02

Why this course??

- These equations describe enormous amount of natural phenomena and of technology
- Any electrical circuit, at fundamental level is governed by these maxwell's equations
- These also describes many natural phenomena eg:lightning
- **In 1887, Heinrich Hertz, discovered for the first time these wave solutions to Maxwell's equations**
- It's these waves that are used by my cell phone to get its signal or to send signals
- They are television signals, Wi-Fi, microwaves, light.

L-02

Atom and charge

- All matter is made up of charged particles– electrons and protons.
- And electrons have negative charge, the protons that positive charge
- If atom is neutral, number of protons is equal to number of electrons
- If you add one electron to neutral atom, you will get **negative ion** and if you remove one electron from neutral atom, you will get **positive ion.**

L-02

Unit of a charge

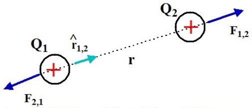
- Unit of charge is coulomb
- One coulomb is a very big amount of charge
- Charge of a single electron: - e (e=1.602*10^{-19} coulombs)
- Charges are quantized and conserved.
  1. **Quantized:** Charge of an any object is integer multiple of e
  2. **Conserved:** We cant make charge out of nowhere; we can charge the an object is by seperating positive and negative charges

Problem 1

Calculate the number of electrons in 1 C?

L-02 **Electric Force: Coulomb's law**

Consider two positive charges Q_1 and Q_2 seperated by distance 'r' as shown below



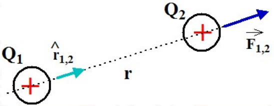
French army engineer, Colonel Charles Coulomb stated that the force between two very small objects seperated in vaccum or free space by **a distance which is large compared to their size** is as given below

$$F \propto \frac{Q_1 Q_2}{r^2} \tag{5}$$

Force 'F' is Central. A central force is a force that points along the direction joining the two objects

L-02

Electric Force: Coulomb's law



The force on the charge Q_2 due to the interaction between the charges, $\vec{F_{12}}$ is given by

$$\vec{F_{12}} = k\frac{Q_1 Q_2}{r^2}\hat{r_{12}} \tag{6}$$

where, $\hat{r_{12}}$ is the unit vector in the direction from Q_1 to Q_2

Problem 2

Value of 'k' and unit of 'k'?

L-02

Electric Force and Gravitational force

Newton's law of gravity

$$F = G * \frac{m_1 m_2}{r^2} \tag{7}$$

where G is the gravitational constant whose value is equal to $6.67*10^{-11}\ \frac{m^2 N}{kg^2}$

Q1) What is the similarity between electric force and gravitational force?

Answer:

L-02

Electric Force and Gravitational force

Newton's law of gravity

$$F = G * \frac{m_1 m_2}{r^2} \tag{8}$$

where G is the gravitational constant whose value is equal to $6.67*10^{-11} \frac{m^2 N}{kg^2}$

Q1) What is the similarity between electric force and gravitational force?

Answer:

- Both are inverse square laws
- Unit of k and unit of G are similar

L-02

Electric force and Gravitational force

It's a beauty that electricity acts in a way that gravity works!!

- Gravitational force is always attractive

P3) Consider two protons seperated by a small distance of 10^{-12}, calculate the ratio of electric force to the gravitational force (mass of the proton is 1.7 * 10^{-27})

Answer:

L-02

Electric force and Gravitational force

It's a beauty that electricity acts in a way that gravity works!!

P3) Consider two protons seperated by a small distance of 10^{-12}, calculate the ratio of electric force to the gravitational force (mass of the proton is $1.7 * 10^{-27}$)

Answer:

$$\frac{F_e}{F_g} \simeq 10^{36}$$

Electric force is 10^{36} times more than the gravitational force!!

L-02

Electric force and Gravitational force

It's a beauty that electricity acts in a way that gravity works!!

P4) Earth and Mars has a charge of about $4 * 10^5\ C$. With the mass of earth ($5.972 * 10^{24} kg$) and mass of mars ($6.39 * 10^{23} kg$), calculate the ratio of electric force to the gravitational force

Answer:

Hint: Distance of seperation is insignificant in calculating the ratio of forces

L-02

Electric force and Gravitational force

It's a beauty that electricity acts in a way that gravity works!!

P4) Earth and Mars has a charge of about $4 * 10^5\ C$. With the mass of earth ($5.972 * 10^{24} kg$) and mass of mars ($6.39 * 10^{23} kg$), calculate the ratio of electric force to the gravitational force

Answer:
Hint: Distance of seperation is insignificant in calculating the ratio of forces

$$\frac{F_e}{F_g} \simeq 10^{-17} => F_g = 10^{17} * F_e$$

L-02

Analogy between Electric force and gravitational force

- Similarities between two laws
  - Both are inverse square laws
  - The unit of k and G are Similar
- Main difference: Gravitational force is always attractive

Force between two protons:

$$\frac{F_e}{F_g} \simeq 10^{36}$$

$$\boxed{F_e = 10^{36} * F_g}$$

Force between earth and mars:

$$\frac{F_e}{F_g} \simeq 10^{-17}$$

$$\boxed{F_g = 10^{17} * F_e}$$

L-02

Electric force and Gravitational force

- On an atomic scale, it is the electric forces that holds our world together
- But on much larger scale, planets, stars and galaxy, it is the gravity that holds our world together
- Eventhough our immediate surroundings are dominated by electric forces, the behaviour of universe on a large scale is dictated by gravity

END-LECTURE-02

Subject: Electromagnetic field theory

Subject Code: EEPC203

Semester: III

Lecture No: 03

Topic: Coordinate systems

L-03

Review of L-02

- Importance of Electric and magnetic fields
- Types of charges : Quantized and conserved
- Coulomb's law
- Analogy between electric force and gravitational force
- What keeps our world together?

L-03

Coordinate system

- A vector has both magnitude and direction.
- A coordinate system consists of four basic elements
  - Choice of origin
  - Choice of axes
  - Choice of positive direction for each axis
  - Choice of unit vectors for each axis
- Three coordinate system which you are going to study in this course are
  1. Rectangular or rectangular cartesian coordinate system
  2. Cylindrical coordinate system
  3. Spherical coordinate system
- Three mutually perpendicular axes

L-03

Unit Vectors

- A vector in any coordinate system is represented by sum of three component vectors (corresponding to three axes) lying along the three coordinate axes.
- This leads to the usage of **unit vectors** which are defined as the vectors of unit magnitude and directed along the coordinate axes in the direction of the increasing coordinate values
- In cartesian coordinate system, the three unit vectors are a_x, a_y and a_z

P5) Write the vector r_p pointing from the origin to point P(1,2,3) in cartesian coordinate system

Answer:

L-03

Unit Vectors

- A vector in any coordinate system is represented by sum of three component vectors (corresponding to three axes) lying along the three coordinate axes.
- This leads to the usage of **unit vectors** which are defined as the vectors of unit magnitude and directed along the coordinate axes in the direction of the increasing coordinate values
- In cartesian coordinate system, the three unit vectors are a_x, a_y and a_z

P5) Write the vector r_p pointing from the origin to point P(1,2,3) in cartesian coordinate system

Answer: $r_p = a_x + 2a_y + 3a_z$

L-03

Vectors in Cartesian coordinate system

- For any vector $\mathbf{B}=B_x\mathbf{a_x}+B_y\mathbf{a_y}+B_z\mathbf{a_z}$, the magnitude of **B** is written as $|B|$,

$$|B| = \sqrt{{B_x}^2 + {B_y}^2 + {B_z}^2} \tag{9}$$

- The unit vector in the direction of B is given by

$$\mathbf{a}_B = \frac{\mathbf{B}}{\sqrt{{B_x}^2 + {B_y}^2 + {B_z}^2}} = \frac{\mathbf{B}}{|\mathbf{B}|} \tag{10}$$

P6) Find the magnitude of r_p and the unit vector a_r

Answer:

L-03

Vectors in Cartesian coordinate system

- For any vector $\mathbf{B}=B_x\mathbf{a_x}+B_y\mathbf{a_y}+B_z\mathbf{a_z}$, the magnitude of **B** is written as $|B|$,

$$|B| = \sqrt{{B_x}^2 + {B_y}^2 + {B_z}^2} \tag{11}$$

- The unit vector in the direction of B is given by

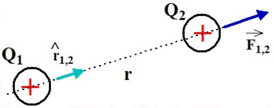
$$\mathbf{a}_B = \frac{\mathbf{B}}{\sqrt{{B_x}^2 + {B_y}^2 + {B_z}^2}} = \frac{\mathbf{B}}{|\mathbf{B}|} \tag{12}$$

P6) Find the magnitude of r_p and the unit vector a_r

Answer: 3.742; unit vector is $\frac{a_x+2a_y+3a_z}{3.742}$

L-03

Unit Vectors and Coulomb's Law



The force on the charge Q_2 due to the interaction between the charges, $\vec{F_{12}}$ is given by

$$\vec{F_{12}} = k\frac{Q_1 Q_2}{r_{12}^2}\hat{r_{12}} \tag{13}$$

$$\hat{r_{12}} = \frac{\vec{r_{12}}}{|\vec{r_{12}}|} \tag{14}$$

$$\vec{F_{12}} = k\frac{Q_1 Q_2}{r_{12}^3}\vec{r_{12}} \tag{15}$$

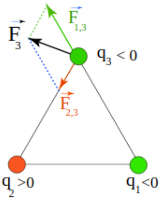
In this expression, the force is divided by the distance cubed but the magnitude of the force still follows an inverse square distance law.

L-03

Superposition of Coulomb's Law

- When more than two charges are present, the total force on any one charge is the vector sum of the forces exerted on it by the other charges
- if three charges q_1 , q_2 and q_3 are present, the resultant force experienced by charge charge 3 is:

$$\vec{F_3} = \vec{F_{13}} + \vec{F_{23}} \tag{16}$$



L-03

Dot Product of Vectors

- Given two vectors **A** and **B**, the *dot product or scalar product* is defined as the product of the magnitude of A, magnitude of B, and the cosine of the smaller angle between them

$$\mathbf{A.B} = |\mathbf{A}||\mathbf{B}|cos(\theta_{AB}) \tag{17}$$

P7) If A=$A_x a_x + A_y a_y + A_z a_z$ and B=$B_x a_x + B_y a_y + B_z a_z$; Calculate **A.B**

Answer:

L-03

Dot Product of Vectors

- Given two vectors **A** and **B**, the *dot product or scalar product* is defined as the product of the magnitude of A, magnitude of B, and the cosine of the smaller angle between them

$$\mathbf{A.B} = |\mathbf{A}||\mathbf{B}|cos(\theta_{AB}) \tag{18}$$

P7) If A=$A_x a_x + A_y a_y + A_z a_z$ and B=$B_x a_x + B_y a_y + B_z a_z$; Calculate **A.B**

Answer: $A_x B_x + A_y B_y + A_z B_z$

L-03

Dot Product of Vectors

- One of the important applications of dot product is the finding of the component of a vector in a direction.
- The component of vector **B** in the direction specified by unit vector **a** is

$$\mathbf{B}.\mathbf{a} = |\mathbf{B}|\mathbf{a}|cos(\theta_{Ba}) = |\mathbf{B}|cos(\theta_{Ba}) \tag{19}$$

L-03

Cross Product of Vectors

- Cross product of two vectors is a vector
- The cross product of A $\times$ B is equal to the product of magnitude of A, B and the sine of smaller angle between A and B;
- The direction of A $\times$ B is perpendicular to the plane containing A and B, and is in the direction of advance of a right handed screw as A is turned into B

$$\mathbf{A} \text{ x } \mathbf{B} = |\mathbf{A}||\mathbf{B}| sin(\theta_{AB})\ a_N \tag{20}$$

P7) If A=$A_x a_x + A_y a_y + A_z a_z$ and B=$B_x a_x + B_y a_y + B_z a_z$; Calculate **A x B**

Answer:

L-03

Cross Product of Vectors

- Cross product of two vectors is a vector
- The cross product of A $\times$ B is equal to the product of magnitude of A, B and the sine of smaller angle between A and B;
- The direction of A $\times$ B is perpendicular to the plane containing A and B, and is in the direction of advance of a right handed screw as A is turned into B

$$\mathbf{A} \text{ x } \mathbf{B} = |\mathbf{A}||\mathbf{B}|sin(\theta_{AB})\ a_N \tag{21}$$

L-03

Cross Product of Vectors

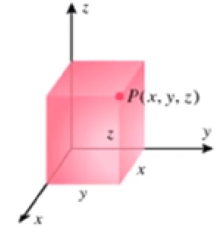
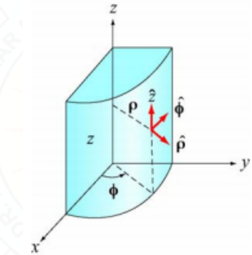
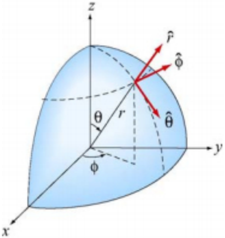
P7) If $A = A_x a_x + A_y a_y + A_z a_z$ and $B = B_x a_x + B_y a_y + B_z a_z$; Calculate **A x B**

Answer: $(A_y B_z - A_z B_y) a_x + (A_z B_x - A_x B_z) a_y + (A_x B_y - A_y B_x) a_z$

$$A \times B = \begin{vmatrix} a_x & a_y & a_z \\ A_x & A_y & A_z \\ B_x & B_y & B_z \end{vmatrix}$$

L-03

Coordinate system

Cartesian	Cylindrical	Spherical
		

	Cartesian	Cylindrical	Spherical
Axis:	x, y, z	ρ, ϕ, z	r, θ, ϕ
UnitVectors:	$\hat{a}_x, \hat{a}_y, \hat{a}_z$	$\hat{a}_\rho, \hat{a}_\phi, \hat{a}_z$	$\hat{a}_r, \hat{a}_\theta, \hat{a}_\phi$
Range:	$-\infty < x < \infty$	$0 \leq \rho < \infty$	$0 \leq r < \infty$
	$-\infty < y < \infty$	$0 \leq \phi < 2\pi$	$0 \leq \theta \leq \pi$
	$-\infty < z < \infty$	$-\infty < z < \infty$	$0 \leq \phi < 2\pi$

L-03

Coordinate system

Cartesian	Cylindrical	Spherical
Vector:		
$A_x a_x + A_y a_y + A_z a_z$	$A_\rho a_\rho + A_\phi a_\phi + A_z a_z$	$A_r a_r + A_\theta a_\theta + A_\phi a_\phi$
Magnitude:		
$\|A\| = \sqrt{{A_x}^2 + {A_y}^2 + {A_z}^2}$	$\|A\| = \sqrt{{A_\rho}^2 + {A_\phi}^2 + {A_z}^2}$	$\|A\| = \sqrt{{A_r}^2 + {A_\theta}^2 + {A_\phi}^2}$
Dot product- unit vectors:		
$a_x.a_x = a_y.a_y = a_z.a_z = 1$	$a_\rho.a_\rho = a_\phi.a_\phi = a_z.a_z = 1$	$a_r.a_r = a_\theta.a_\theta = a_\phi.a_\phi = 1$
$a_x.a_y = a_y.a_z = a_z.a_x = 0$	$a_\rho.a_\phi = a_\phi.a_z = a_z.a_\rho = 0$	$a_r.a_\theta = a_\theta.a_\phi = a_\phi.a_r = 0$
Cross product- unit vectors:		
$a_x \times a_y = a_z;$	$a_\rho \times a_\phi = a_z;$	$a_r \times a_\theta = a_\phi;$
$a_y \times a_z = a_x;$	$a_\phi \times a_z = a_\rho;$	$a_\theta \times a_\phi = a_r;$
$a_z \times a_x = a_y;$	$a_z \times a_\rho = a_\phi;$	$a_\phi \times a_r = a_\theta;$
$a_x \times a_x = a_y \times a_y = a_z \times a_z = 0$	$a_\rho \times a_\rho = a_\phi \times a_\phi = 0$	$a_r \times a_r = a_\phi \times a_\phi = 0$

END-LECTURE-03

Subject: Electromagnetic field theory

Subject Code: EEPC203

Semester: III

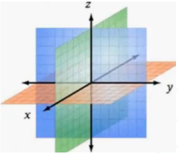
Lecture No: 04

Topic: Coordinate systems (Part-2)

L-04

Constant-Coordinate surfaces

Cartesian

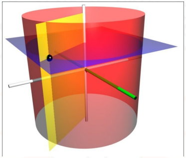


x=constant
Infinite plane

y=constant
Infinite plane

z=constant
Infinite plane

Cylindrical

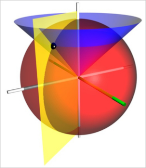


ρ = constant
Circular Cylinder

ϕ = constant
Semi infinite plane
(with edge along Z-axis)

z=constant
Infinite plane

Spherical



r = constant
Sphere-radius 'ρ'
(with center at the origin)

θ = constant
Circular cone
(Z-axis as axis, origin as vertex)

ϕ = constant
Semi infinite plane
(with edge along Z-axis)

Subject: Electromagnetic field theory

Subject Code: EEPC203

Semester: III

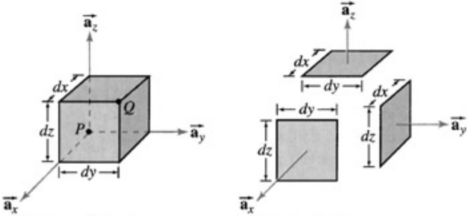
Lecture No: 05

Topic: Coordinate systems (part-3)

L-05

Differential length, area, volume

Cartesian coordinate system



a) Differential volume b) Exploded view

Differential length

$$dl = dx\, a_x + dy\, a_y + dz\, a_z$$

Differential Area

$$ds_x = dy\ dz\ a_x$$
$$ds_y = dx\ dz\ a_y$$
$$ds_z = dx\ dy\ a_z$$

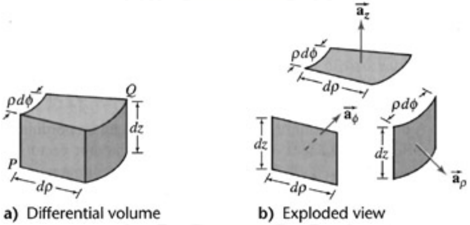
Differential Volume

$$dv = dxdydz$$

L-05

Differential length, area, volume

Cylindrical coordinate system



Differential length

dl= $d\rho a_\rho + \rho d\phi a_\phi + dz a_z$

Differential Area

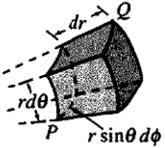
$$ds_\rho = \rho\ d\phi\ dz a_\rho$$
$$ds_\phi = d\rho\ dz\ a_\rho$$
$$ds_z = d\rho\ \rho d\phi\ a_z$$

Differential Volume

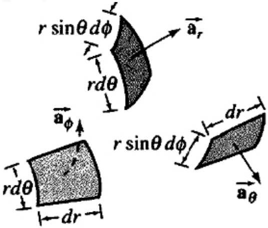
$$dv = \rho\ d\rho\ d\phi\ dz$$

Differential length, area, volume

Spherical coordinate system



a) Differential volume



b) Exploded view

Differential length

dl = $dr\,a_r + rd\theta a_\theta + r sin\theta d\phi a_\phi$

Differential Area

$ds_r = r^2 sin\theta d\theta d\phi a_r$

$ds_\theta = r sin\theta dr d\phi a_\theta$

$ds_\phi = r\ dr\ d\theta\ a_\phi$

Differential Volume

$dv = r^2 sin\theta dr d\theta d\phi$

L-05

Cartesian & Cylindrical coordinates

Point to Point Transformation

Cartesian to Cylindrical

$$\rho = \sqrt{x^2 + y^2}$$
$$\phi = tan^{-1}\frac{y}{x}$$
z=z

Cylindrical to cartesian

$$x = \rho\ cos\phi$$
$$y = \rho\ sin\phi$$
$$z = z$$

Vector Transformation

Cartesian to Cylindrical

$$\begin{bmatrix} a_\rho \\ a_\phi \\ a_z \end{bmatrix} = \begin{bmatrix} cos\phi & sin\phi & 0 \\ -sin\phi & cos\phi & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} a_x \\ a_y \\ a_z \end{bmatrix}$$

Cylindrical to cartesian

$$\begin{bmatrix} a_x \\ a_y \\ a_z \end{bmatrix} = \begin{bmatrix} cos\phi & -sin\phi & 0 \\ sin\phi & cos\phi & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} a_\rho \\ a_\phi \\ a_z \end{bmatrix}$$

L-05

Cartesian & Spherical coordinates

Point to Point Transformation

Cartesian to Spherical

$$r = \sqrt{x^2 + y^2 + z^2}$$
$$\theta = cos^{-1} \frac{z}{\sqrt{x^2+y^2+z^2}}$$
$$\phi = tan^{-1}\left(\frac{y}{x}\right)$$

Spherical to cartesian

$$x = r\ sin\theta\ cos\phi$$
$$y = r\ sin\theta\ sin\phi$$
$$z = rcos\theta$$

Vector Transformation

Cartesian to Spherical

$$\begin{bmatrix} a_r \\ a_\theta \\ a_\phi \end{bmatrix} = \begin{bmatrix} sin\theta cos\phi & sin\theta sin\phi & cos\theta \\ cos\theta cos\phi & cos\theta sin\phi & -sin\theta \\ -sin\phi & cos\phi & 0 \end{bmatrix} \begin{bmatrix} a_x \\ a_y \\ a_z \end{bmatrix}$$

Spherical to cartesian

$$\begin{bmatrix} a_x \\ a_y \\ a_z \end{bmatrix} = \begin{bmatrix} sin\theta cos\phi & cos\theta cos\phi & -sin\phi \\ sin\theta sin\phi & cos\theta sin\phi & cos\phi \\ cos\theta & -sin\theta & 0 \end{bmatrix} \begin{bmatrix} a_r \\ a_\phi \\ a_\theta \end{bmatrix}$$

END-LECTURE-05

Subject: Electromagnetic field theory

Subject Code: EEPC203

Semester: III

Lecture No: 06

Topic: Electric field intensity

L-06

Electric field of a positive point charge

- Consider a test charge at the point of interest in the vicinity of source (q_s)
- Electric field intensity is the vector force on a unit positive test charge

$$E_p = \frac{F_{ts}}{Q_t} = k * \frac{Q_s Q_t}{r_{ts}^2 Q_t} \hat{r_{ts}} \tag{22}$$

$$E_p = k * \frac{Q_s}{r_{ts}^3} \vec{r_{ts}} \tag{23}$$

- The direction of electric field doesnot depend on the sign of the test charge.
- The magnitude of electric field is independent of magnitude of test charge

L-06

Superposition of Electric field

For 'N' charges located at $r_1, r_2, ...r_N$, the electric field intensity at point r is given by

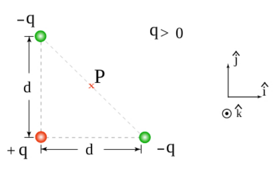
$$E = \frac{Q_1(\vec{r} - \vec{r_1})}{4\pi\epsilon_0|r - r_1|^3} + \frac{Q_2(\vec{r} - \vec{r_2})}{4\pi\epsilon_0|r - r_2|^3} + ... + \frac{Q_N(\vec{r} - \vec{r_N})}{4\pi\epsilon_0|r - r_N|^3} \qquad (24)$$

Electric field intensity is measured in units of **Volts per meter (V/m)**

L-06

Superposition of Electric field

P8) Three charged objects lie on the corners of an isosceles right triangle of side d as shown. Calculate the x and y components of the electric field produced by the three charges at point P , the midpoint between the two negative charges. Express your answer in terms of k , q, and d .



L-06

Superposition of Electric field

P9) A charge of -0.3μC is located at A(25,-30,15) (in cm), and a second charge of 0.5μC is at B(-10,8,12) (in cm). Find E at origin

Ans: 92.3 a_x-77.6a_y-94.2a_z kV/m

END-LECTURE-06

Subject: Electromagnetic field theory

Subject Code: EEPC203

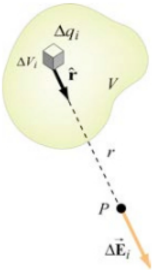
Semester: III

Lecture No: 07

Topic: Continuous Charge Distribution

L-07

Electric field due to continuous charge distributions



Electric field due to Δq_i at the point P is

$$\overrightarrow{dE}(P) = \frac{kdq}{r^2}\hat{r}$$

Total electric field at point P is

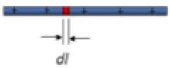
$$\vec{E}(P) = \int_v dE(P)$$

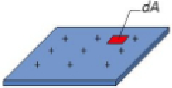
Total charge (Q),

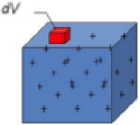
$$Q = \sum_{i=1}^{n} \Delta q_i$$

$$\boxed{\vec{E}(P) = \int_v \frac{kdq}{r^2}\hat{r}}$$

Charge Densities







Line Charge	Surface Charge	Volume Charge
$dQ = \lambda dL$	$dQ = \sigma\, dA$	$dQ = \rho\, dV$
$Q = \int_\ell \lambda dL$	$Q = \int_s \sigma\, dA$	$Q = \int_v \rho\, dV$
λ is the Line charge density (C/m)	σ is the surface charge density (C/m^2)	ρ is the volume charge density (C/m^3)
$\boxed{\vec{E}(P) = \int_\ell k\frac{\lambda dl}{r^2}\hat{r}}$	$\boxed{\vec{E}(P) = \int_s k\frac{\sigma dA}{r^2}\hat{r}}$	$\boxed{\vec{E}(P) = \int_v k\frac{\rho dv}{r^2}\hat{r}}$

L-07

Total charge calculation

P10) Find the total charge in each of the following cases

(i) A line charge of infinite extent in the z direction with charge density distribution

$$\lambda = \frac{\lambda_0}{1 + (z/a)^2}$$

(ii) A line charge λ_0 uniformly distributed in a circular loop of radius a.

(iii) Surface charge σ_0 unifromly distributed on a circular disk of radius a.

(iv) Volume charge ρ_0 uniformly distributed throughout a sphere of radius R.

END-LECTURE-07

Subject: Electromagnetic field theory

Subject Code: EEPC203

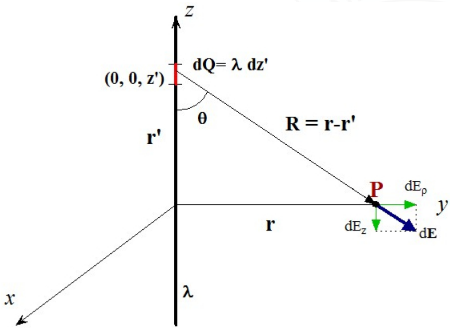
Semester: III

Lecture No: 08

Topic: Electric field due to infinite line and sheet of charge

L-08

Electric field due to infinite line charge



$$dE = \frac{\lambda\ dz\ (\mathbf{r} - \mathbf{r}')}{4\pi\epsilon_0|\mathbf{r} - \mathbf{r}'|^3}$$

$$d\vec{E} = \frac{\lambda\ dz'\ (\rho\mathbf{a}_\rho - \mathbf{z}'\mathbf{a_z})}{4\pi\epsilon_0(\rho^2 + z'^2)^{3/2}}$$

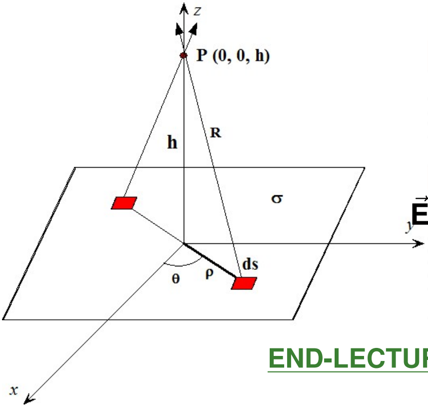
$$d\vec{E}_\rho = \frac{\lambda\ dz'\ (\rho)}{4\pi\epsilon_0(\rho^2 + z'^2)^{3/2}}$$

$$\vec{E}_\rho = \frac{\lambda}{2\pi\epsilon_0\ \rho}$$

$$\boxed{\vec{E} = \frac{\lambda}{2\pi\epsilon_0\rho}\mathbf{a}_\rho}$$

L-08

Electric field due to infinite sheet of charge



$$d\vec{\mathbf{E}} = \frac{dQ}{4\pi\epsilon_0 R^2}\mathbf{a}_R$$

$$d\vec{\mathbf{E}} = \frac{\sigma\ \rho\ d\phi\ d\rho(h\mathbf{a}_z - \rho\mathbf{a}_\rho)}{4\pi\epsilon_0(\rho^2 + h^2)^{3/2}}$$

$$\vec{\mathbf{E}} = \frac{\sigma}{4\pi\epsilon_0}\int_{\phi=0}^{2\pi}\int_{\rho=0}^{\infty}\frac{h\ \rho\ d\rho\ d\phi}{(\rho^2 + h^2)^{3/2}}\mathbf{a}_z$$

$$\vec{\mathbf{E}} = \frac{\sigma}{2\epsilon_0}\mathbf{a}_N$$

END-LECTURE-08

Subject: Electromagnetic field theory

Subject Code: EEPC203

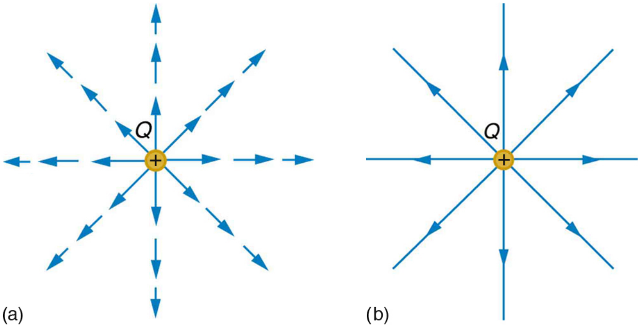
Semester: III

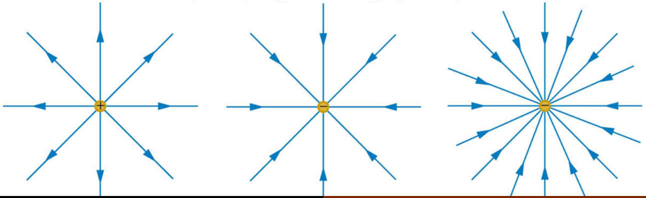
Lecture No: 09

Topic: Electric field lines

L-09

Electric field lines



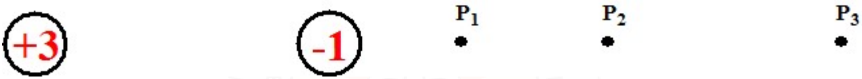


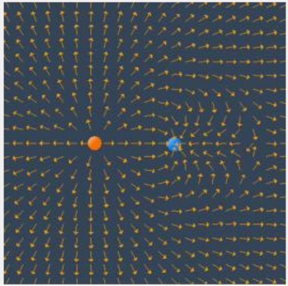
Properties of Electric field lines

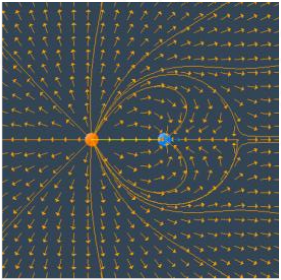
- Field lines must begin on positive charges and terminate on negative charges, or at infinity in the hypothetical case of isolated charges.
- The number of field lines leaving a positive charge or entering a negative charge is proportional to the magnitude of the charge.
- The strength of the field is proportional to the closeness of the field lines more precisely, it is proportional to the number of lines per unit area perpendicular to the lines.
- The direction of the electric field is tangent to the field line at any point in space.
- Field lines can never cross.

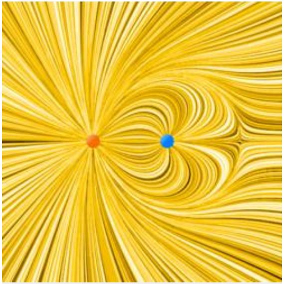
L-09

Electric field lines



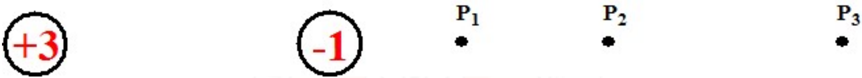






L-09

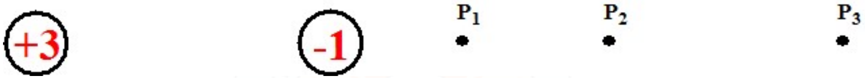
Electric field lines



P10) A point charge Q_1=-2μC is located at x=0, and a point charge Q_2=+8μC is located at x=-0.5m on the x-axis of the cartesian coordinate system. (i) Determine the electric field (in N/C) for x=-76.5 m (ii) At what point (apart from $|x| = \infty$) is electric field becomes zero.

L-09

Electric field lines



P10) A point charge Q_1=-2μC is located at x=0, and a point charge Q_2=+8μC is located at x=-0.5m on the x-axis of the cartesian coordinate system. (i) Determine the electric field (in N/C) for x=-76.5 m (ii) At what point (apart from $|x| = \infty$) is electric field becomes zero.

$$E = 9.389\ N/C \quad r = -0.5\ \&\ r = 1.5$$

END-LECTURE-09

Subject: Electromagnetic field theory

Subject Code: EEPC203

Semester: III

Lecture No: 10

Topic: Divergence, Curl and Gauss Law

L-10

Divergence of a vector

The divergence at a given point P is the outward flux per unit volume as the volume shrinks about P.

$$div\mathbf{D} = \nabla.\mathbf{D} = lim_{\Delta v \to 0} \frac{\oint \mathbf{A.ds}}{\Delta v} \tag{25}$$

where Δv is the volume enclosed by the closed surface S in which point P is located.
∇ is the vector operator which is given by,

$$\nabla = \frac{\partial}{\partial x} + \frac{\partial}{\partial y} + \frac{\partial}{\partial z}$$

L-10

Divergence of a vector

In cartesian coordinate system,

$$\nabla.A = \frac{\partial A_x}{\partial x} + \frac{\partial A_y}{\partial y} + \frac{\partial A_z}{\partial z}$$

In cylindrical coordinate system,

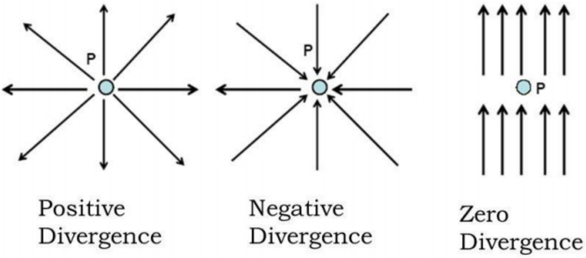
$$\nabla.A = \frac{1}{\rho}\frac{\partial}{\partial \rho}(\rho A\rho) + \frac{1}{\rho}\frac{\partial A_\phi}{\partial \phi} + \frac{\partial A_z}{\partial z}$$

In spherical coordinate system,

$$\nabla.A = \frac{1}{r^2}\frac{\partial}{\partial r}(r^2 A_r) + \frac{1}{rsin\theta}\frac{\partial}{\partial \theta}(A_\theta sin\theta) + \frac{1}{rsin\theta}\frac{\partial A_\phi}{\partial \phi}$$

L-10

Divergence of a vector



Divergence Theorem

$$\oint_s \mathbf{A.ds} = \int_v \nabla.A\,dv \tag{26}$$

L-10

Curl of a vector

The curl of a vector A is an axial vector whose magnitude is the maximum circulation of A per unit area as the area tend to zero and whose direction is the normal direction of the area when the area is oriented to make the circulation maximum.

$$curl\ \mathbf{A} = \nabla \times \mathbf{A} = \left(lim_{\nabla s -> 0} \frac{\oint_L \mathbf{A.dl}}{\nabla S} \right)_{max} \mathbf{a}_n \qquad (27)$$

Stroke's Theorem

$$\oint_L \mathbf{A.dl} = \int_s (\nabla \times \mathbf{A}).\mathbf{ds} \qquad (28)$$

L-10

Curl of a vector

In cartesian coordinate system, curl of A is given by

$$\nabla \times A = \begin{vmatrix} a_x & a_y & a_z \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ A_x & A_y & A_z \end{vmatrix}$$

In cylindrical coordinate system, curl of A is given by

$$\nabla \times A = \frac{1}{\rho} \begin{vmatrix} a_\rho & \rho a_\phi & a_z \\ \frac{\partial}{\partial \rho} & \frac{\partial}{\partial \phi} & \frac{\partial}{\partial z} \\ A_\rho & \rho A_\phi & A_z \end{vmatrix}$$

In spherical coordinate system, curl of A is given by

$$\nabla \times A = \frac{1}{r^2 sin\theta} \begin{vmatrix} a_r & r a_\theta & rsin\theta\, a_\phi \\ \frac{\partial}{\partial r} & \frac{\partial}{\partial \theta} & \frac{\partial}{\partial \phi} \\ A_r & rA_\theta & rsin\theta\, A_\phi \end{vmatrix}$$

L-10

Electric flux density

- In SI units, one line of electric flux emanates from +1C and terminates on -1 C. Therefore, the electric flux is measured in coulombs.
- The total electric flux ψ coming out of a closed surface is given by

$$\boxed{\psi = \int_s \mathbf{D}.ds}$$

The vector **D** is called electric flux density and it is measured in coulombs per square meter (C/m^2)

L-10

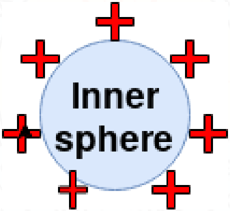
Electric flux density

- The electric flux density is independent of medium
- The relation between the electric flux density (**D**) and electric field intensity (**E**) is given by

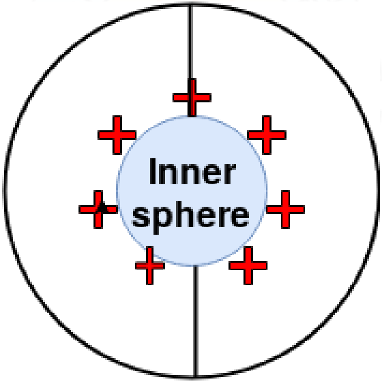
$$\boxed{\mathbf{D} = \epsilon_0 \mathbf{E}} \text{ (for free space only)}$$

Faraday's Experiment:Review

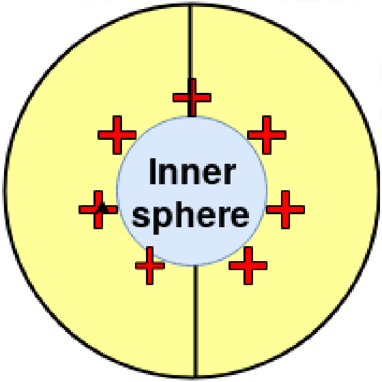
Faraday's Experiment:Review



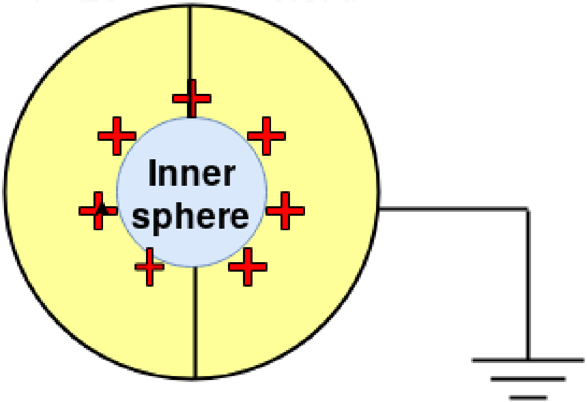
Faraday's Experiment:Review



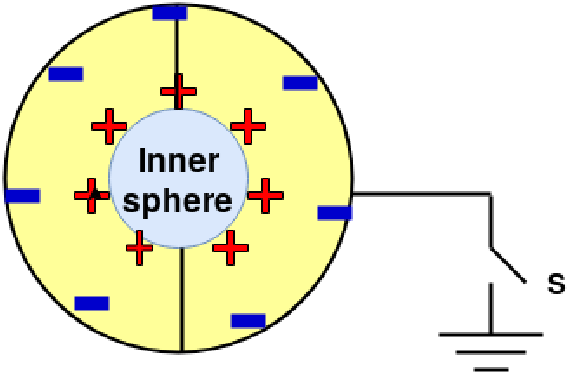
Faraday's Experiment:Review



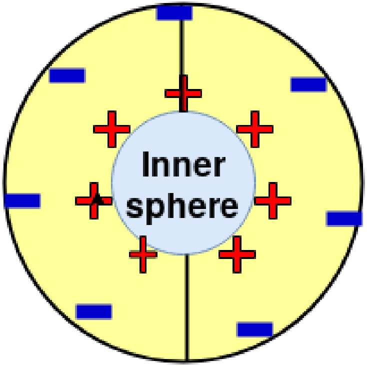
Faraday's Experiment:Review



Faraday's Experiment:Review

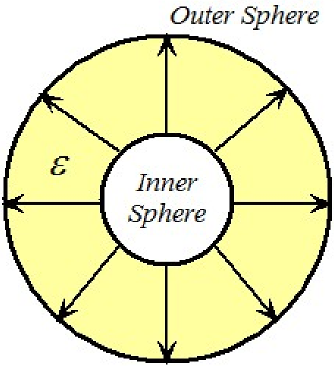


Faraday's Experiment:Review



L-10

Faraday's Experiment:Review



$$\boxed{\psi_{outer} = Q_{inner}}$$

There was some sort of displacement from the inner sphere to outer sphere and refer to this quantity as electric flux ψ.

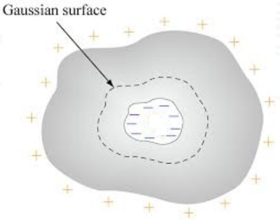
Generalisation: Gauss Law (Maxwell's Equation)

- The **flux** passing through the any closed surface located in between two spheres is equal to the **charge distributed on the surface** of the inner sphere
- Inner conductor may be irregular, still the **induced charges** on the outer sphere is **same** as previous one.
- Going further, if we change the outer hemisphere to some other irregular shape, still **+Q** charge from inner sphere will create $\psi = \mathbf{Q}$ lines of electric flux will induce **-Q** C of charge on the outer sphere

"The electric flux passing through any closed surface is equal to the total charge enclosed by that surface"

Generalisation: Gauss Law (Maxwell's Equation)

- Going further, if we change the outer hemisphere to some other irregular shape, still **+Q** charge from inner sphere will create $\psi = \mathbf{Q}$ lines of electric flux will induce **-Q** C of charge on the outer sphere



"The electric flux passing through any closed surface is equal to the total charge enclosed by that surface"

L-10

Gauss Law : Maxwell's Equation

"The electric flux passing through any closed surface is equal to the total charge enclosed by that surface"

$$\psi = Q_{enclosed}$$

$$\psi = \oint_s d\psi = \oint_s \mathbf{D.dS}$$

$$\oint_s \mathbf{D.dS} = Q$$

$$\boxed{\oint_s \mathbf{E.dS} = \frac{Q}{\epsilon_0}}$$

Gauss Law : Maxwell's Equation

Integral form of Gauss law:

$$\boxed{\oint_s \mathbf{D.dS} = Q}$$

Applying divergence theorem,

$$\oint_s \mathbf{D.dS} = \int_v \nabla.\mathbf{D}\, dv$$

$$\int_v \nabla.\mathbf{D} dv = Q = \int_v \rho_v dv$$

$$\boxed{\nabla.\mathbf{D} = \rho_v}$$

The above equation is the differential form of Gauss law

Applications: Procedure

- Find whether the symmetrical charge distribution exists
- Construct a mathematical closed surface known as **Gaussian Surface** around the charge such that **D** is normal (or tangential) to the surface.

Note:

- Gauss law holds good irrespective of symmetry of charge distribution. However, we **cannot use this law to determine E or D when there is non-symmetric charge distribution**

END-LECTURE-10

Subject: Electromagnetic field theory

Subject Code: EEPC203

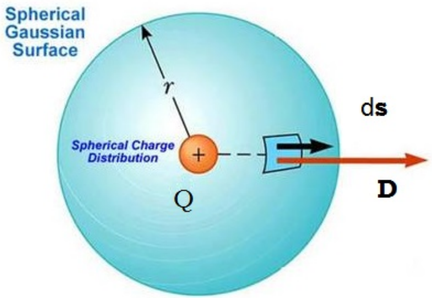
Semester: III

Lecture No: 11

Topic: Applications of Gauss Law

L-11

Electric field due to the point charge



$$Q=\oint_{\mathbf{s}} \mathbf{D}.\mathbf{dS}=D_r \oint_s ds$$

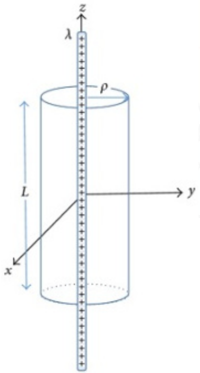
$$Q=D_r\ 4\pi r^2$$

$$\mathbf{D}=\frac{Q}{4\pi r^2}a_r$$

$$\boxed{\mathbf{E}=\frac{Q}{4\pi\epsilon_0 r^2}a_r}$$

L-11

Electric field due to infinite line of charge



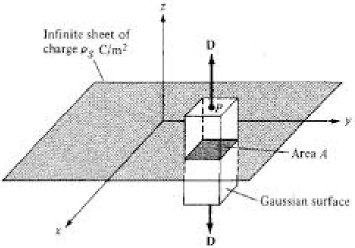
$$Q = \oint_s \mathbf{D}.\mathbf{dS} = D_\rho \oint_s ds$$

$$\lambda l = D_\rho(2\pi\rho l)$$

$$\mathbf{D} = \frac{\lambda}{2\pi\rho} a_\rho$$

$$\boxed{\mathbf{E} = \frac{\lambda}{2\pi\epsilon_0\rho} a_\rho}$$

Electric field due to infinite sheet of charge



$$\mathbf{D} = D_z \mathbf{a}_z$$

$$Q = \oint_s \mathbf{D.ds} = D_z(\oint_{top} ds + \oint_{bottom} ds)$$

$$Q = D_z(A + A)$$

$$\sigma\ A = D_z(A + A)$$
$$D_z = \frac{\sigma}{2}$$
$$\mathbf{D} = \frac{\sigma}{2}\mathbf{a}_z$$

"Electric field is indenpendent of the distance from the sheet of charge"

$$\boxed{\mathbf{E} = \frac{\sigma}{2\epsilon_0}\mathbf{a}_z}$$

L-11

P12) Find the electric field in all regions of space where two charged planes with charges $+\sigma$ and $-\sigma$ are seperated by a distance 'd'

L-11

P12) Find the electric field in all regions of space where two charged planes with charges $+\sigma$ and $-\sigma$ are seperated by a distance 'd'

solution:

$$E_A = 0$$

$$E_B = E_{+\sigma} + E_{-\sigma}$$

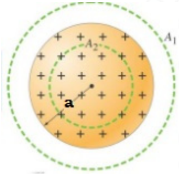
$$E_B = \frac{\sigma}{2\epsilon_0} + \frac{\sigma}{2\epsilon_0}$$

$$E_B = \frac{\sigma}{\epsilon_0}$$

$$E_C = 0$$

L-11

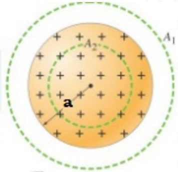
Electric field due to Uniformly charged sphere



- Insulating sphere
- Volume charge density of ρ_0 C/m^3

L-11

Electric field due to Uniformly charged sphere



For $r \le a$: (inside the sphere)

$$\mathbf{D} = D_r \mathbf{a}_r$$

$$Q_{enc} = \int_v \rho_v dv = \rho_0 \left(\frac{4}{3} \pi r^3 \right)$$

$$Q = \oint_s \mathbf{D.ds} = D_r \int_s ds$$

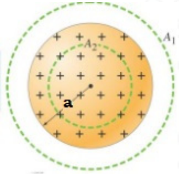
$$Q = D_r (4\pi r^2)$$

$$D_r (4\pi r^2) = \rho_0 (\tfrac{4}{3} \pi r^3)$$

$$\boxed{\mathbf{D} = \frac{r}{3} \rho_0 \mathbf{a}_r} \quad 0 \le r \le a$$

L-11

Electric field due to Uniformly charged sphere



$$\mathbf{D} = D_r \mathbf{a}_r$$

$$Q_{enc} = \int_v \rho_v dv = \rho_0 \left(\frac{4}{3} \pi r^3 \right)$$

For $r \geq a$: (outside the sphere)

$$Q = \oint_s \mathbf{D.ds} = D_r \int_s ds$$

$$Q = D_r (4\pi r^2)$$

$$D_r (4\pi r^2) = \rho_0 (\tfrac{4}{3} \pi a^3)$$

$$\boxed{\mathbf{D} = \frac{a^3}{3r^2} \rho_0 \mathbf{a}_r} \quad r \geq a$$

L-11

Electric field due to Uniformly charged sphere

For $0 \leq r \leq a$:

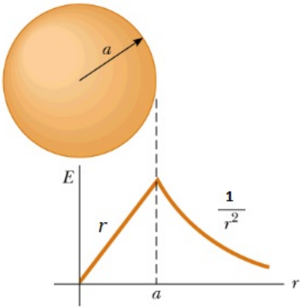
$$\boxed{\mathbf{D} = \frac{r}{3}\rho_0 \mathbf{a}_r}$$

$$\boxed{\mathbf{E} = \frac{r}{3\epsilon_0}\rho_0 \mathbf{a}_r}$$

For $r \geq a$:

$$\boxed{\mathbf{D} = \frac{a^3}{3r^2}\rho_0 \mathbf{a}_r}$$

$$\boxed{\mathbf{E} = \frac{a^3}{3r^2\epsilon_0}\rho_0 \mathbf{a}_r}$$



L-11

Electric field due to Hollow sphere

For $0 \leq r \leq a$:

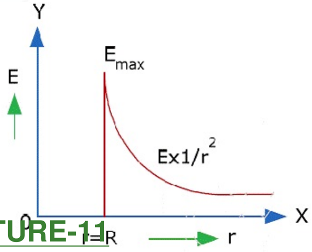
$$\boxed{\mathbf{D} = 0}$$

$$\boxed{\mathbf{E} = 0}$$

For $r \geq a$:

$$\boxed{\mathbf{D} = \frac{a^2}{r^2}\sigma \mathbf{a}_r}$$

$$\boxed{\mathbf{E} = \frac{a^2}{r^2 \epsilon_0}\sigma \mathbf{a}_r}$$



END-LECTURE-11

Subject: Electromagnetic field theory

Subject Code: EEPC203

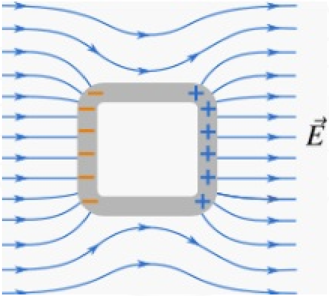
Semester: III

Lecture No: 12

Topic: Electric field in metals and Electric Potential

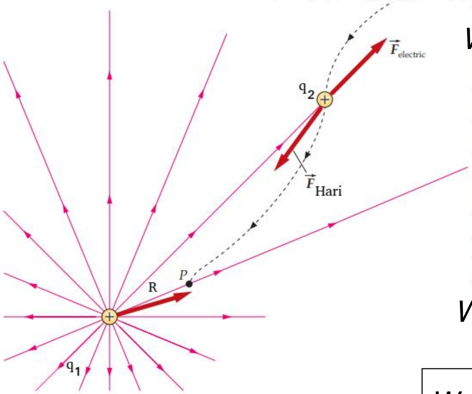
L-12

Electric field inside a metal conductor



$$\boxed{E_{inside} = 0}$$

Potential Energy



$$W_{Hari} = \int_{\infty}^{R} F_{Hari} . \vec{dr}$$

$$= -\int_{\infty}^{R} F_{+q1} \, dr$$

$$= -q_2 \int_{\infty}^{R} E_{+q1} \, dr$$

$$W_{Hari} = \frac{q_1 q_2}{4\pi\epsilon_0} \int_{R}^{\infty} \frac{dr}{r^2}$$

$$\boxed{W_{Hari} = \frac{q_1 q_2}{4\pi\epsilon_0 R}} \quad (Joules)$$

Potenital Energy

The potential energy (U) is

$$U = \frac{q_1 q_2}{4\pi\epsilon_0 R} \quad (Joules)$$

- q_1 & q_2 are positive or both negative, then the work done will be positive, that is external agent (hari) has to perform the work.
- q_1 & q_2 are different signed then work done is negative, that is work is being done by the field.

If we have collection of charges, then total workdone is the sum of workdone in bringing a particular charge from infinity.

Potenital Energy

The potential energy (U) is

$$U = \frac{q_1 q_2}{4\pi\epsilon_0 R} \quad (Joules)$$

- q_1 & q_2 are positive or both negative, then the work done will be positive, that is external agent (hari) has to perform the work.
- q_1 & q_2 are different signed then work done is negative, that is work is being done by the field.

If we have collection of charges, then total workdone is the sum of workdone in bringing a particular charge from infinity.

Potenital of a point charge

It is the workdone in bringing the unit charge from infinity to that position.

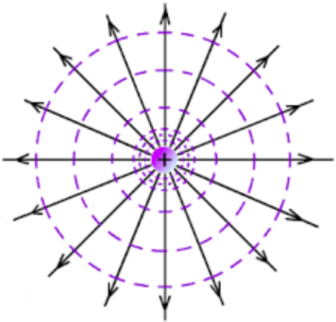
$$\boxed{V_p = \frac{Q}{4\pi\epsilon_0 R}}$$

$$V = -\int_{\infty}^{R} \vec{E}.\vec{dl}$$

The potential at point B with reference to point A

$$V_{AB} = V_B - V_A = -\int_{A}^{B} \vec{E}.\vec{dl} = \frac{Q}{4\pi\epsilon_0}\left\{\frac{1}{r_B} - \frac{1}{r_A}\right\}$$

Equipotential surfaces



Properties:

- They are $\perp$ to field lines
- Wont intersect with each other
- Workdone in moving a charge from one point to another on an Equipotential surface is zero

- They are closer to each other in regions of strong electric field and farther apart in regions of weaker electric field

Electric Potenital

$$dV = \frac{dU}{q_0} = -\vec{E}.\vec{dl} \ \ (J/C)$$

$$dV = -\vec{E}.\vec{dl} = -E_x dx - E_y dy - E_z dz$$

From vector calculus,

$$dV = \frac{\partial V}{\partial x} dx + \frac{\partial V}{\partial y} dy + \frac{\partial V}{\partial z} dz$$

$$E_x = -\frac{\partial V}{\partial x}; E_y = -\frac{\partial V}{\partial y}; E_z = -\frac{\partial V}{\partial z};$$

$$\boxed{\mathbf{E} = -\nabla V}$$

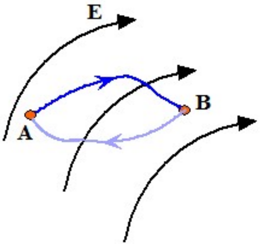
$$\boxed{\mathbf{E} = -\nabla V}$$

- Electric field is gradient of voltage
- Negative sign indicates that the direction of **E** is opposite to the direction in which V increases
- Electric field is directed from higher to lower level of voltage

$$\boxed{V_{AB} = V_B - V_A = -\int_A^B \vec{E}.\vec{dl}}$$

- Potential between the two points can be obtained by integrating the electric field between the points.
- Think of this as voltage difference going from A (initial point) to B (final point) and you get that by integrating the electric field from the initial to final point.

Relation between E and V: Maxwell's Equation



$$V_{AB} = -\int_A^B \vec{\mathbf{E}}.\vec{\mathbf{dl}} \quad V_{BA} = \int_A^B \vec{\mathbf{E}}.\vec{\mathbf{dl}}$$

$$V_{AB} = -V_{BA}$$

$$V_{AB} + V_{BA} = 0$$

$$\int_A^B \vec{\mathbf{E}}.\vec{\mathbf{dl}} + \int_B^A \vec{\mathbf{E}}.\vec{\mathbf{dl}} = 0$$

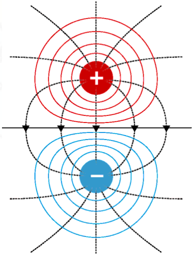
$$\boxed{\oint \vec{\mathbf{E}}.\vec{\mathbf{dl}} = 0}$$

$$\boxed{\nabla X \mathbf{E} = 0}$$

"This equation is more general form of Kirchoff's circuit law for voltages"

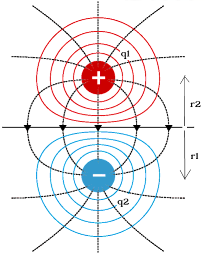
Electric dipole

- Positive charge and negative charge seperated by a small distance.
- The entire object is neutral



Why interested in dipole??

- Everything around us is neutral
- We rarely encounter isolated positive or negative charges
- Strength of dipole $\Rightarrow$ electric dipole moment



$$\vec{p} = q_1\vec{r_1} + q_2\vec{r_2}$$

$$\vec{p} = -qa\hat{-j} + qa\hat{j}$$

$$\vec{p} = 2qa\hat{j}$$

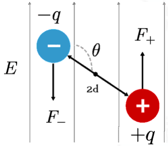
Dipole moment is independent of choice of origin.

For system of charges with total zero magnitude,

$$\vec{p} = q_1\vec{r_1} + q_2\vec{r_2} + ... + q_n\vec{r_n}$$

Torque in Electric dipole

- Dipole placed in external electric field connected with mass less rod



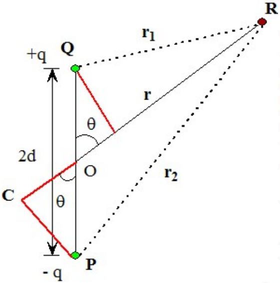
$$F_{external} = q\vec{E}_{ext} - q\vec{E}_{ext} = 0$$

Net force is zero but it will experience torque

$$\vec{\tau} = \sum_{j=1}^{N} \vec{r}_j \times \vec{F}_j \qquad \vec{\tau} = \sum_{j=1}^{N} \vec{r}_j \times q_j\vec{E}_{ext}$$

$$\vec{\tau} = \left(\sum_{j=1}^{N} q_j\vec{r}_j\right) \times \vec{E}_{ext} \qquad \boxed{\vec{\tau} = \vec{p} \times \vec{E}_{ext}}$$

Electric field due to dipole



$$V = \frac{Q}{4\pi\epsilon_0}\left[\frac{1}{r_1} - \frac{1}{r_2}\right]$$

$$V = \frac{Q}{4\pi\epsilon_0}\left[\frac{2dcos\theta}{r^2}\right]$$

$$E = -\nabla V$$

$$E = -\left(\frac{\partial V}{\partial r}\mathbf{a}_r + \frac{1}{r}\frac{\partial V}{\partial \theta}\mathbf{a}_\theta + \frac{1}{rsin\theta}\frac{\partial V}{\partial \phi}\mathbf{a}_\phi\right)$$

$$E = \frac{Qd}{4\pi\epsilon_0 r^3}(2cos\theta\mathbf{a}_r + sin\theta\mathbf{a}_\theta)$$

Electric field due to dipole

- A point charge is a monopole and its electric field varies inversely as r^2 while its potential field varies inversely as r
- In case of dipole (two point charges), the electric field varies inversely as r^3 while its potential field varies inversely as r^2
- Similarly, the electric field due to successive higher-order multipoles (such as quadrupole consisting of two dipoles or an octapole consisting of four dipoles) vary inversely as $r^4, r^5, r^6..$ while their corresponding potentials vary inversely as $r^3, r^4, r^5, ..$

END-LECTURE-13